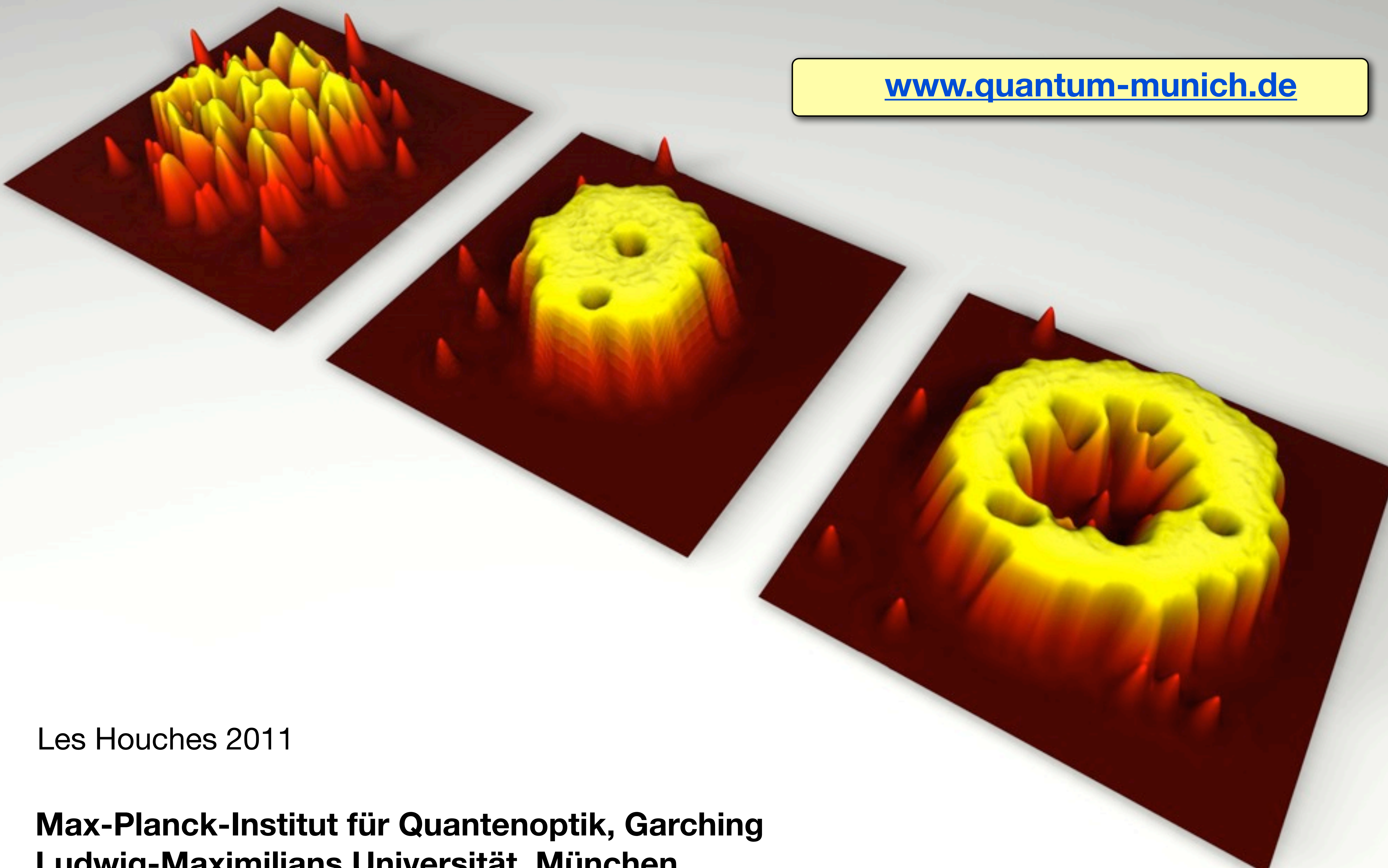
Exploring Quantum Matter with Ultracold Atoms



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Les Houches 2011

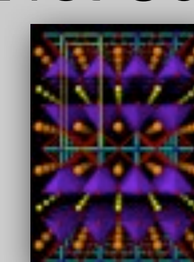
Max-Planck-Institut für Quantenoptik, Garching
Ludwig-Maximilians Universität, München

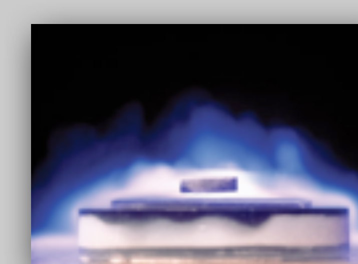
Course Outline

- **Introduction to UCG**
- **Interactions (Scattering, Feshbach, ...)**
- **Introduction to Optical Lattices**
- **Detection Techniques**
- **Many-Body Physics in Optical Lattices**
  - Bose Hubbard Model
  - Fermi Hubbard Model
- **Controlling Few Body Physics**
  - Repulsively Bound Pairs, Correlated Atom Tunneling
  - Superexchange Interactions
  - Creating & Probing Entangled Atom States
  - Minimal Versions of Topologically Ordered States (RVB, d-Wave,...)
- **Non-Equilibrium Dynamics**
- **Outlook**
  - Polar Molecules, Rydberg Atoms

The Challenge of Quantum Many Body Systems

- **Understand and Design Quantum Materials** -
  one of the biggest challenge of Quantum Physics in the 21st Century





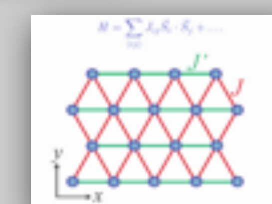
- **Technological Relevance**

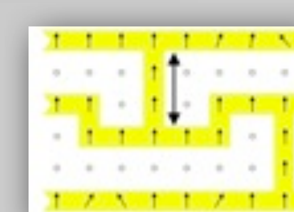
  High-Tc Superconductivity (Power Delivery)

  Magnetism (Storage, Spintronics...)

  Novel Quantum Sensors (Precision Detectors)

  Quantum Computing





Many cases: lack of basic understanding of underlying processes

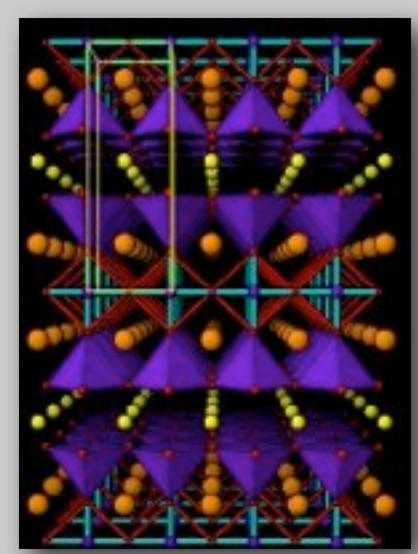
Difficulty to separate effects: probe impurities, complex interplay, masking of effects...

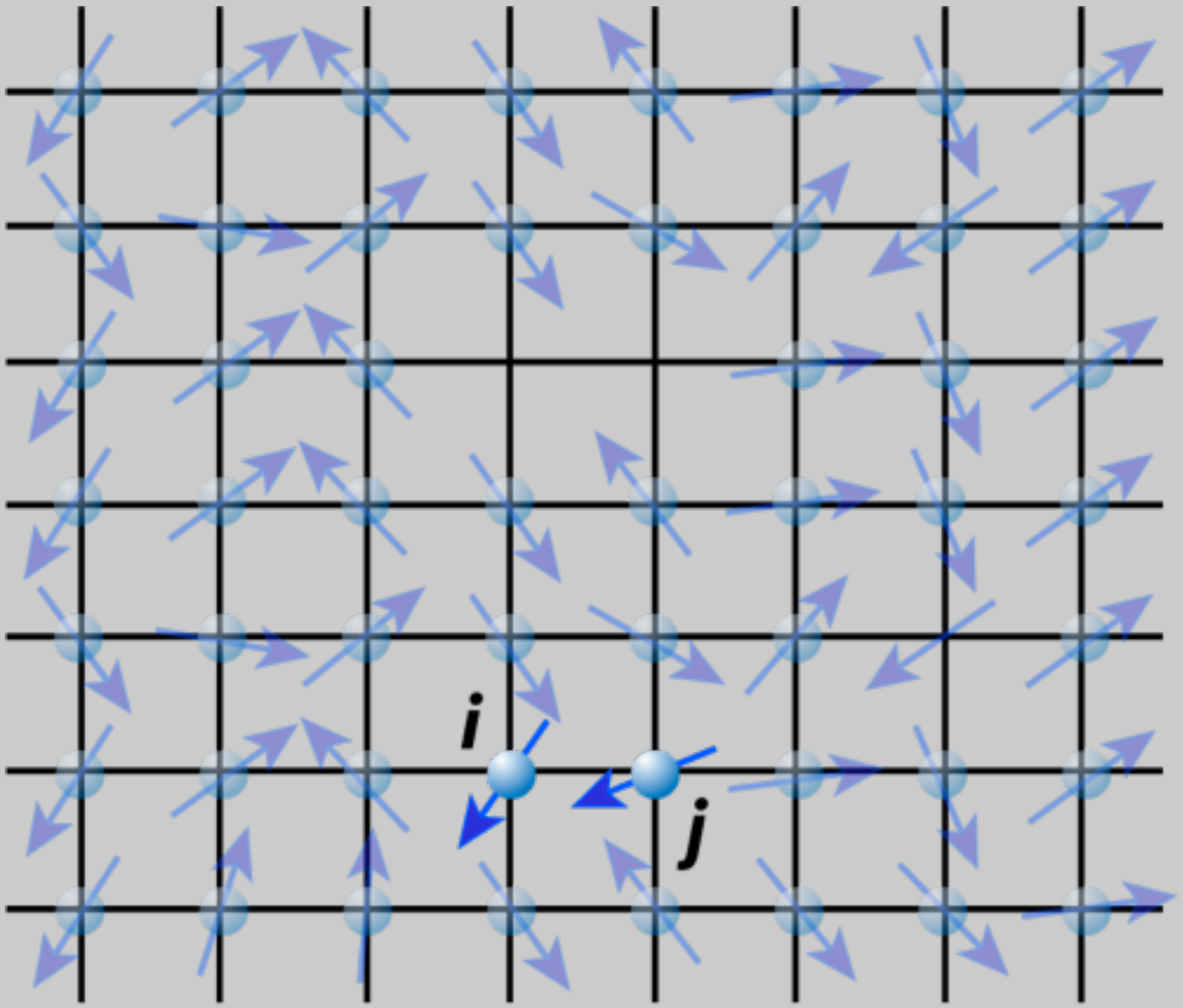
Many cases: even simple models “not solvable”

Need to synthesize new material **to analyze effect of parameter change**

Strongly Correlated Electronic Systems

$$H = -J \sum_{\langle i,j \rangle,\sigma} \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + V_0\sum_{i,\sigma}R_i^2\hat{n}_{i,\sigma}$$





In strongly correlated electron system spin-spin interactions exist.

$$-J_{ex}\vec{S}_i \cdot \vec{S}_j$$

Underlying many solid state & material science problems:
Magnets, High-Tc Superconductors, Spintronics ....

Roadrunner – Los Alamos

1.1 Petaflops/s
2000 t
3.9 MW

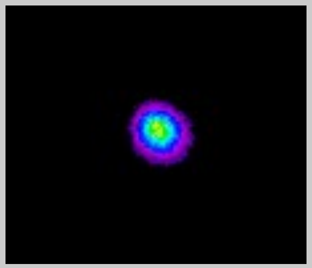
State of the art: < 40 spins (2^{40}x 2^{40}) (what does it take to simulate 300 spins ?)

each doubling allows for one more spin 1/2 only

2^{300} estimated number of protons in the universe

Introduction

- **Controlling Single Quantum Systems**

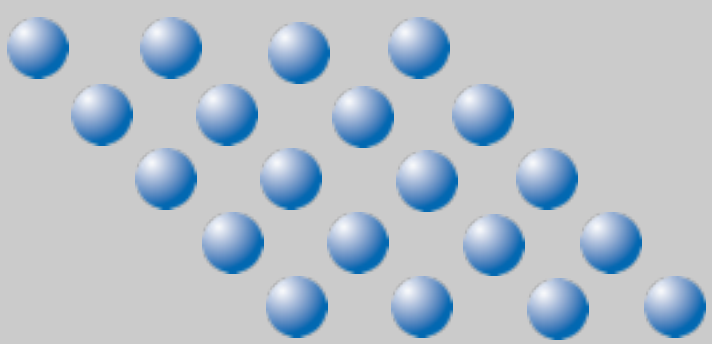


Single Atoms and Ions

Photons

Quantum Dots

- **New challenges ahead: control, engineer and understand complex quantum system**
  quantum computers, quantum simulators, novel (states of) quantum matter, advanced materials, multi-particle entanglement



R. P. Feynman‘s Vision

A Quantum Simulator to study the quantum behaviour of another system.

