Quantum measurement and control II.

G J Milburn

Centre for Engineered Quantum Systems, The University of Queensland

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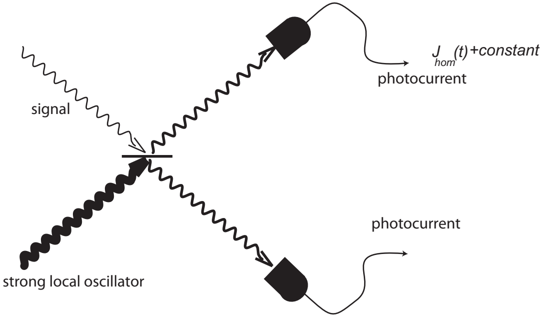
1. Homodyne detection

2. Homodyne mediated direct feedback.

3. Continuous error correction by feedback.

4. Feedback creation of entanglement.

The local oscillator



The local oscillator

A beam splitter of transmittance η, described by he transformation

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$$\hat{o} \rightarrow \sqrt{\eta}\hat{o} - \sqrt{1-\eta}\hat{a}$$

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For homodyne detection, $\hat{o}$ is a very strong coherent field modelled as $\hat{o} = \gamma/\sqrt{1-\eta} + \hat{\nu}$,

$\hat{\nu}$ is a continuum field that satisfies $[\hat{\nu}(t), \hat{\nu}^\dagger(t')] = \delta(t-t')$ and acts on the vacuum state.

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For η very close to one,

$$\hat{a} \rightarrow \hat{a} + \gamma,$$

The local oscillator

Let γ be real. The rate of photodetections:

$$\mathrm{E}[dN(t)/dt] = \kappa \mathrm{Tr}[(\gamma^2 + \gamma\hat{x} + \hat{a}^\dagger\hat{a})\rho_I(t)].$$

where we define two system quadrature phase operators

$$\hat{x} = \hat{a} + \hat{a}^\dagger \ ; \ \ \hat{y} = -i(\hat{a} - \hat{a}^\dagger).$$

In the limit $\gamma >> \langle\hat{a}^\dagger\hat{a}\rangle$, the rate is a large constant term plus a term proportional to $\hat{x}$, plus a small term.

The continuum limit.

In the limit that $|\gamma|^2 >> \langle a^\dagger a \rangle$, the photocount is mostly due to the LO photons.

Approximate the photocurrent by a continuous function of time, and derive a smooth evolution equation for the system.

Key idea: approximate a Poisson process with large rate by a Gaussian stochastic process for the number of counts $\delta N(t)$ in time δt.

The continuum limit.

substitute δN as a Gaussian random variable and keeping only the lowest order terms in $\gamma^{-1/2}$ and letting $\delta t \to dt$ yields the SME:

$$d\rho_J(t) = -i[\hat{H}, \rho_J(t)]dt + \kappa dt\mathcal{D}[\hat{a}]\rho_J(t) + \sqrt{\kappa}dW(t)\mathcal{H}[\hat{a}]\rho_J(t),$$

where the J subscript refers to a homodyne conditioning (see next slide)

$dW(t)$ is an infinitesimal Wiener increment satisfying

$$\begin{aligned} dW(t)^2 &= dt, \\ \mathcal{E}[dW(t)] &= 0. \end{aligned}$$

That is, the jump conditional evolution has been replaced by diffusive evolution.

The continuum limit.

Exercise: *Consider a Poisson process, $dN(t)$, defined by*

$$\mathcal{E}[dN(t)] = \kappa dt$$

The probability to detect m photons in time interval δt is given by

$$Pr(m) = \frac{(\kappa\delta t)^m}{m!} e^{-\kappa\delta t}$$

which has a mean of $\kappa\delta t$, which also equals the variance. Define the zero-mean random variable $x = m - \kappa\delta t$. Using Stirling's formula show that, for $\kappa\delta t >> 1$, the probability distribution for x may be well approximated by the Gaussian

$$P(x) = (2\pi\kappa\delta t)^{-1/2} e^{-\frac{(x-\kappa\delta t)^2}{2\kappa\delta t}}$$

The continuum limit.

The $\gamma \to \infty$ also changes the point process photocount into a continuous photocurrent with white noise.

Removing the constant local oscillator contribution gives

$$J_{\mathrm{hom}}(t) \equiv \lim_{\gamma\to\infty} \frac{\delta N(t) - \kappa\gamma^2\delta t}{\gamma\delta t} = \kappa\langle\hat{x}\rangle_J(t) + \sqrt{\kappa}\xi(t),$$

where $\xi(t) = dW(t)/dt$.

Homodyne photocurrent correlations.

The mean is

$$\mathrm{E}[J_{\mathrm{hom}}(t)] = \kappa \mathrm{Tr}[\rho(t)\hat{x}],$$

where $\hat{x} = \hat{a} + \hat{a}^\dagger$ as usual, and $\rho(t)$ is assumed given.

The autocorrelation function is defined as

$$F^{(1)}_{\mathrm{hom}}(t, t+\tau) = \mathrm{E}[J_{\mathrm{hom}}(t+\tau)J_{\mathrm{hom}}(t)].$$

this function is related to Glauber's *first*-order coherence function.

Glauber, The quantum theory of optical coherence, Phys. Rev. 130, 2529, (1963).

Homodyne photocurrent correlations.

Using the Itô rules for $dW(t)$

$$F^{(1)}_{\rm hom}(t,t+\tau)=\kappa^2{\rm Tr}\left[\hat{x}e^{\mathcal{L}\tau}\left(\hat{a}\rho(t)+\rho(t)\hat{a}^\dagger\right)\right]+\kappa\delta(\tau).$$

Homodyne photocurrent correlations.

Measure time in units of κ^{-1}

Experimentally, it is more common to use the *spectrum* of the homodyne photocurrent,

$$\begin{aligned} S(\omega) &= \lim_{t\to\infty} \int_{-\infty}^{\infty} d\tau F^{(1)}_{\rm hom}(t, t+\tau) e^{-i\omega\tau} \\ &= 1 + \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \mathrm{Tr}\left[\hat{x} e^{\mathcal{L}\tau}\left(\hat{a}\rho_{\rm ss} + \rho_{\rm ss}\hat{a}^\dagger\right)\right]. \end{aligned}$$

The unit contribution is known as the local oscillator shot-noise or vacuum noise because it is present even when there is no light from the system.

Homodyne mediated feedback.

SME for homodyne detection of efficiency η,

$$d\rho_J(t) = -i[\hat{H}, \rho_J(t)]dt + dt\mathcal{D}[\hat{a}]\rho_J(t) + \sqrt{\eta}dW(t)\mathcal{H}[\hat{a}]\rho_J(t).$$

The homodyne photocurrent, normalized so that the deterministic part does not depend on the efficiency,

$$J_{\rm hom}(t) = \langle \hat{x} \rangle_J (t) + \xi(t)/\sqrt{\eta},$$

where $\xi(t) = dW(t)/dt$ and $\hat{x} = \hat{a} + \hat{a}^\dagger$.

Note: from now on we measure time in units of κ^{-1}.

Homodyne mediated feedback.

$$[\dot{\rho}_J(t)]_{\rm fb} = J_{\rm hom}(t-\tau)\mathcal{K}\rho_J(t),$$

Homodyne mediated feedback.

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But a homodyne current may be negative because the constant local oscillator background has been subtracted, so the feedback superoperator $\mathcal{K}$ must be such as to give valid evolution irrespective of the sign of time.

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But a homodyne current may be negative because the constant local oscillator background has been subtracted, so the feedback superoperator $\mathcal{K}$ must be such as to give valid evolution irrespective of the sign of time.