R.P. Feyman, Int. J. Theo. Phys. (1982)
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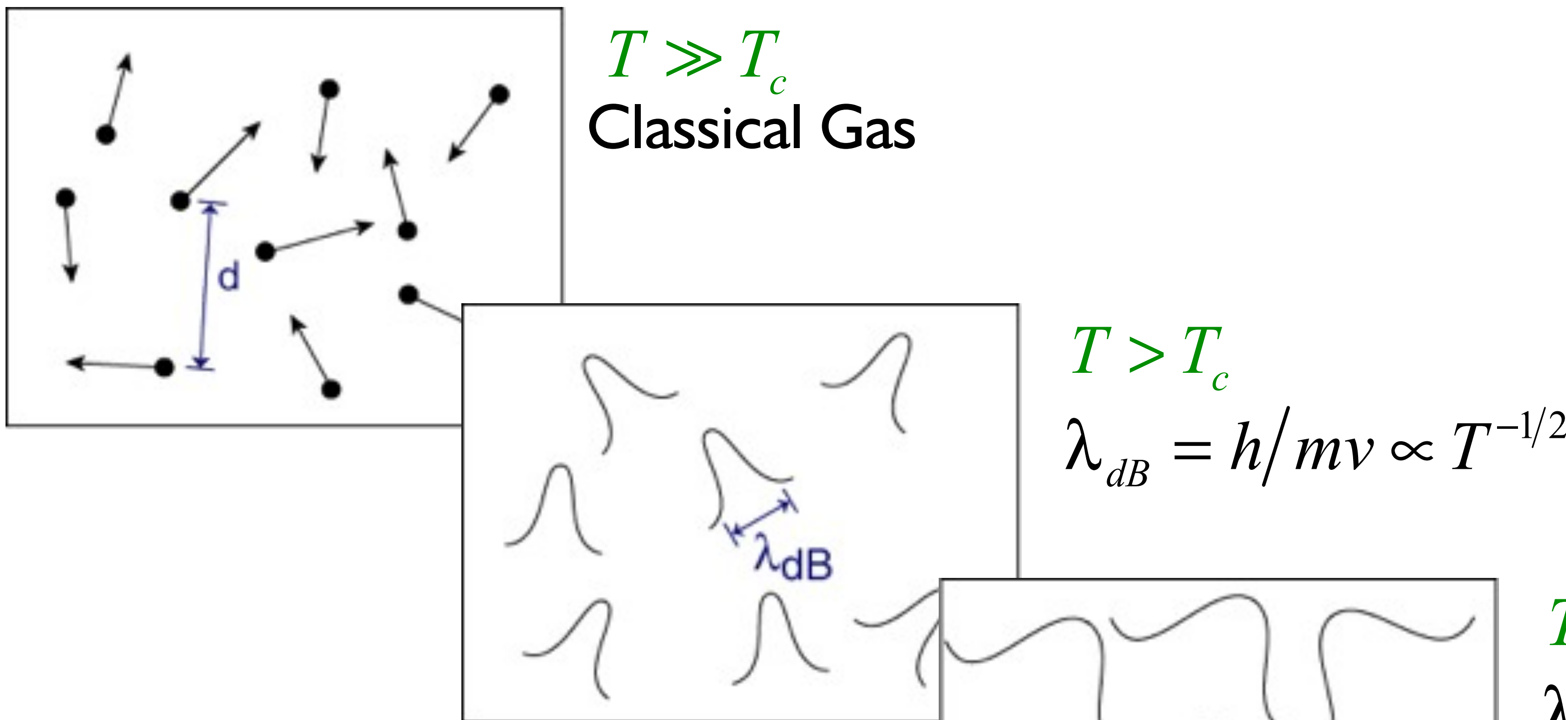
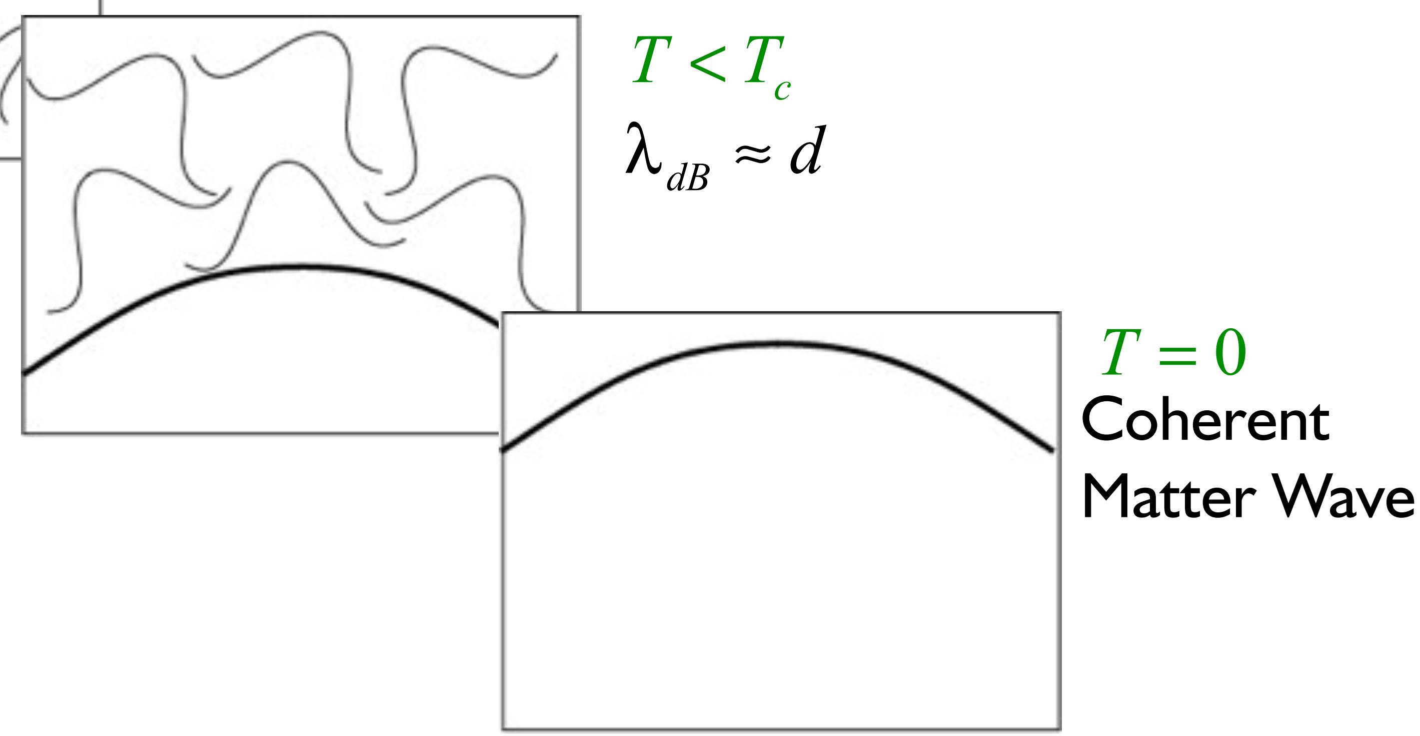
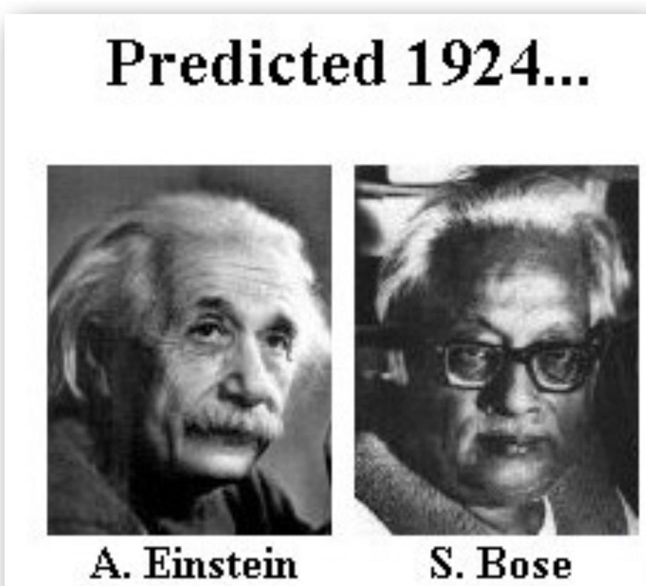
From a Classical Gas to a Bose-Einstein-Condensate







Why is it Difficult to Reach BEC?

Condition for BEC:

$$n \cdot \lambda^3 \approx 1$$

e.g. Water

For a typical density of water n_{H20} one obtains $T_c = 1K$

Problem: Water is Ice @ 1K

Solution: Reduced densities by several orders of magnitude, such that the solid is only formed very slowly!

Even Lower Temperatures are Necessary

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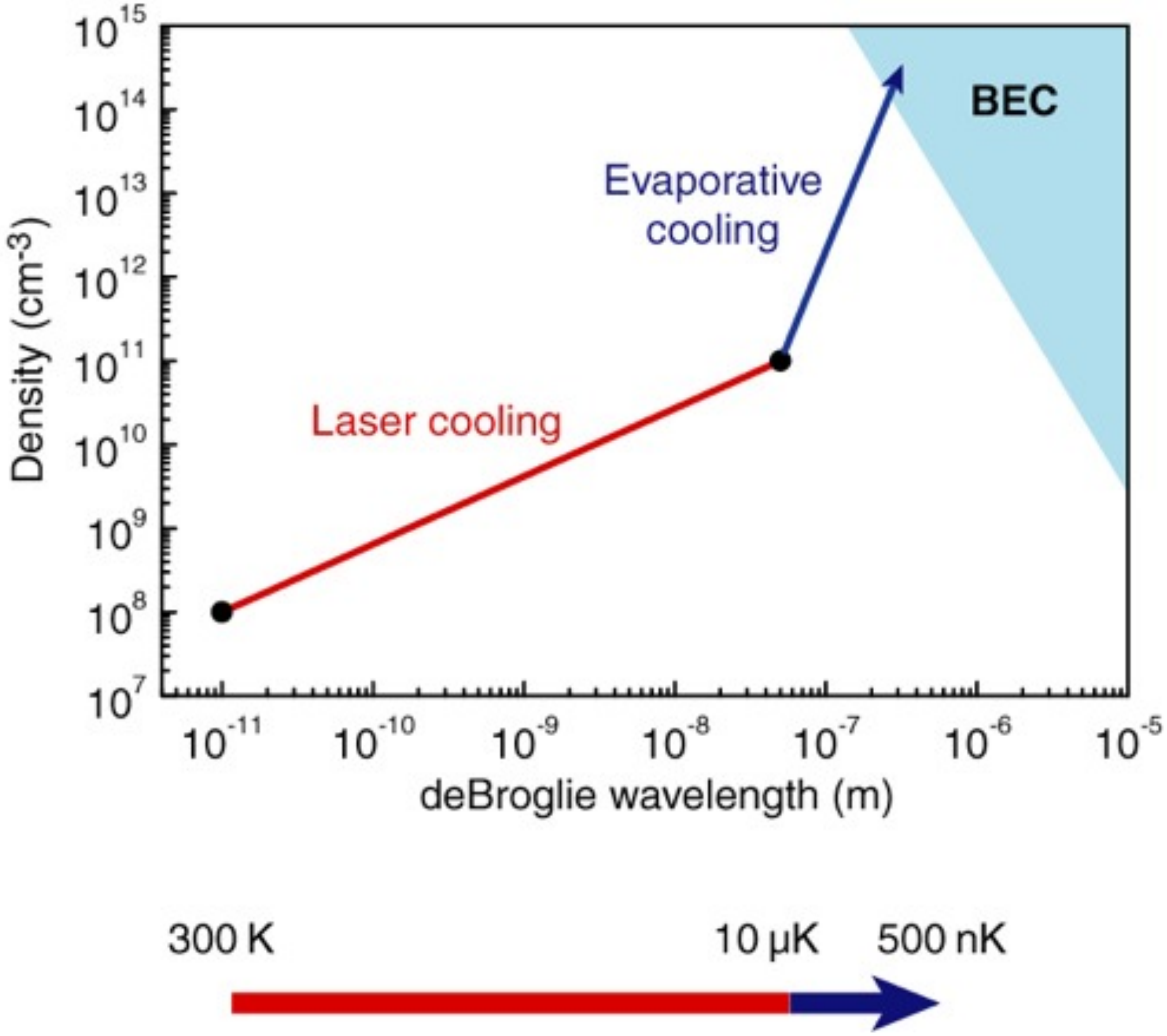
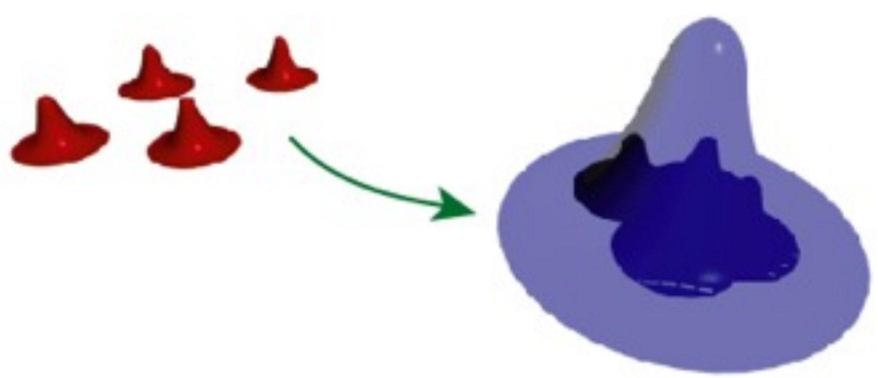
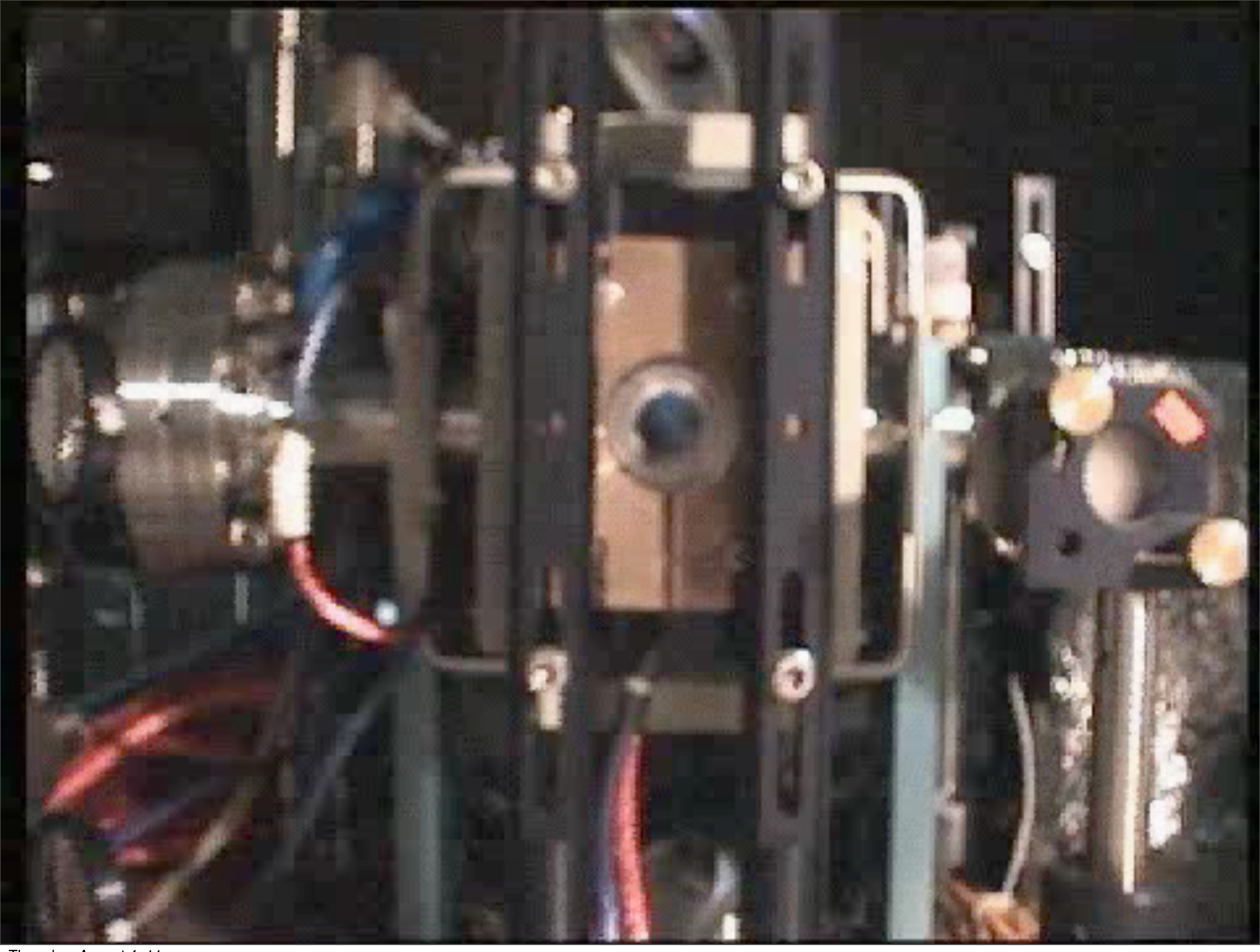
The Path to Bose-Einstein Condensation



$$n \cdot \lambda^3 \approx 1$$





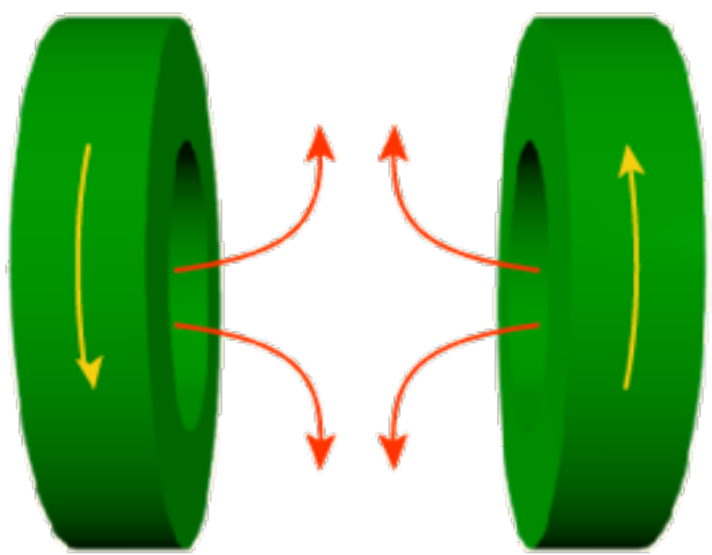
Magnetic Traps for Neutral Atoms

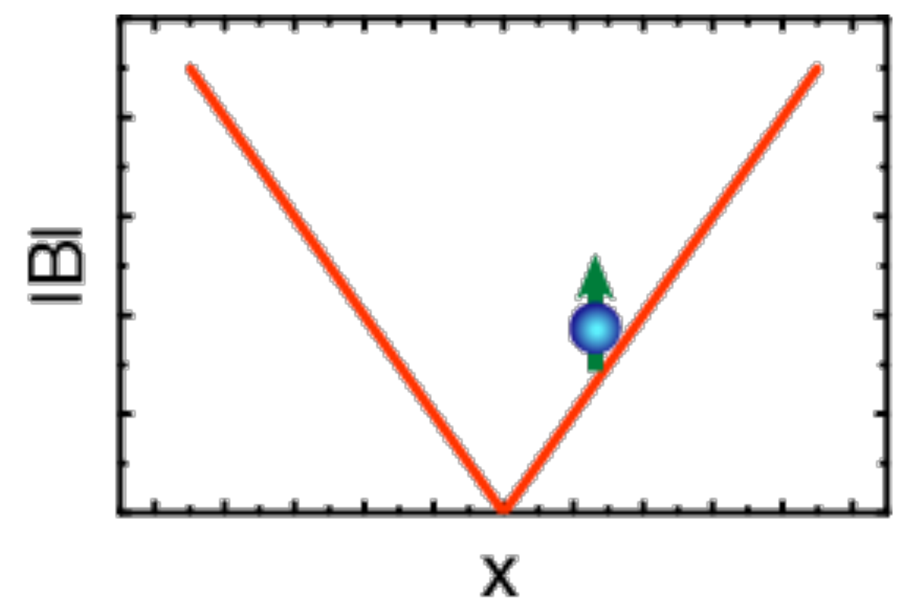
Energy of an atom in an external magnetic field