It must give reversible evolution with

$$\mathcal{K}\rho \equiv -i[\hat{F}, \rho]$$

for some Hermitian operator $\hat{F}$.

Homodyne mediated feedback: the outside view.

Average over all measurement records with feedback to give the homodyne feedback master equation

$$\dot{\rho} = -i[\hat{H}, \rho] + \mathcal{D}[\hat{a}]\rho - i[\hat{F}, \hat{a}\rho + \rho\hat{a}^\dagger] + \frac{1}{\eta}\mathcal{D}[\hat{F}]\rho.$$

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The first feedback term, linear in $\hat{F}$, is the desired effect of the feedback which would dominate in the classical regime.

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The second feedback term causes diffusion in the variable conjugate to $\hat{F}$. It can be attributed to the noise in the measurement. The lower the efficiency, the more noise.

Homodyne mediated feedback.

The homodyne feedback master equation can be rewritten in the Lindblad form

$$\dot{\rho} = -i\left[\hat{H} + \tfrac{1}{2}(\hat{a}^\dagger \hat{F} + \hat{F}\hat{a}), \rho\right] + \mathcal{D}[\hat{a} - i\hat{F}]\rho + \frac{1-\eta}{\eta}\mathcal{D}[\hat{F}]\rho \equiv \mathcal{L}\rho.$$

Homodyne mediated feedback.

The two-time correlation function ($\eta = 1$) of the current is

$$\mathrm{E}[J_{\mathrm{hom}}(t')J_{\mathrm{hom}}(t)] = \mathrm{Tr}\left\{(\hat{a}+\hat{a}^\dagger)e^{\mathcal{L}(t'-t)}[(\hat{a}-i\hat{F})\rho(t)+\mathrm{H.c.}]\right\} + \delta(\tau).$$

The feedback affects the term in square brackets, as well as the evolution by $\mathcal{L}$ for time $t'-t$ so the in-loop photocurrent may have a sub-shot-noise spectrum, even if the light in the cavity dynamics is classical.

The feedback will not produce nonclassical dynamics for a damped harmonic oscillator if $\hat{F}$ is a Hamiltonian corresponding to linear optical processes (quadratic in $a, a^\dagger$).

A linear quantum system.

Exercise: *Show that, for the ground state of a simple harmonic oscillator, the variance of $\hat{x} = a + a^\dagger$, defined by*

$$V = \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$$

is unity

States for which $V < 1$ are called quadrature squeezed states.

Exercise: *Consider the state $|r\rangle = \hat{S}|0\rangle$ where the unitary transformation $\hat{S}$ is defined by*

$$\hat{S} = \exp[\frac{r}{2}(a^2 - a^{\dagger 2})]$$

Show that for this state the variance in $\hat{x}$ is

$$V = e^{-2r}$$

Find the variance in the canonically conjugate operator $\hat{y} = -i(a - a^\dagger)$

A linear quantum system.

A single mode cavity, with damping, driving and parametric amplification.

$$H_I = -i(\epsilon_0 + \eta(t))(a - a^\dagger) - \frac{\chi}{4}(a^2 - a^{\dagger\ 2})$$

where $\eta(t)$ is a delta correlated fluctuating force term,

$$\mathcal{E}(\eta(t+\tau)\eta(t)) = L\delta(\tau)$$

$$\dot{\rho} = \mathcal{D}[\hat{a}]\rho + \tfrac{1}{4}L\mathcal{D}[\hat{a}^\dagger - \hat{a}]\rho + \tfrac{1}{4}\chi[\hat{a}^2 - \hat{a}^{\dagger 2}, \rho] \equiv \mathcal{L}_0\rho,$$

A linear quantum system.