$$E = -\vec{\mu} \cdot \vec{B}$$

Force on an atom in an inhomogeneous field

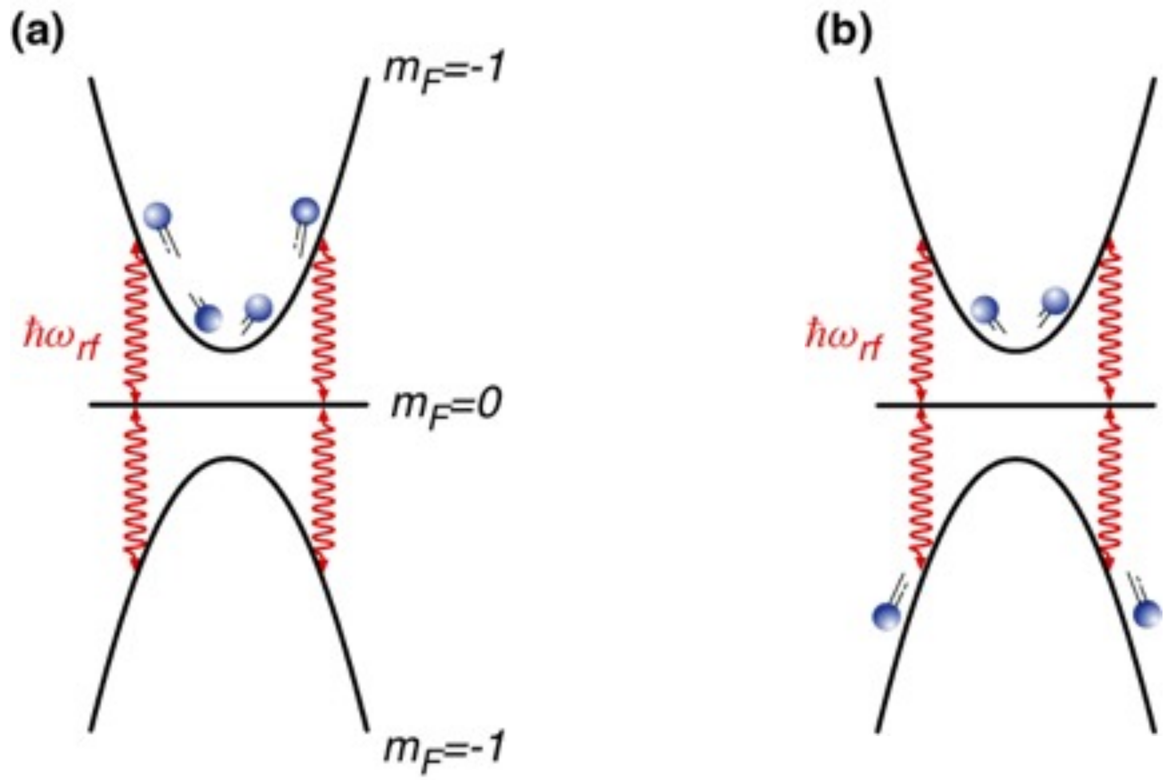
$$\vec{F} = -\mu \cdot \nabla B$$



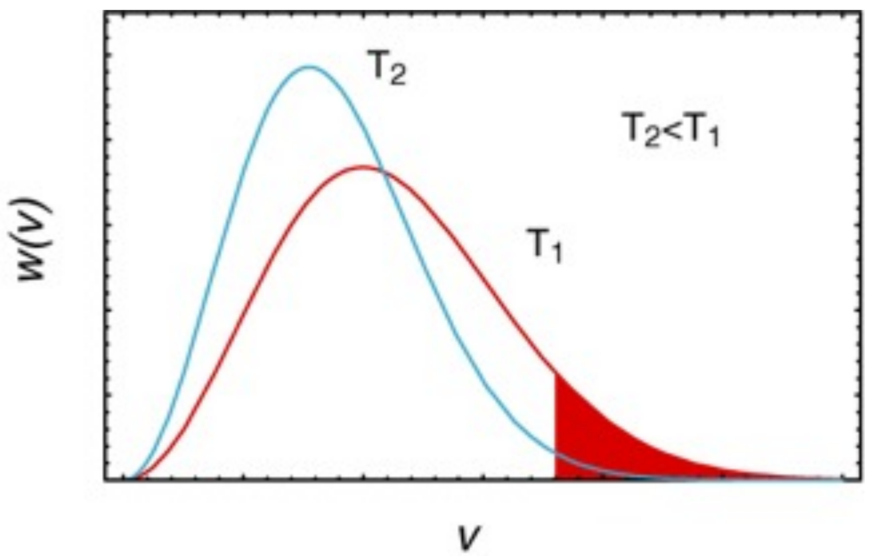


Evaporative Cooling

With the help of RF-transitions between neighbouring magnetic sublevels, the hottest atoms can be selectively removed from the trap.



Elastic collisions rethermalize the atoms resulting in a cooler and denser atomic distribution.



Phase space density is increased

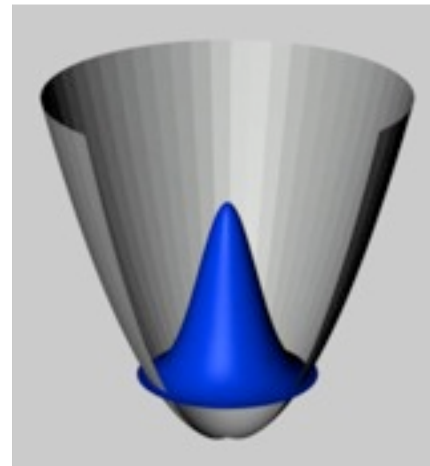
The Path to Bose-Einstein-Condensation

1. Magneto Optical Trap (MOT) (10^9 atoms)
2. Compressed MOT to increase density of atom cloud
3. Optical molasses mooling
4. Optical pumping to spin polarize atoms
5. Magnetic trapping
6. Evaporative cooling
7. Bose-Einstein condensation (10^5-10^6 atoms) around temperatures of 1μK and densitied of 10^{14} cm^{-3}

From a Bose Gas without Interactions to a Strongly Correlated Bose System

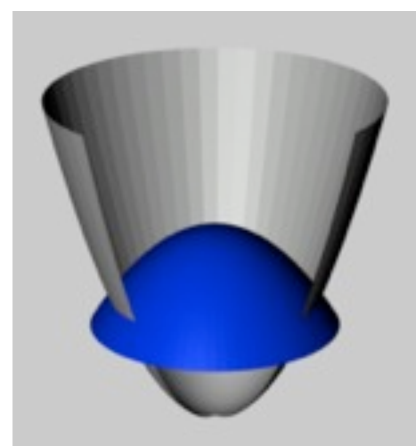
Many-Body State

No Interactions



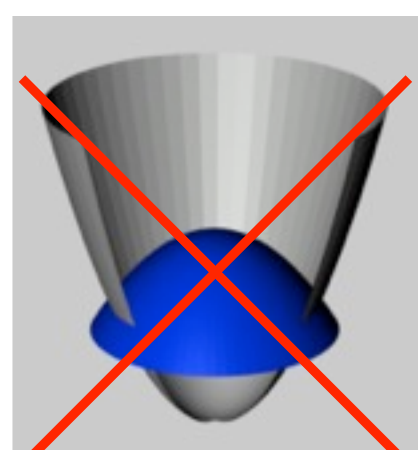
$$|\Psi\rangle \propto |\psi\rangle^{\otimes N}$$