The mean and variance of $\hat{x} = a + a^\dagger$

$$\begin{aligned} \frac{d\langle \hat{x}\rangle}{dt} &= -k\langle \hat{x}\rangle\,, \\ \frac{dV}{dt} &= -2kV + D. \end{aligned}$$

For a *stable* system with $k > 0$, there is a steady state with $\langle x\rangle = 0$ and

$$V = \frac{D}{2k}.$$

Exercise *Show that for the particular master equation above (the properties of which will be denoted by the subscript* 0*) these equations hold, with*

$$\begin{aligned} k_0 &= \tfrac{1}{2}(1+\chi), \\ D_0 &= 1 + L. \end{aligned}$$

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Here, $V_0 = (1 + l)/(1 + \chi)$. If this is less than unity, the system exhibits squeezing of the x quadrature.

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Define the the normally ordered variance

$$U \equiv \left\langle \hat{a}^\dagger \hat{a}^\dagger + 2\hat{a}^\dagger \hat{a} + \hat{a}\hat{a} \right\rangle - \left\langle \hat{a}^\dagger + \hat{a} \right\rangle^2 ,$$

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$$U_0 = \frac{l - \chi}{1 + \chi}.$$

If the variance in the y quadrature does not become unbounded, then the maximum value for χ is one. (**Show this**)

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At this value, $U_0 = -1/2$ when the x-diffusion rate $l = 0$, which is half of the theoretical minimum of $U_0 = -1$.

A linear quantum system with feedback.

Add feedback to try to reduce the fluctuations in x.

$$\hat{F} = -\lambda\hat{y}/2.$$

a translation in the negative x direction for $\lambda > 0$.

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$$\dot{\rho} = \mathcal{L}_0\rho + \frac{\lambda}{2}[\hat{a} - \hat{a}^\dagger, \hat{a}\rho + \rho\hat{a}^\dagger] + \frac{\lambda^2}{4\eta}\mathcal{D}[\hat{a} - \hat{a}^\dagger]\rho.$$

η is the proportion of output light used in the feedback loop, multiplied by the efficiency of the detection.

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$$\begin{aligned} k &= k_0 + \lambda, \\ D &= D_0 + 2\lambda + \lambda^2/\eta. \end{aligned}$$

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Minimizing U_λ with respect to λ one finds

$$U_{\min} = \eta^{-1} \left(-k_0 + \sqrt{k_0^2 + 2\eta k_0 U_0} \right) ,$$

when

$$\lambda = -k_0 + \sqrt{k_0^2 + 2\eta k_0 U_0} .$$

Note that this λ has the same sign as U_0.

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If system produces squeezed light, then to enhance the squeezing, add a force which displaces the state in the direction of the difference between the measured photocurrent and the desired mean photocurrent.

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The best intracavity squeezing will be when $\eta = 1$, in which case the intracavity squeezing can be simply expressed as

$$U_{\min} = k_0 \left(-1 + \sqrt{1 + R_0} \right) .$$

Errors, noise and decoherence.

Bit flip errors on qubits.

$$X|0\rangle = |1\rangle, \qquad X|1\rangle = |0\rangle$$

X is a Pauli operator, σ_x.

Phase flip errors on qubits.

$$Z(|0\rangle + |1\rangle) = |0\rangle - |1\rangle$$

Z is a Pauli operator, σ_z

Quantum Markov process.

Assume errors act randomly in time: *quantum Poisson process*.
Quantum Markov process

$$\frac{d\rho(t)}{dt} = \gamma(X\rho(t)X - \rho(t))\gamma \equiv \mathcal{D}[X]\rho$$

Phase flip errors.

Poisson process at rate κ

$$\frac{d\rho}{dt} = \kappa(Z\rho Z - \rho) \equiv \kappa\mathcal{D}[Z]\rho$$

Quantum information and noise.

Can we protect quantum information from classical noise ?

Continuous bit flip errors.

How to correct for a single bit flip ?

Classical: bit flip errors can be corrected by a simple repetition code:

$1 \rightarrow 111 \quad , \quad 0 \rightarrow \quad 000$

examine each bit, then take a majority vote.

Quantum code ?

Need to protect an arbitrary superposition state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

Problem: quantum *no-cloning* theorem.