Weak Interactions



$$|\Psi\rangle \propto |\psi_{\text{int}}\rangle^{\otimes N}$$

Strongly Correlated System



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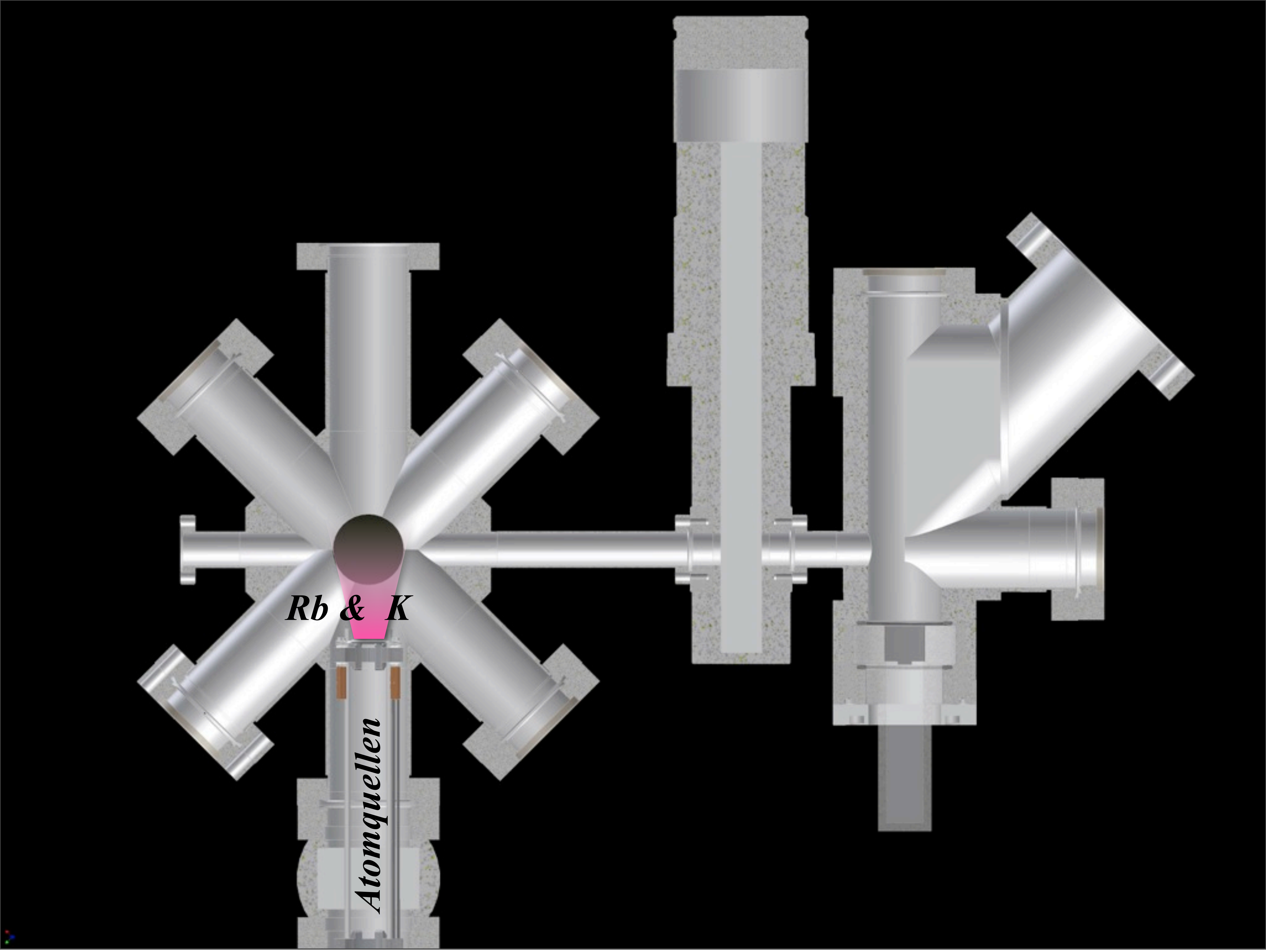
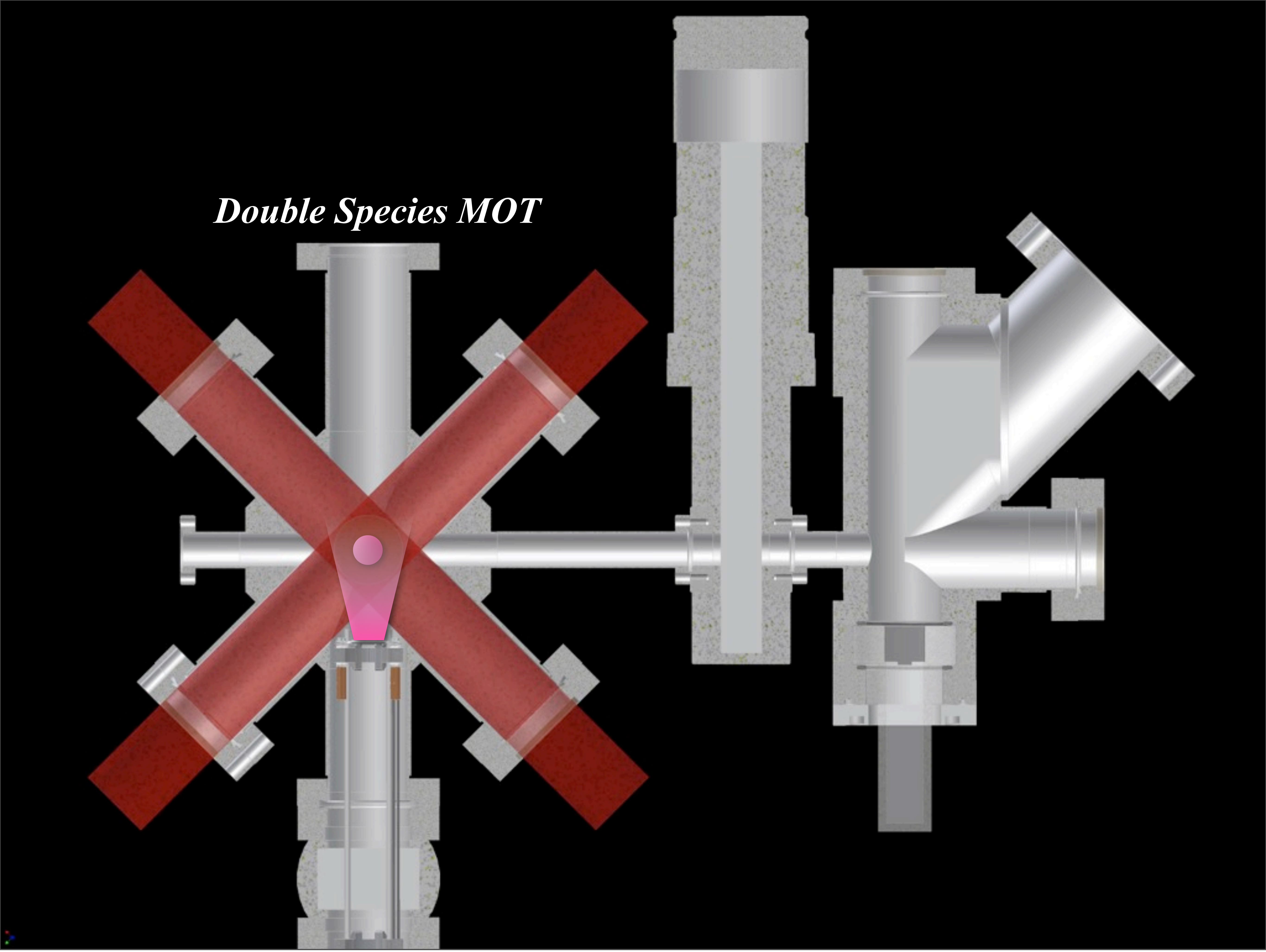
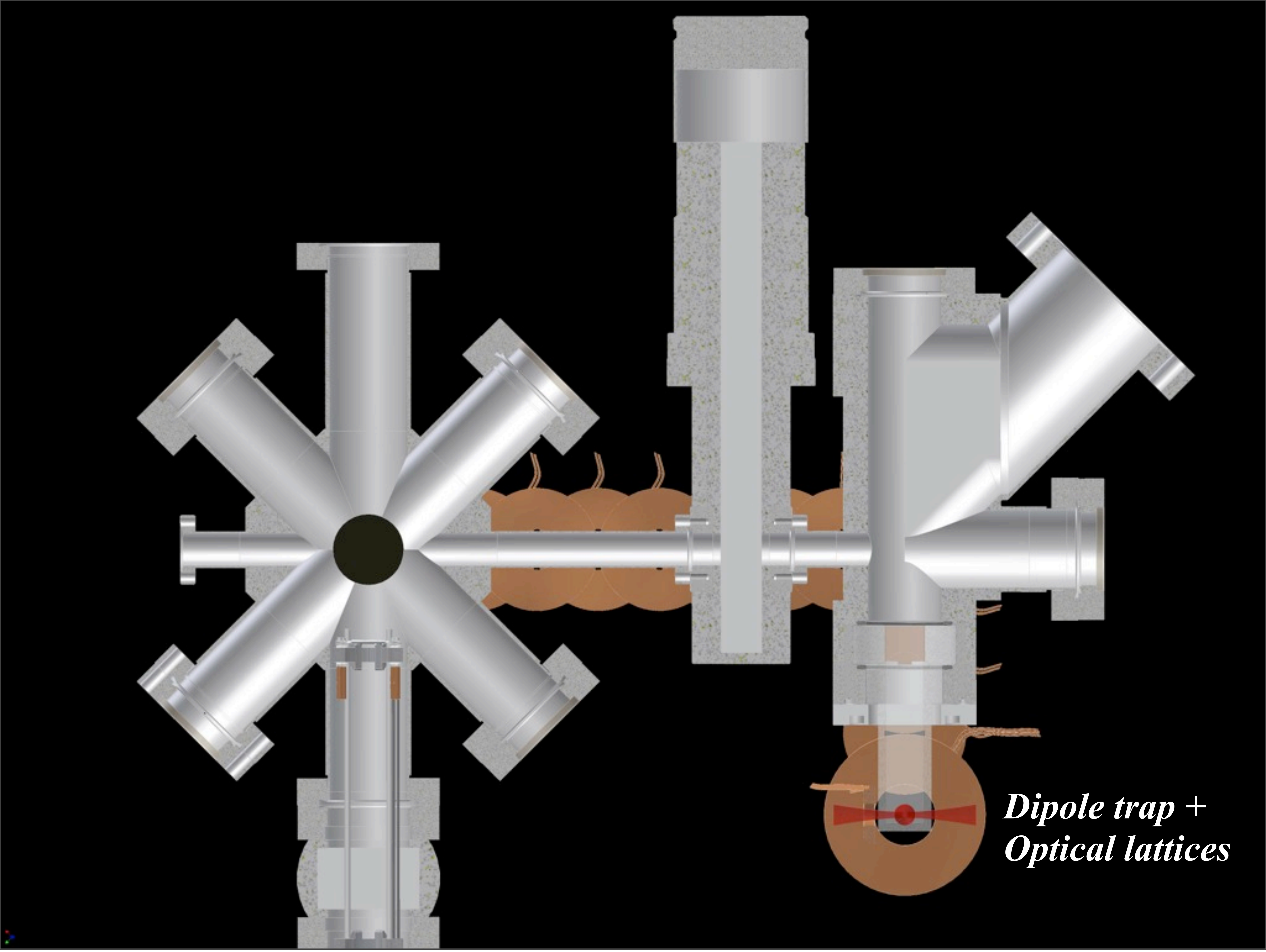
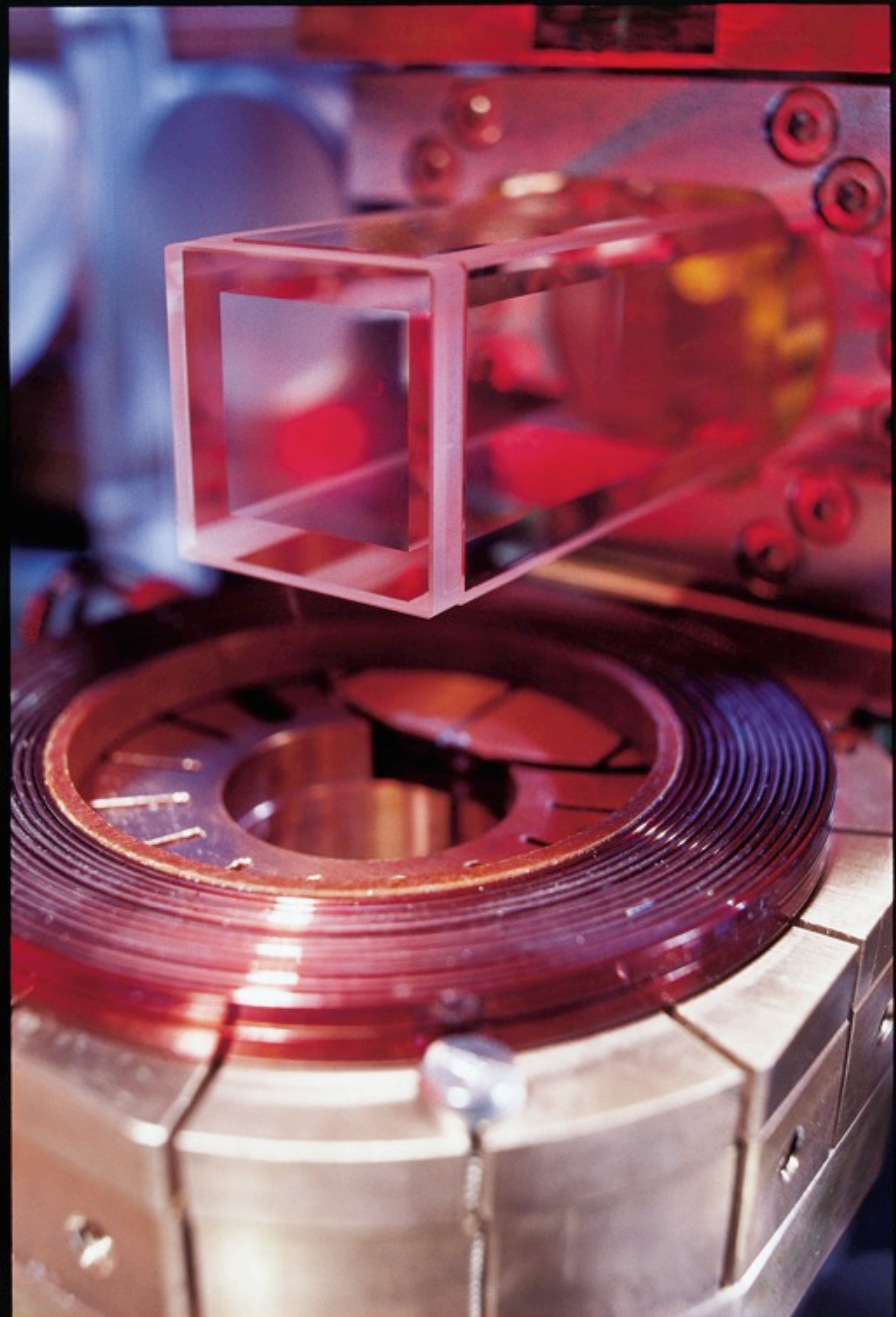

Rb & K
Atomquellen


Double Species MOT

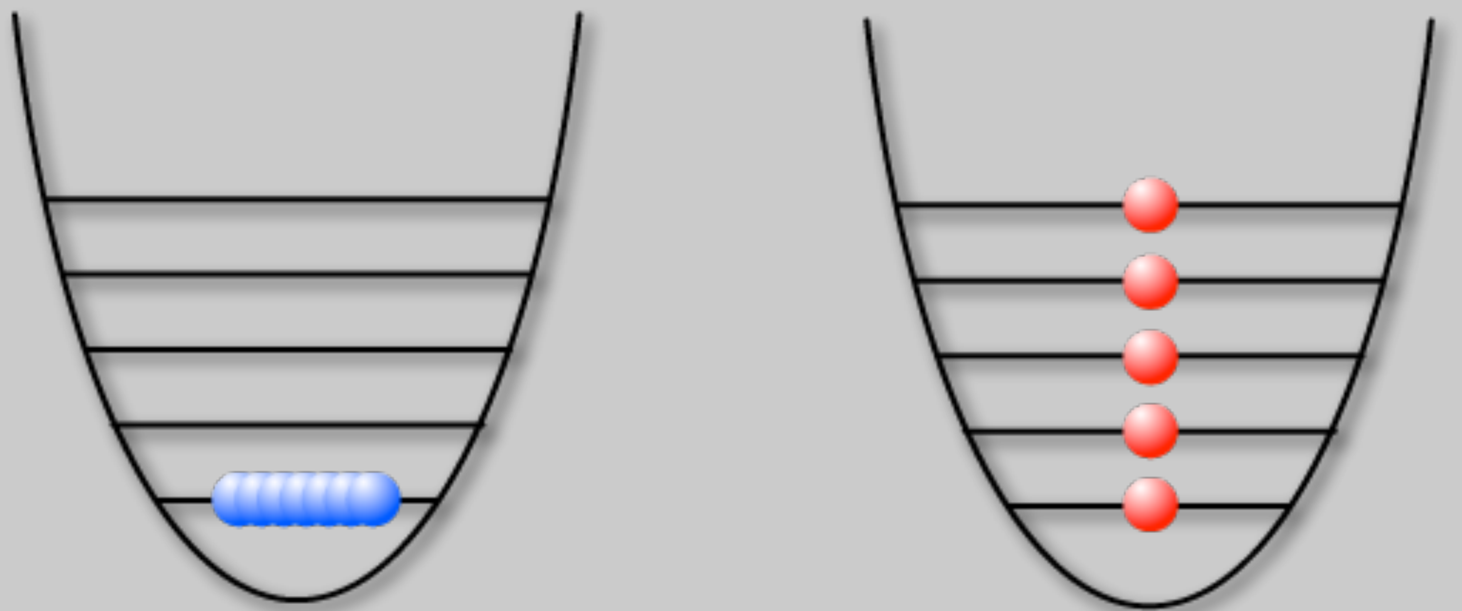

Dipole trap +
Optical lattices



Our Starting Point – Ultracold Quantum Gases

Parameters:
Densities: 10^{15} cm^{-3}
Temperatures: Nano Kelvin
Atom Numbers 10^6

Ground States at T=0



Bose-Einstein Condensates
e.g. ^{87}Rb

Degenerate Fermi Gases
e.g. ^{40}K

Introduction

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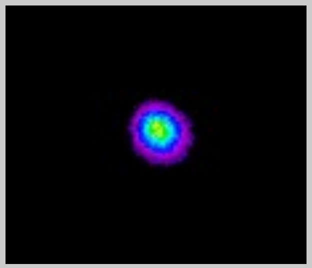

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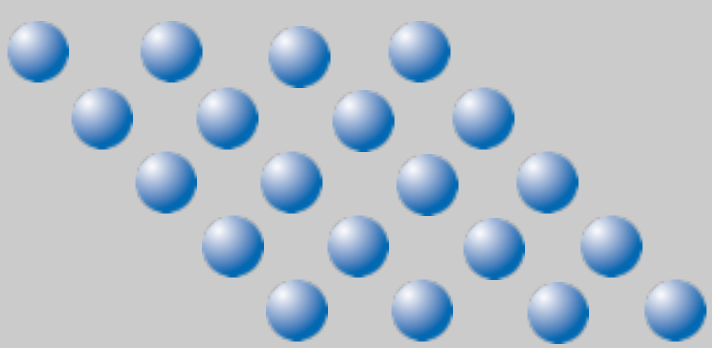

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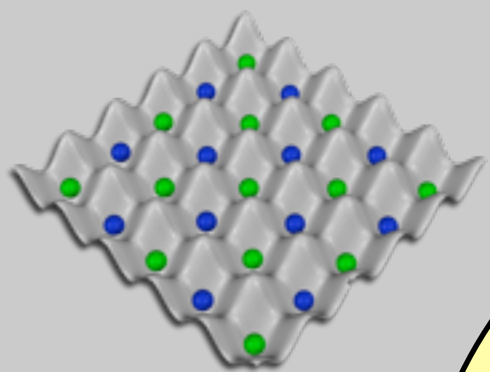
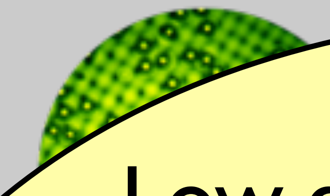
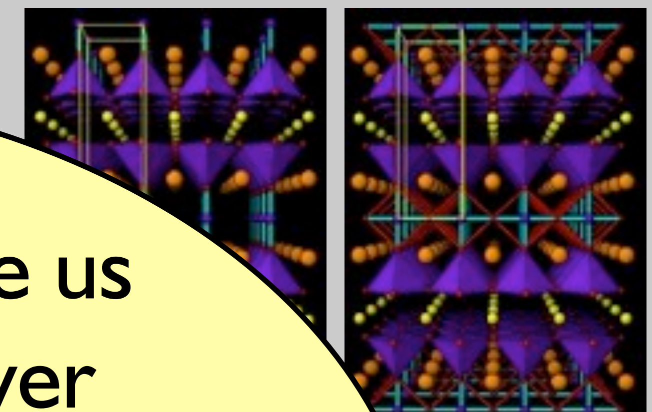
From Artificial Quantum Matter to Real Materials

Ultracold Quantum Gases in Optical Lattices





Real Materials



ductors (YBCO)

Low densities require us
to work at even lower
temperatures
but
we gain the control & manipulations
techniques of the atomic physics
toolbox

• ***Densities:***
(100000 times thinner t

10^{24}-10^{25}/cm^3

• ***Temperatures:***
(100 millionen times lower

mK – several hundred K

• ***Crystal Structu***
Material Para ers can
be changed dynamically and
in-situ.

res and
ameters given by Material
possible via e.g.
external parameters like e.g. pressure, B-fields
or via synthesis)