Quantum bit flip code.

Solution: use entangled states of three physical qubits !

$$\begin{aligned} |0\rangle_L &= |0\rangle|0\rangle|0\rangle \equiv |000\rangle \\ |1\rangle_L &= |1\rangle|1\rangle|1\rangle \equiv |111\rangle \end{aligned}$$

Encode using two CNOT gates:

$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle|0\rangle \rightarrow \alpha|0\rangle_L + \beta|1\rangle_L$$

Quantum bit flip correction.

Example: suppose first bit flips.

$$\alpha|000\rangle + \beta|111\rangle \rightarrow \alpha|100\rangle + \beta|011\rangle$$

Recall that

$$\begin{aligned} Z|0\rangle &= |0\rangle \\ Z|1\rangle &= -|1\rangle. \end{aligned}$$

ZZI	**IZZ**	**Error**	**Correcting unitary**
+1	+1	None	None
-1	+1	on qubit 1	XII
+1	-1	on qubit 3	IIX
-1	-1	on qubit 2	IXI

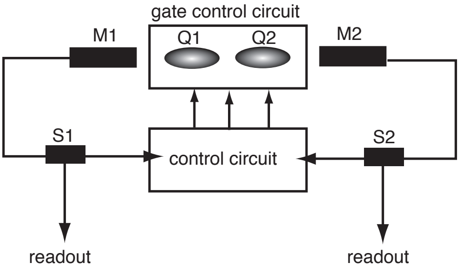
Summary of bit flip correction.

- code logical qubit as three physical qubits
- measure ZZI and IZZ *instantaneously with perfect accuracy*
- apply a unitary correction operation

Can this work as a closed loop process, with continuous measurement and feedback ?

Feedback protection of quantum information.

example: can we protect a memory from accidental readout?



Feedback protection of quantum information.

Three qubit code for bit flip errors.

$$\alpha|0\rangle_L + \beta|1\rangle_L = \alpha|000\rangle + \beta|111\rangle$$

Need to measure: ZZI, IZZ.
Corrections:XII, XIX, IIX

Feedback protection of quantum information.

$$
\begin{aligned}
d\rho_c(t) &= \boxed{\gamma(\mathcal{D}[XII] + \mathcal{D}[IXI] + \mathcal{D}[IIX])\rho_c(t)dt} \text{ error} \\
&\quad +\kappa(\mathcal{D}[ZZI] + \mathcal{D}[IZZ])\rho_c(t)dt \\
&\quad +\sqrt{\kappa}(\mathcal{H}[ZZI]dW_1(t) \\
&\qquad + \mathcal{H}[IZZ]dW_2(t))\rho_c(t) \\
dQ_1(t) &= 2\kappa\langle ZZI\rangle_c(t)dt + \sqrt{\kappa}dW_1(t) \\
dQ_2(t) &= 2\kappa\langle IZZ\rangle_c(t)dt + \sqrt{\kappa}dW_2(t)
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$\mathcal{H}[A]$ depends on $\rho_c(t) \rightarrow$ non-linear dynamics.

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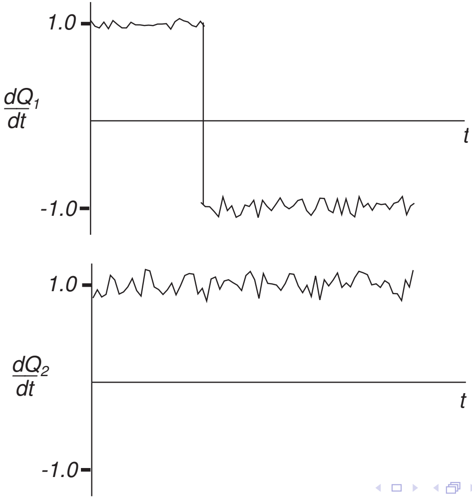
observed processes

Error signal.

Example: bit flip error on first qubit.



Error signal processing.

Smooth observed measurement process.

$$R_i(t) = \frac{1}{\mathcal{N}} \int_{t-T}^{t} e^{-r(t-t')} dQ_i(t') \qquad i = 1, 2$$

r →controls smoothing (filter spectral width).

1. If $R_1(t) < 0$ and $R_2(t) > 0$, apply XII.
2. If $R_1(t) > 0$ and $R_2(t) < 0$, apply IIX.
3. If $R_1(t) < 0$ and $R_2(t) < 0$, apply IXI.
4. If $R_1(t) > 0$ and $R_2(t) > 0$, do not apply any feedback.

Feedback process.

Control hamiltonian terms:

$$\begin{aligned} d\rho_c(t) &= \ldots - i\lambda(G_1(t)[XII,\rho_c(t)] + G_2(t)[IXI,\rho_c(t)] \\ &\quad + G_3(t)[IIX,\rho_c(t)])dt \end{aligned}$$

λ strength of the feedback

$$G_1(t) = \begin{cases} R_1(t) & \text{if } R_1(t) < 0 \text{ and } R_2(t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

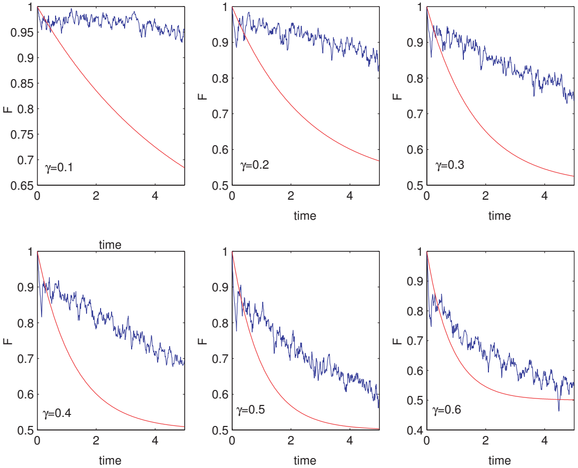
$$G_2(t) = \begin{cases} R_2(t) & \text{if } R_1(t) > 0 \text{ and } R_2(t) < 0 \\ 0 & \text{otherwise} \end{cases}$$

$$G_3(t) = \begin{cases} R_1(t) & \text{if } R_1(t) < 0 \text{ and } R_2(t) < 0 \\ 0 & \text{otherwise} \end{cases}$$

Performance measure.

Fidelity, F, survival probability of initial state versus t.
Compute fidelity for a single simulation of conditional dynamics.

Performance measure.



κ=150 meas. rate

λ=150 feedback rate

r=20 filter

Feedback creation of entanglement.

Can we correct for detected 'jump' errors?

See: Quantum error correction for continuously detected errors, Ahn, Wiseman, GJM, Phys. Rev. A 67, 052310 (2003).

Feedback creation of entanglement.

Measurement on multiple qubits can create entanglement.

Feedback creation of entanglement.

Measurement on multiple qubits can create entanglement.

Can measurement plus feedback create entangled steady states?

The Hamiltonian

Dispersive level shifts in a cavity containing a qubit.

Effective Hamiltonian in the interaction picture.

$$H_I = \chi a^\dagger a(|1\rangle\langle 1| - |0\rangle\langle 0|) \equiv \chi a^\dagger a \sigma_z$$

Conditional frequency shift of cavity.

Two qubit case.