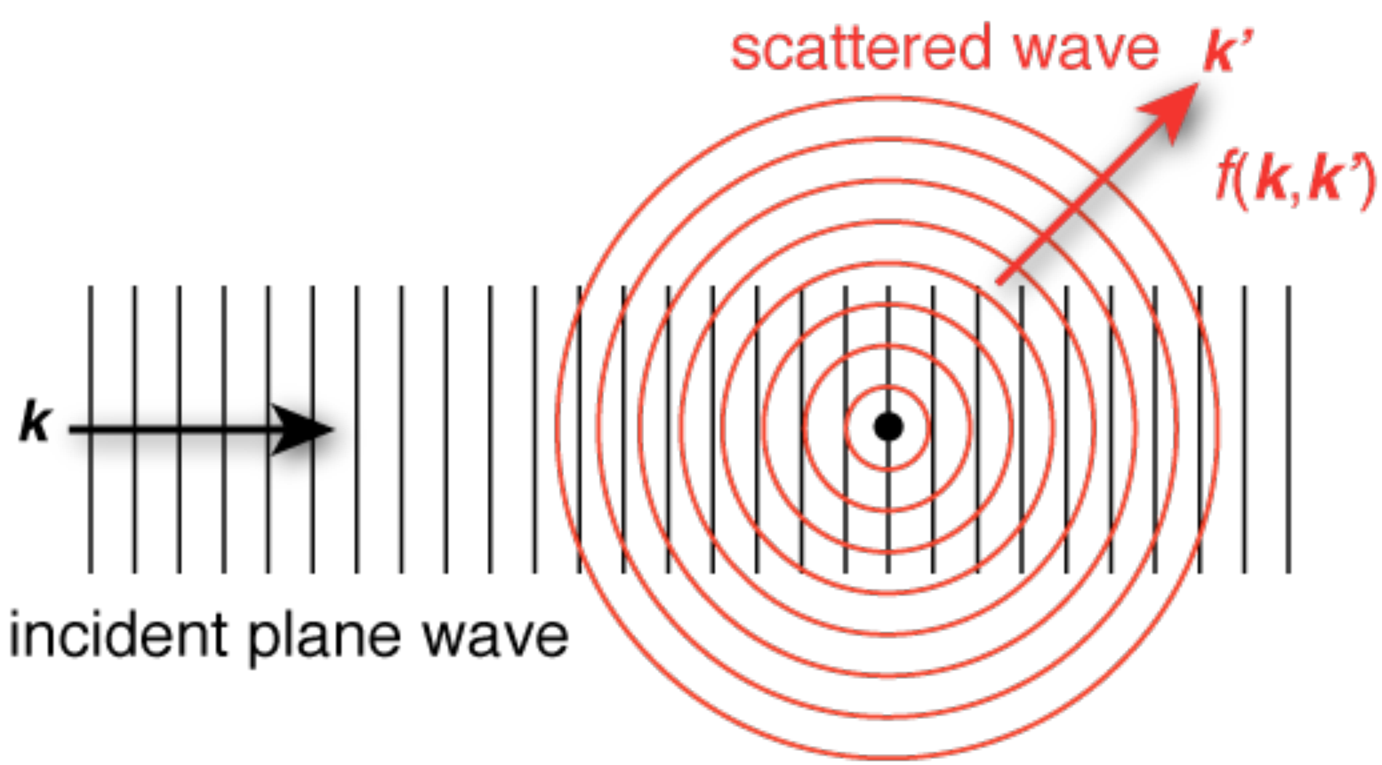
New tunable model systems for many body systems!

Atomic Interactions

Scattering Theory

Schrödinger Equation of Scattering Problem

$$\hat{H}_0 + \hat{U}(\mathbf{r})|\psi_k\rangle = E|\psi_k\rangle$$



Wave function
in far-field (outside
region of scattering
potential)

$$\boxed{\psi_k^+ = e^{i\mathbf{kr}} + f(\mathbf{k},\mathbf{k}')\frac{e^{ikr}}{r}}$$

Scattering Cross Section

Differential Scattering Cross Section

$$\frac{d\sigma}{d\Omega} = \frac{\text{Rate of particles scattered into solid angle } d\Omega}{\text{incident particle flux}}$$

Particle flux

$$\mathbf{j} = \frac{\hbar}{m}\mathrm{Im}\{\Psi^* \nabla \Psi\}$$

we obtain

$$\boxed{\frac{d\sigma}{d\Omega} = |f(\theta)|^2}$$

total scattering cross section

$$\boxed{\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega}$$

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Partial Wave Expansion

For spherically symmetric scattering potential we can write (partial wave decomposition)

$$\psi_k = \sum_{l=0} A_l P_l(\cos\theta) R_l(r)$$

For every angular momentum l, we obtain radial wave equation

$$\left(\frac{\hbar^2}{2m}\left\{-\frac{d^2}{dr^2}-\frac{2}{r}\frac{d}{dr}+\frac{l(l+1)}{r^2}\right\}+U(r)\right)R_l(r)=ER_l(r)$$

For free particle motion (U=0), this corresponds to the differential equation of the spherical Bessel functions.

$$R_l(r) \propto \cos\delta_l j_l(kr) + \sin\delta_l n_l(kr)$$

This yields in the far field limit:

$$\boxed{R_l(r) \underset{r\to\infty}{\propto} \frac{1}{kr}\sin(kr+\delta_l - l\frac{\pi}{2})}$$

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Scattering Phase Shift & Scattering Amplitude

We can relate the scattering phase shift δ_l to the scattering amplitude, via:

$$\boxed{f(\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\theta)}$$

scattering cross section

$$\sigma = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l$$

unitarity limit for partial wave cross sections

$$\sigma_l \leq \frac{4\pi}{k^2}(2l+1)$$

s-Wave Scattering

Far field radial wave function

$$R_0(r) \underset{r\to\infty}{\propto} \frac{1}{kr}\sin(kr+\delta_0)$$

For $r_0 < r < 1/k$ we can approximate the above to

$$R_0 \approx 1 + \frac{\delta_0}{kr} = 1 - \frac{a}{r}$$

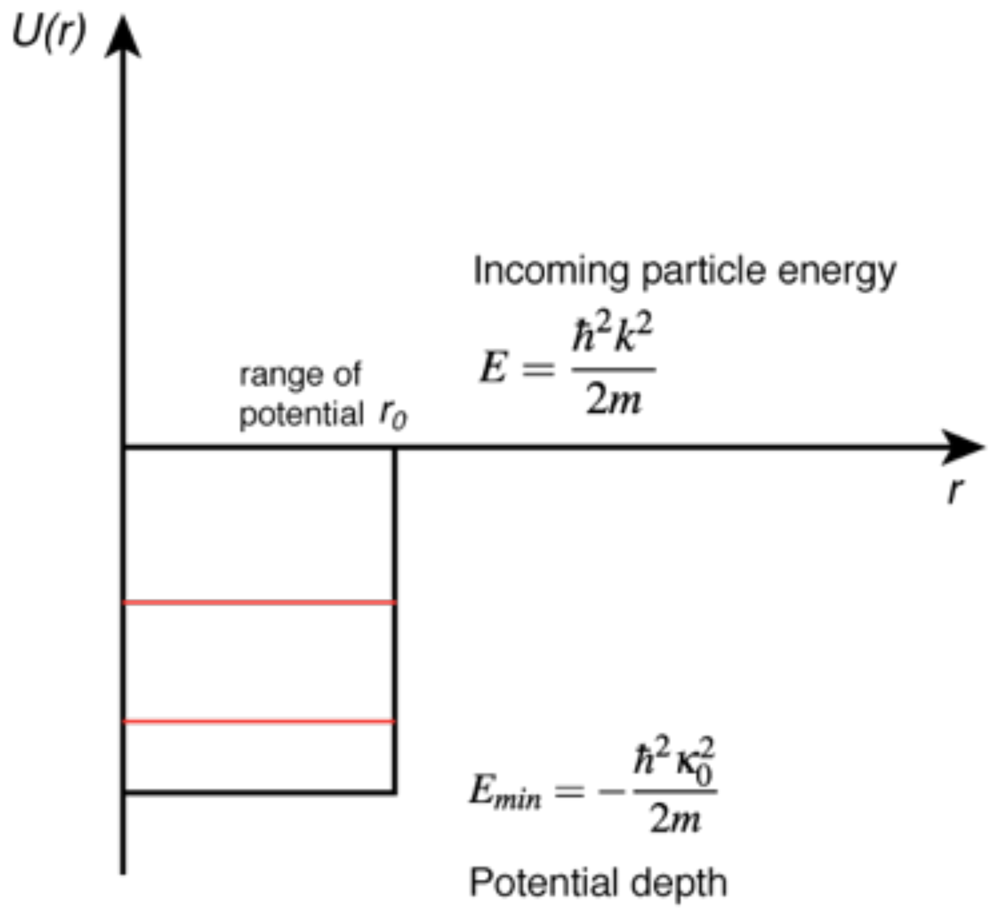
Scattering Length

$$\boxed{a \underset{k\to 0}{=} -\frac{\delta_0}{k}} \qquad \tan\delta_0 \underset{k\to 0}{\simeq} -ka$$

Scattering amplitude (including effective range)

$$\boxed{f(k) = \frac{1}{k\cot\delta_0(k) - ik} \to -\frac{a}{1 - ar_e k^2/2 + ika}}$$

Scattering from Attractive Square Well Potential



$$\chi_0 = rR_0(r)$$

<u>Ansatz:</u>

$$\chi_0(r) = A\,\sin(k_+ r) \qquad \text{for } r < r_0$$

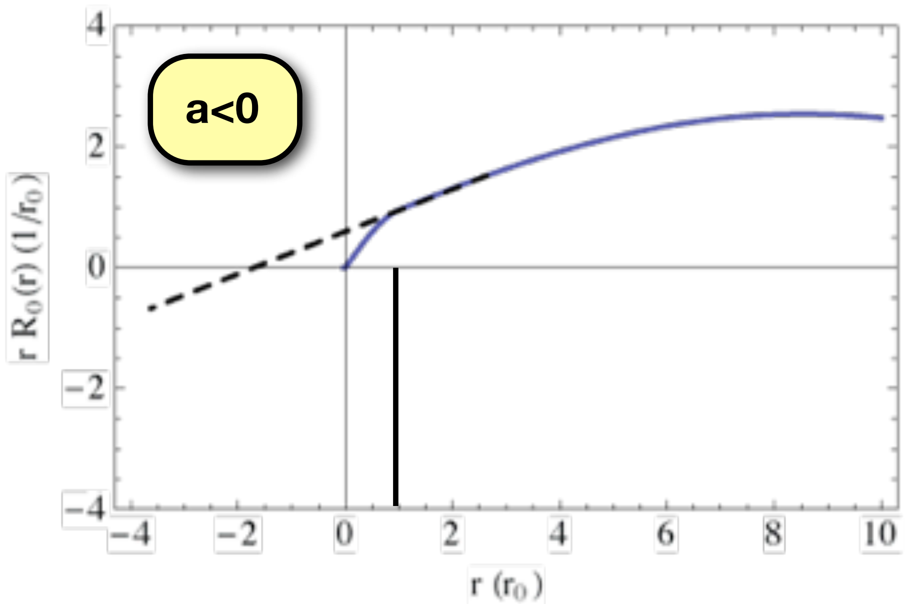
$$\chi_0(r) = B\,\sin(kr + \delta_0) \qquad \text{for } r > r_0$$

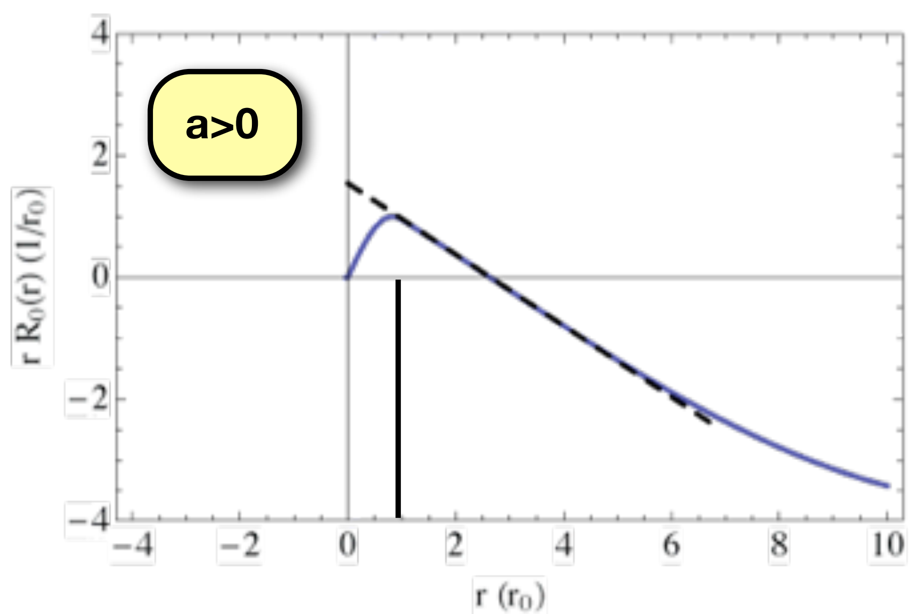
Wave vector in inner region of potential

$$k_+ = \sqrt{\kappa_0^2 + k^2}$$

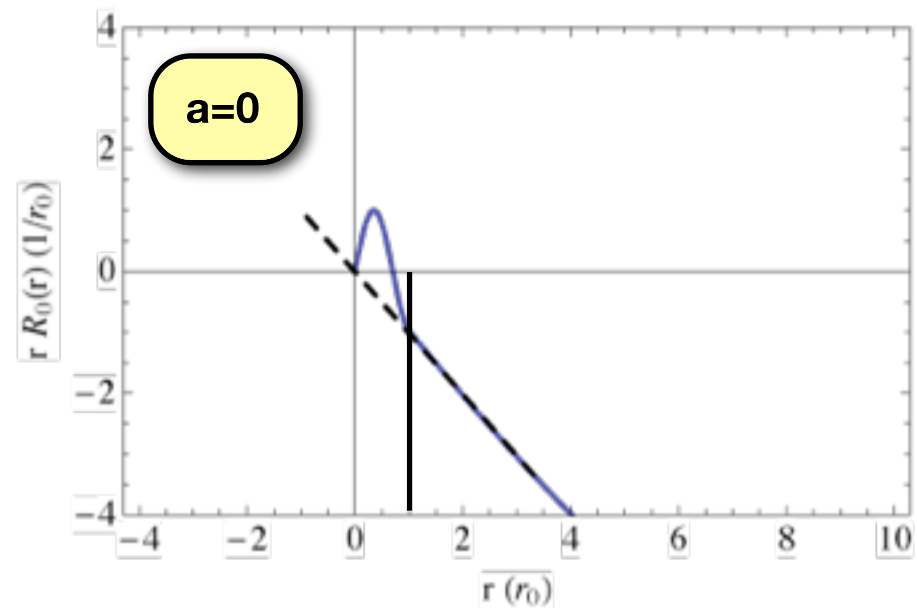
Scattering Wave Functions

no bound state



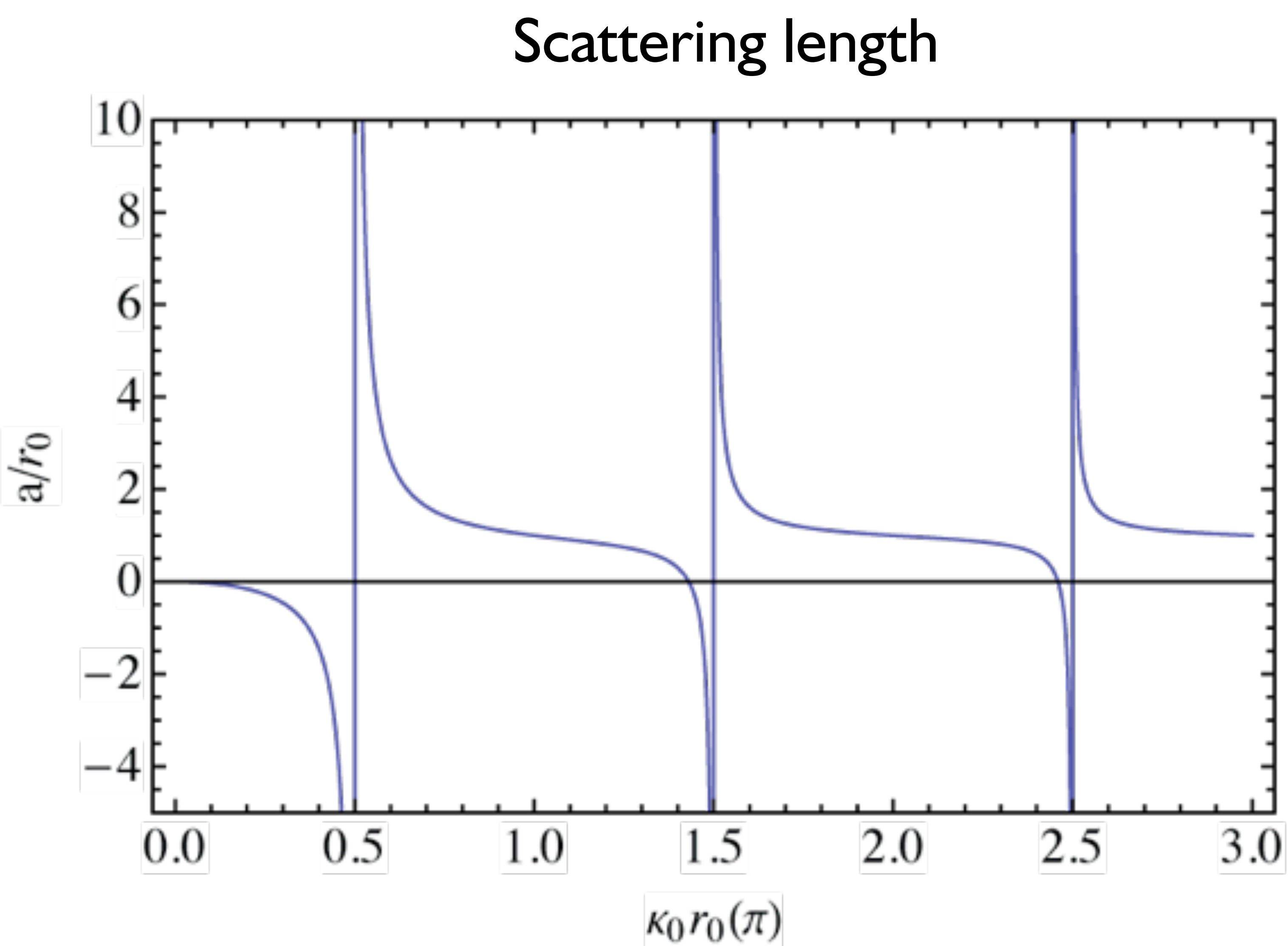


one bound state



$$rR_0(r) \propto r - a$$
for $r_0 < r < 1/k$

Scattering Length (Box Potential)