*Sarovar, Goan, Spiller , GJM, Phys. Rev. A, 72, 062327 (2005)**
Two CPB qubits, dispersive limit.

$$H_I = 2\chi J_z a^\dagger a + \chi(\sigma_1^+\sigma_2^- + \sigma_2^-\sigma_1^+)$$

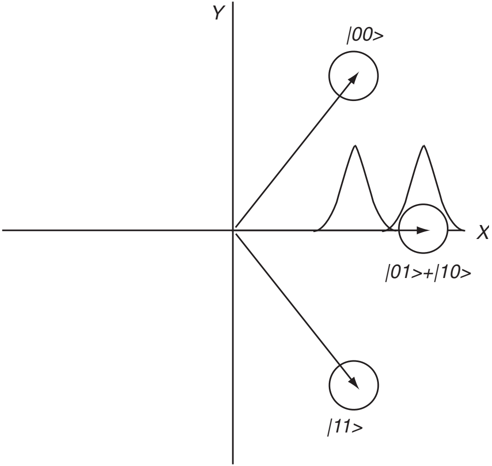
where $J_z = \sigma_{z1} + \sigma_{z2}$.

$$\begin{aligned} & e^{-i\theta J_z a^\dagger a}(|00> + |01> + |10> + |11>)|\alpha\rangle \\ = \quad & |00\rangle|\alpha e^{i\theta}\rangle + |11\rangle|\alpha e^{-i\theta}\rangle + (|10\rangle + |01\rangle)|\alpha\rangle \end{aligned}$$

Measure phase of field by homodyne detection.

* See also "Tunable joint measurements in the dispersive regime of cavity QED", Lalumière, Gambetta, Blais arXiv:0911.5322

CQED as a qubit bus mode.



Nemoto & Munro. PRL 2004.

Measurement induced entanglement.

Add coherent driving

$$H_I = \epsilon(a + a^\dagger) + 2\chi J_z a^\dagger a + \chi(\sigma_1^+\sigma_2^- + \sigma_2^-\sigma_1^+)$$

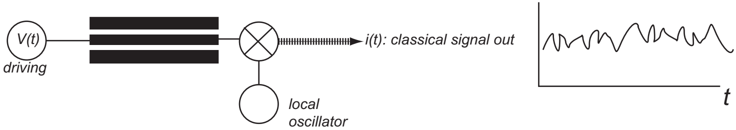
And cavity decay

$$\frac{d\rho}{dt} = -i[H_I, \rho] + \kappa\left(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a\right)$$

and monitor the output field via homodyne detection.

Need $\epsilon \sim \kappa >> \chi$, and strong feedback, $\lambda > \kappa$

Continuous conditional evolution.



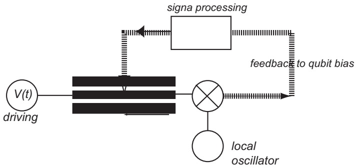
The homodyne current for *quantum limited detection* obeys

$$dI(t) = \kappa\langle a + a^\dagger\rangle + \sqrt{\kappa}dW(t)$$

Assume the only source of noise in the signal comes from the quantum source.

What is the conditional state of the source, conditioned on a particular current history, $i(t)$.

Feedback creation of entanglement.



Feedback homodyne current from SET to change bias conditions of the CPB.
Process signal by low-pass filter:

$$R(t) = \frac{1}{N} \int_{t-T}^{t} e^{-\gamma(t-t')} dI(t')$$

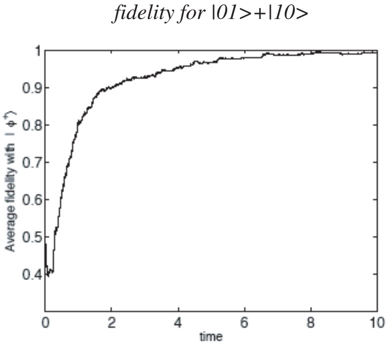
Add control Hamiltonian

$$H_{FB} = \lambda R(t)^3 (\sigma_{x1} + \sigma_{x2})$$

Feedback creation of entanglement.

$$d|\psi_c(t)\rangle = [-iH_I - iH_{FB}(t) - \kappa a^\dagger a]|\psi_c(t)\rangle dt + dI(t)\ a|\psi_c(t)\rangle$$

Feedback creation of entanglement.



99% of trajectories converge to target state.

Sarovar et al., Phys. Rev. A 72, 062327 (2005)