Resonances for:

$$\kappa_0 r_0 = (n+1/2)\pi$$

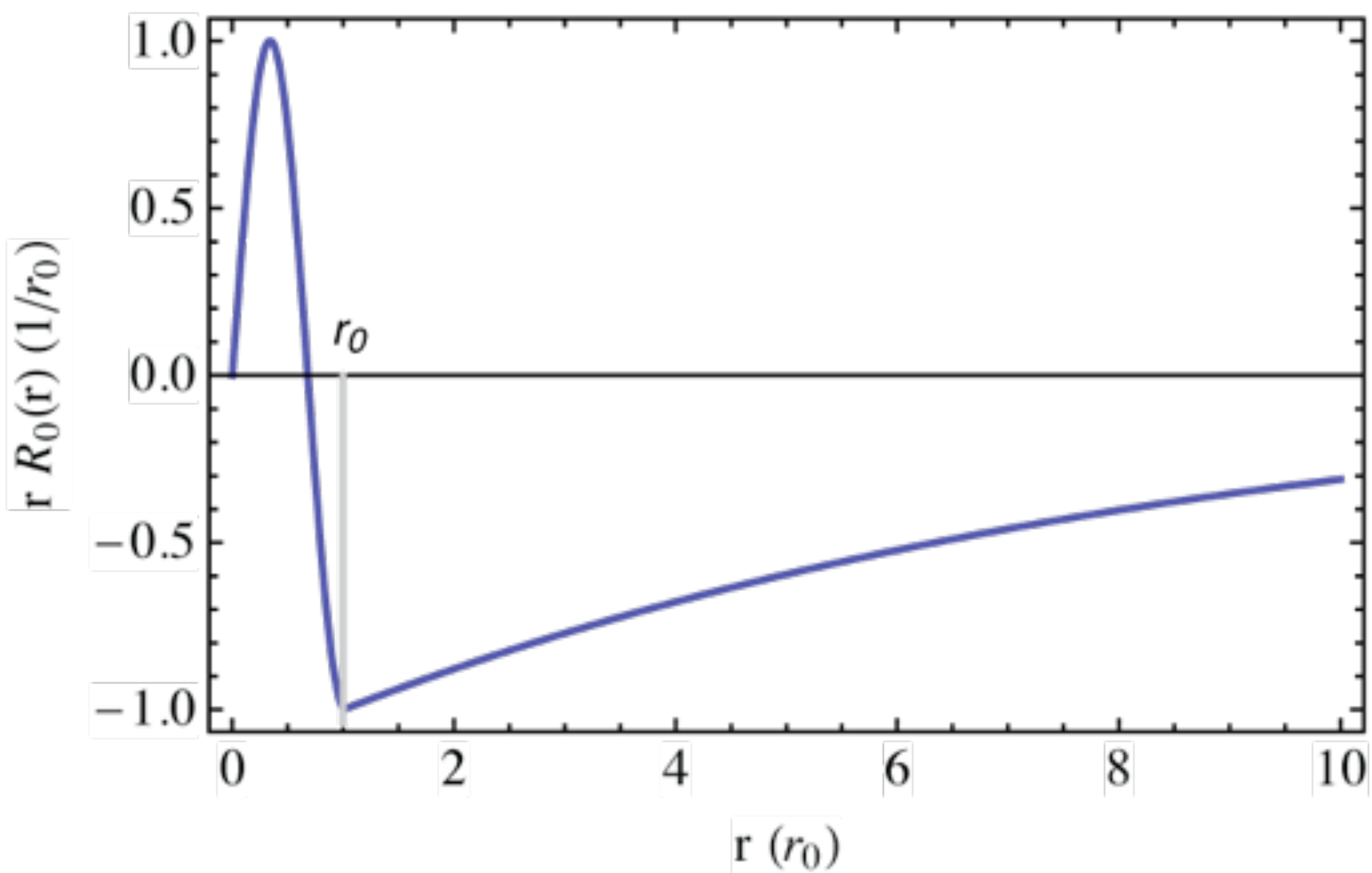
Weakly Bound "Halo" States

Very extended "***Halo states"*** are formed close to a Feshbach Resonance for a>0.
These correspond to weakly bound states that enter the potential well.

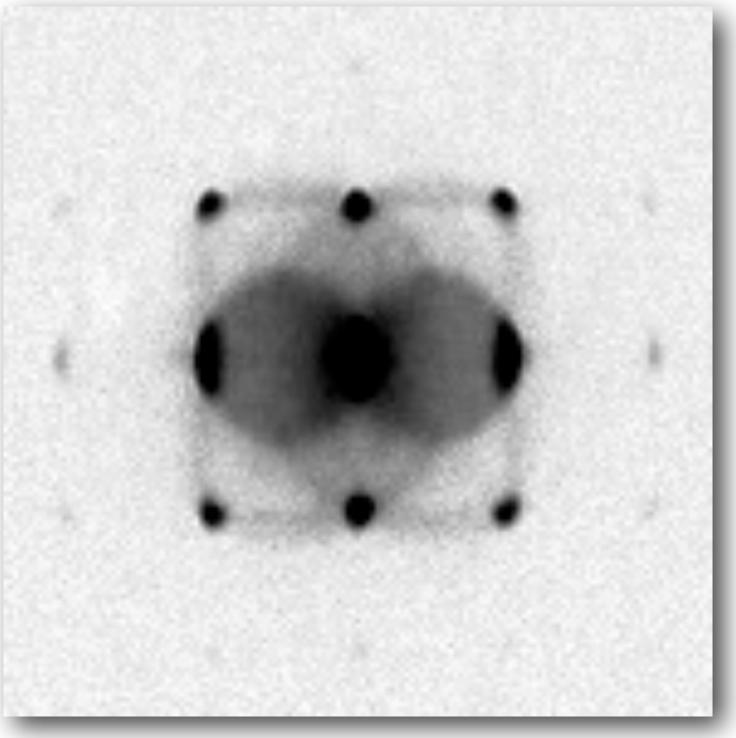
Binding energy of Halo state

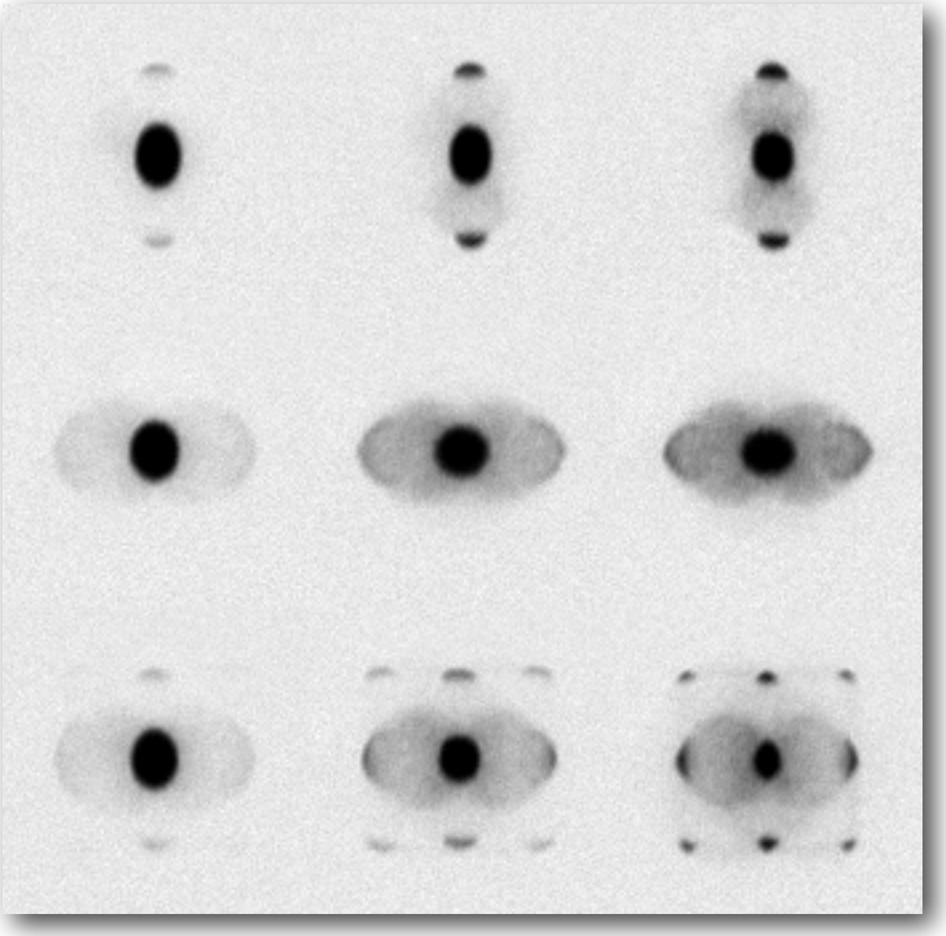
$$E_b = -\frac{\hbar^2}{2ma^2}$$

Wave function of Halo state



Some s-wave Scattering Spheres

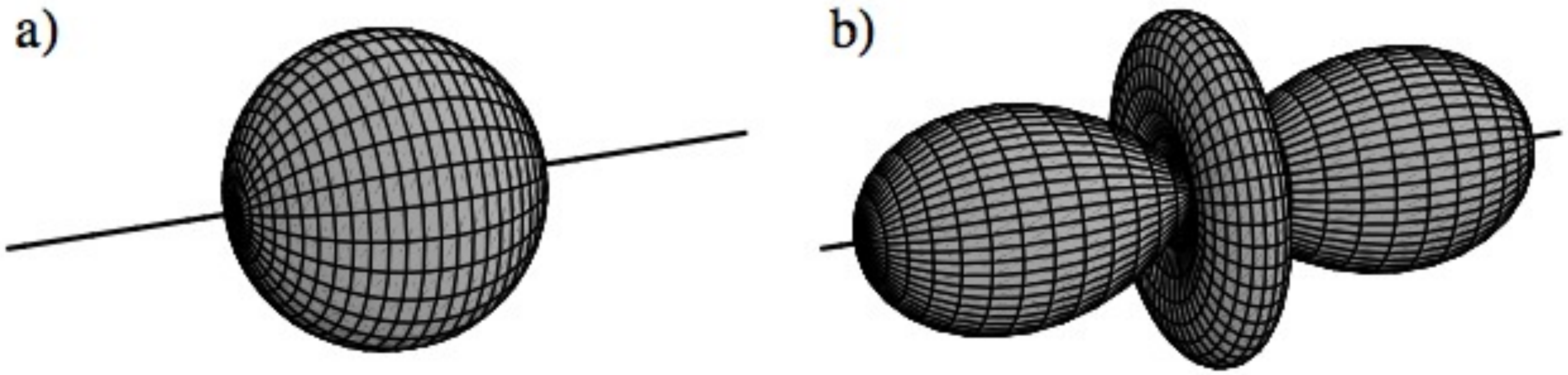




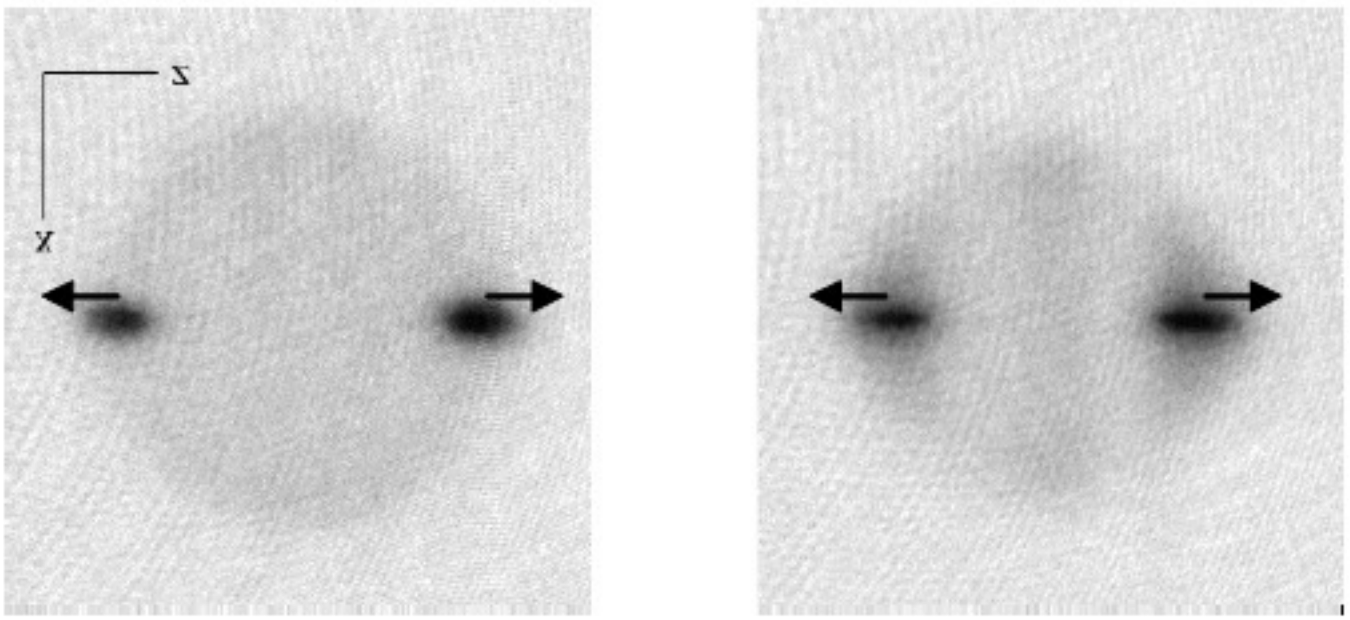
Multiple scattering spheres become between different momentum components become visible!

Images shown after time of flight period.

Scattering of Bosons



s-wave (l=0) d-wave (l=2,m=0)



from. Ch. Buggle (thesis UoA 2005)

Pseudopotential

For ultracold collisions, scattering between particles is characterized by a single parameter - the scattering length.

We can replace the molecular scattering potential with alternative potential that gives same scattering length!

e.g. Pseudopotential

$$U(\mathbf{r})(\cdots) = \frac{4\pi\hbar^2}{2m_r k \cot\delta_0}\delta(\mathbf{r})\frac{\partial}{\partial r}r\cdots \underset{k\to 0}{=} \frac{4\pi\hbar^2 a}{2m_r}\delta(\mathbf{r})\frac{\partial}{\partial r}r\cdots$$

For regular functions at the origin, this latter derivative may be omitted:

$$\boxed{U(\mathbf{r})(\cdots) = \frac{4\pi\hbar^2 a}{2m_r}\delta(\mathbf{r})\cdots}$$

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Identical Particle Scattering

For scattering of identical particles, the scattering wave-function has to obey the right symmetry under particle exchange!

$$\psi_k(\mathbf{r}) = \frac{1}{\sqrt{2}}\left[e^{ikz} \pm e^{-ikz} + (f_k(\theta) \pm f_k(\theta+\pi))\,\frac{e^{ikr}}{r}\right]$$

+ for Bosons, - for Fermions

Leads to constructive or destructive interference in partial wave amplitudes!

Identical Boson: s,d,f... wave scattering (even partial waves)
Identical Fermions: p,g,h... wave scattering (odd partial waves)

$$\sigma = \int |f(\theta) \pm f(\theta \pm \pi)|^2 \, d\Omega = \frac{8\pi}{k^2} \sum_{l=e,o} (2l+1)^2 \sin^2 \delta_l$$

<u>s-wave scattering</u>

distinguishable particles $\sigma = 4\pi a^2$

indistinguishable particles $\sigma = 8\pi a^2$

Consequence: no s-wave scattering for identical fermions!

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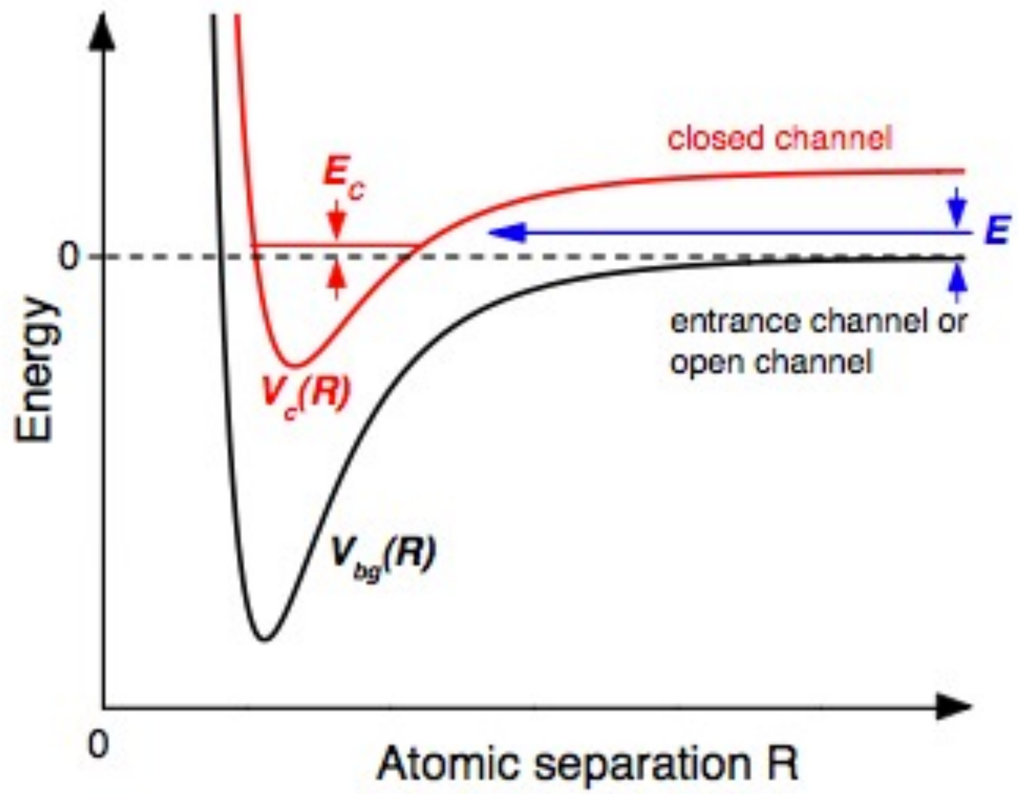
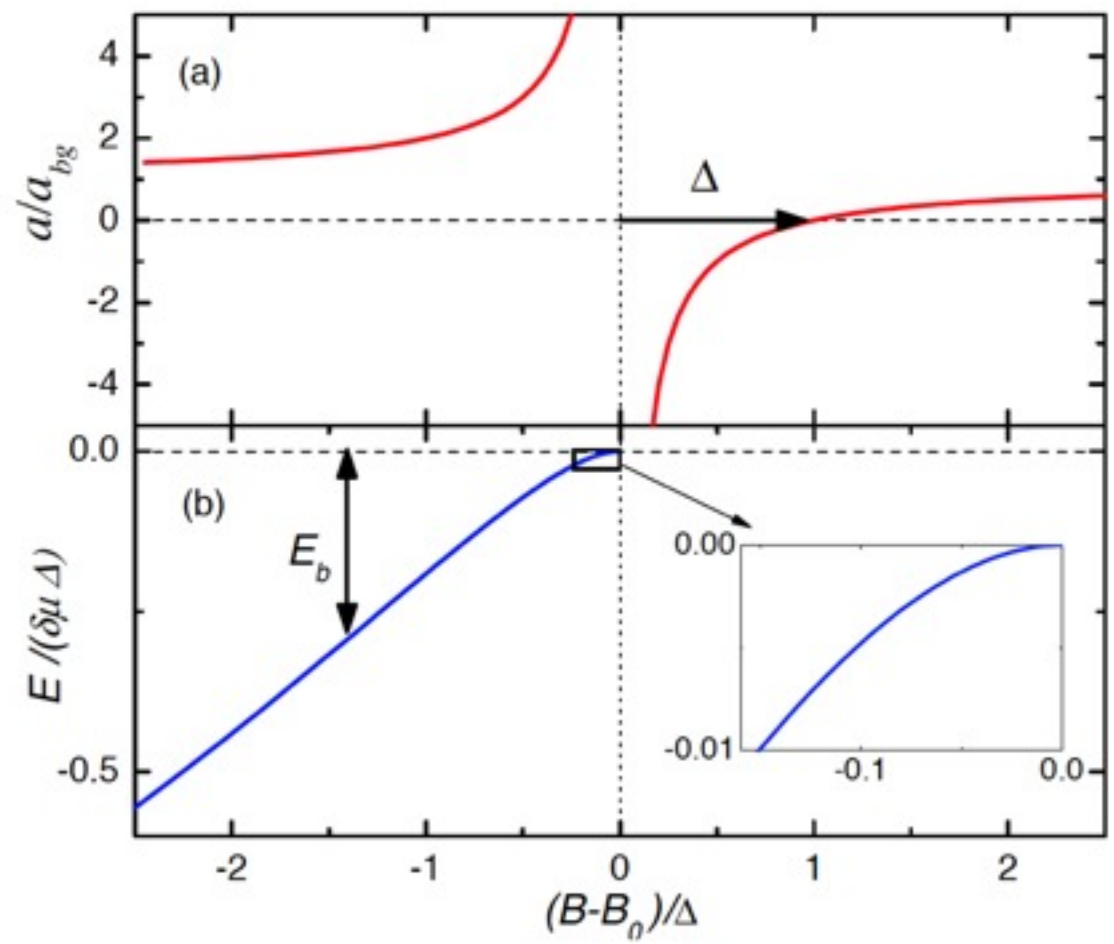
Feshbach Resonance

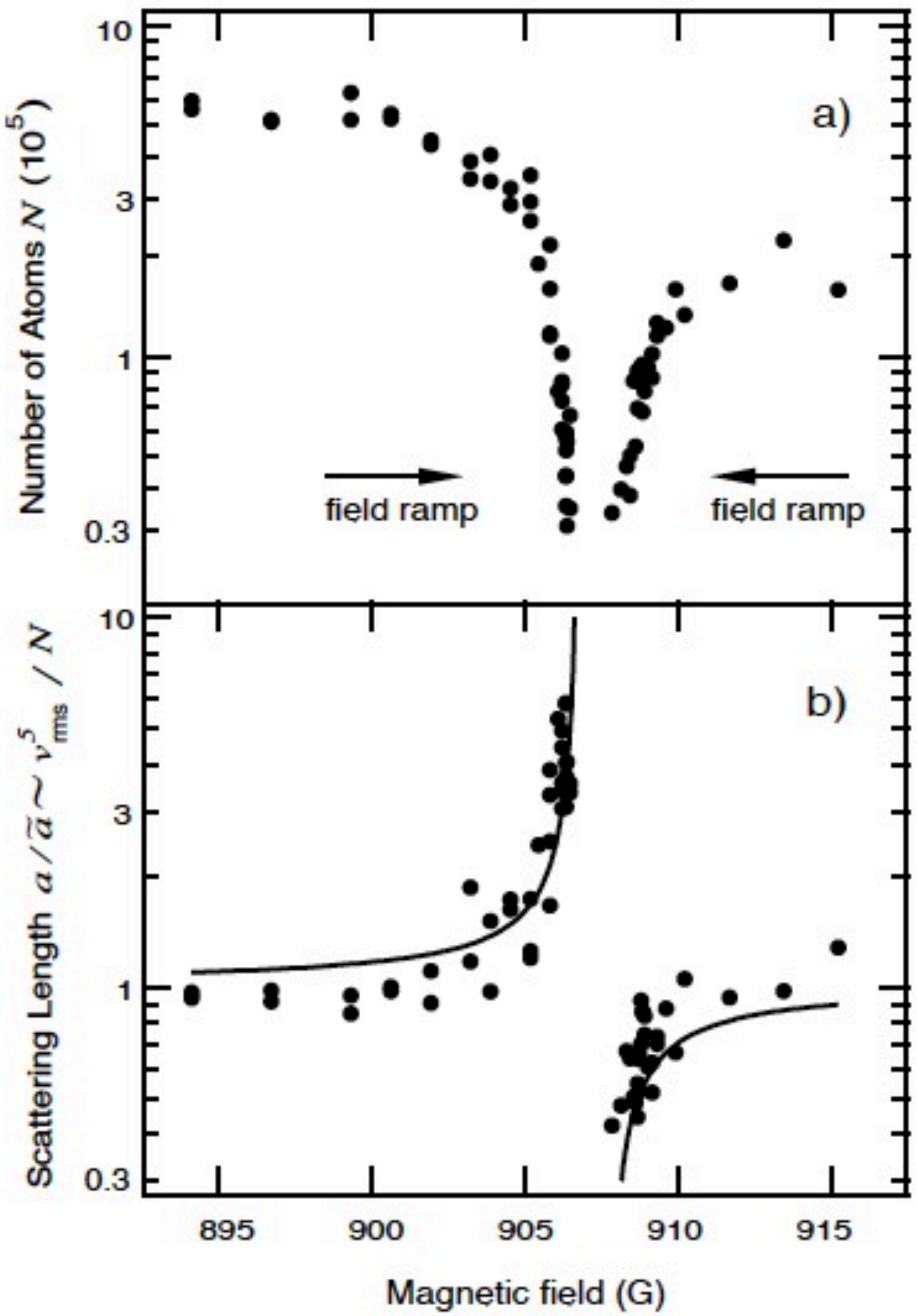




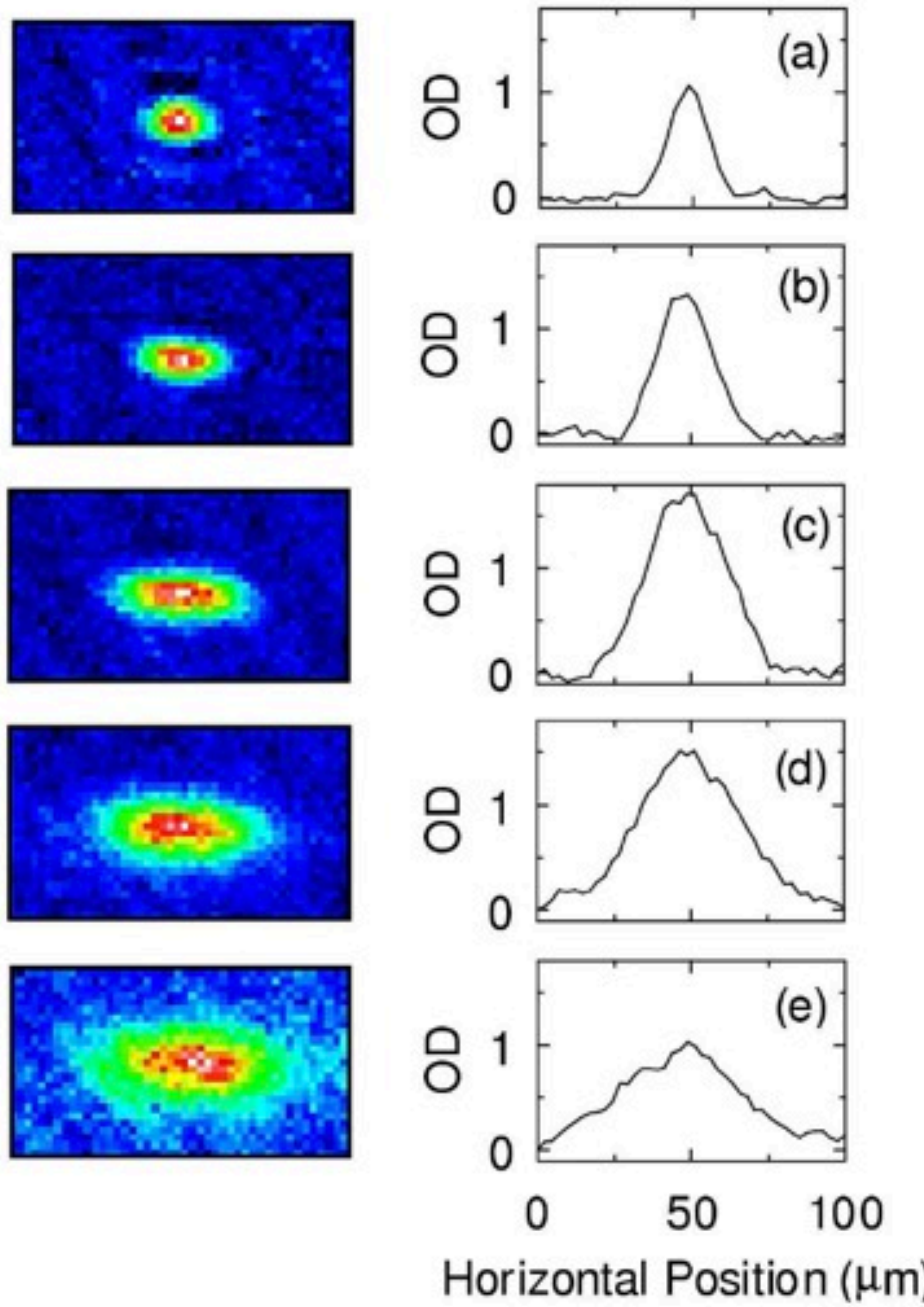
Potential curves of open and closed scattering channels.

Scattering length and binding energy of weakly bound state across Feshbach resonance.

Feshbach Resonances - Experiment

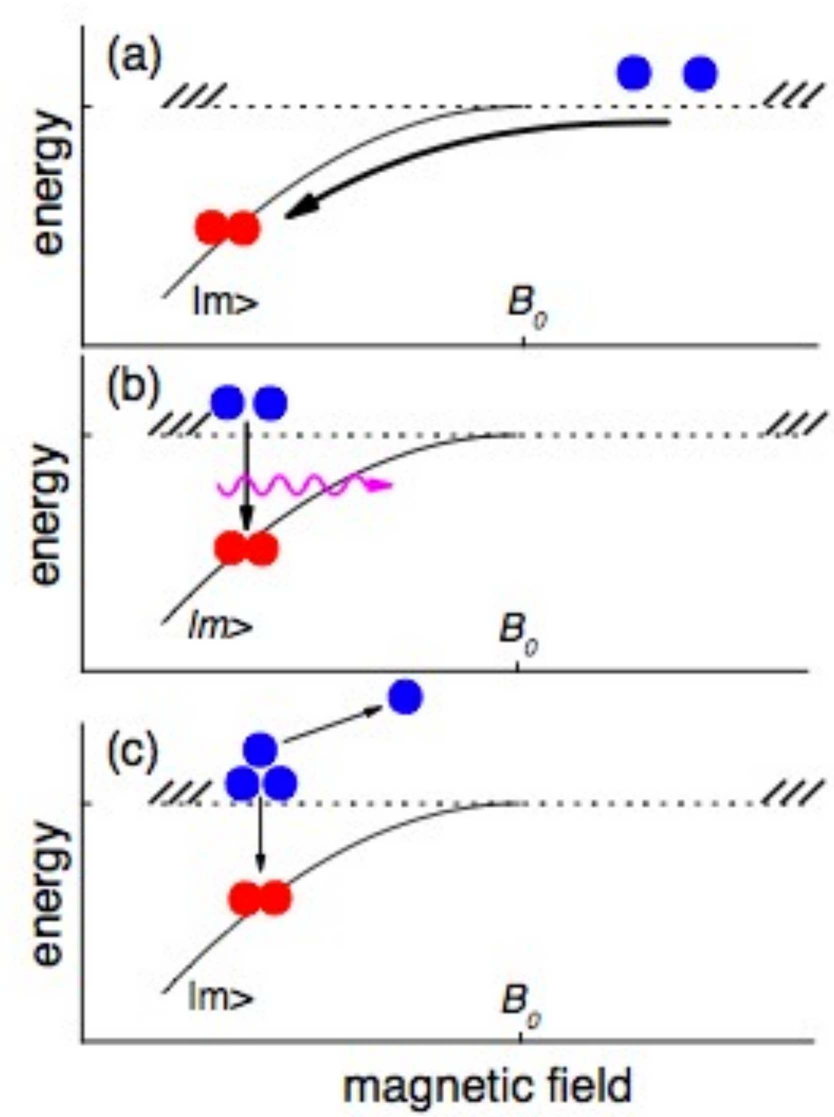


S. Inouye et al. Nature



S. Cornish et al. PRL

Converting Atoms Pairs into Bound Molecules

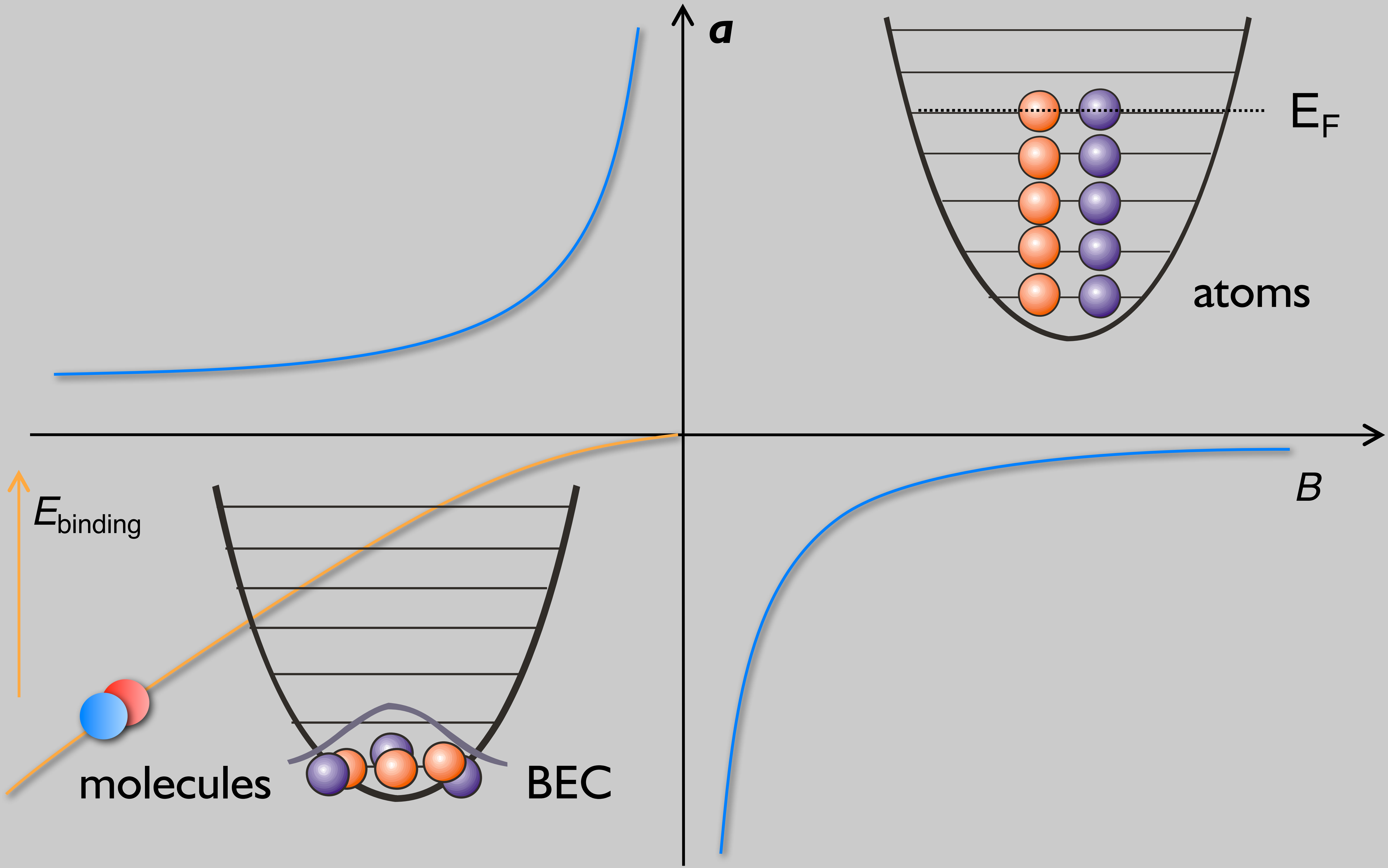


Adiabatic Feshbach Ramp

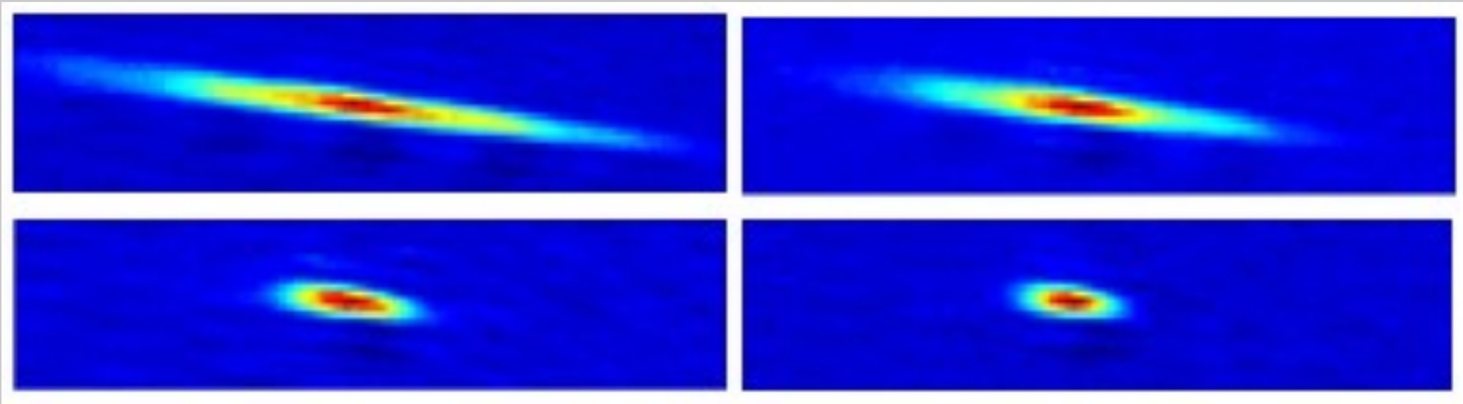
RF Association

Three-Body Recombination

Creating a MBEC out of a Fermi Gas

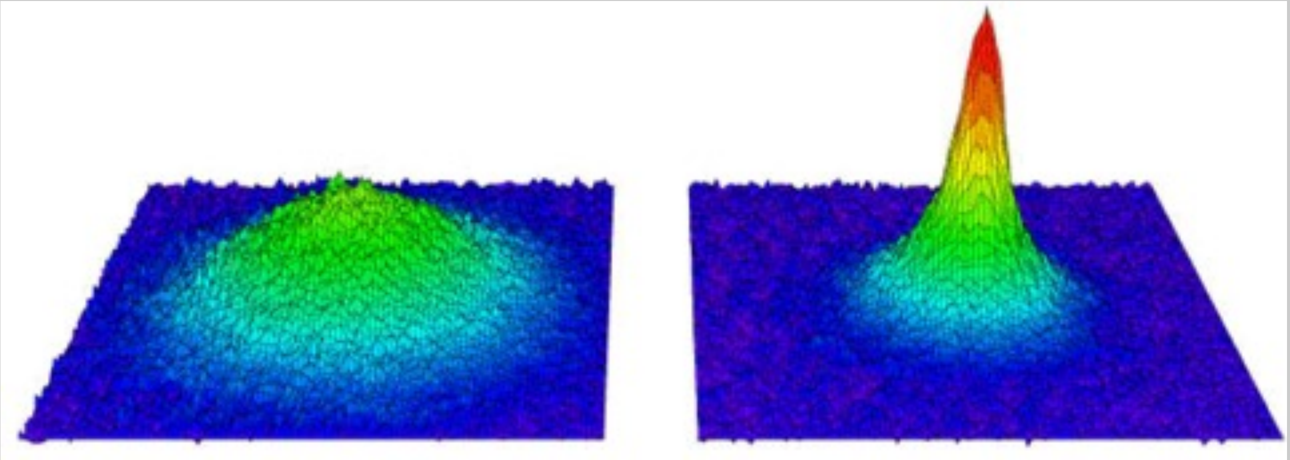


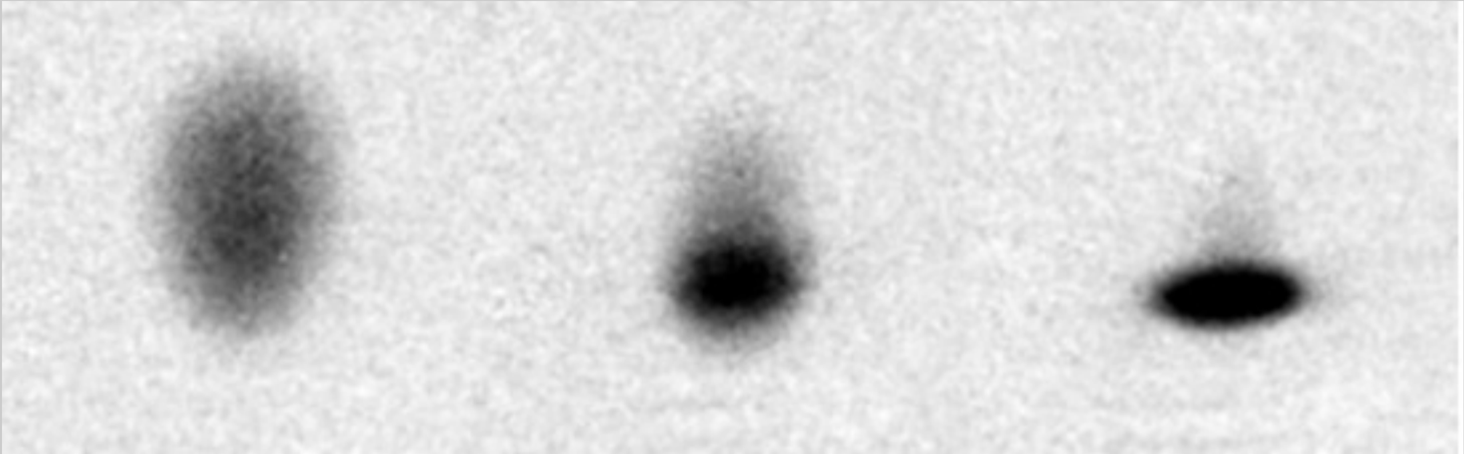
Molecular Bose-Einstein Condensates



S. Jochim et al.,
Science, 2003
(Innsbruck)

M. Greiner, C. Regal and D. Jin
Nature, 2003
(JILA)



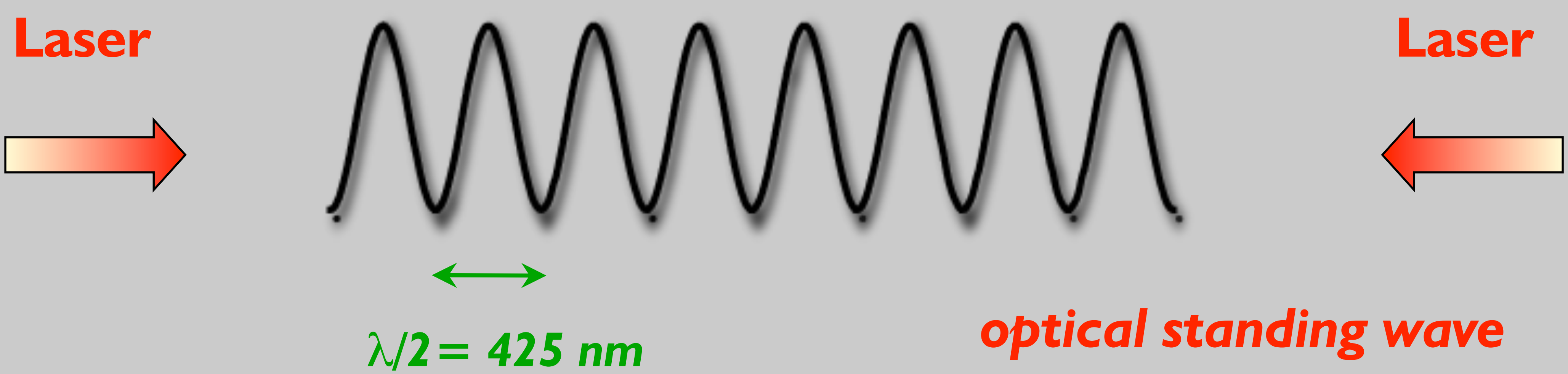


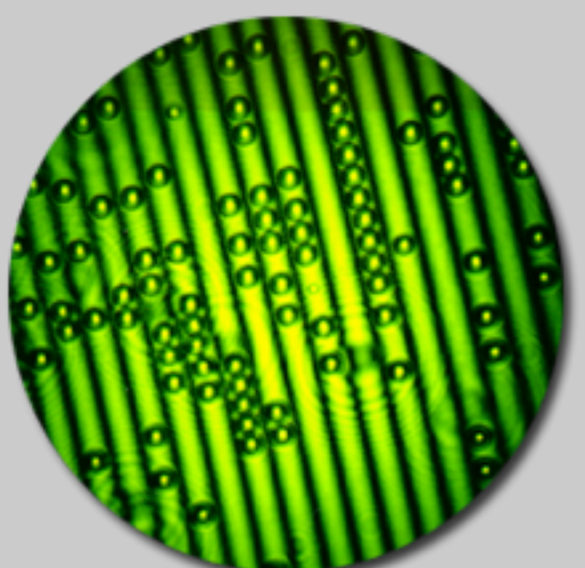
M. W. Zwierlein et al.,
Phys. Rev. Lett, 2003
(MIT)

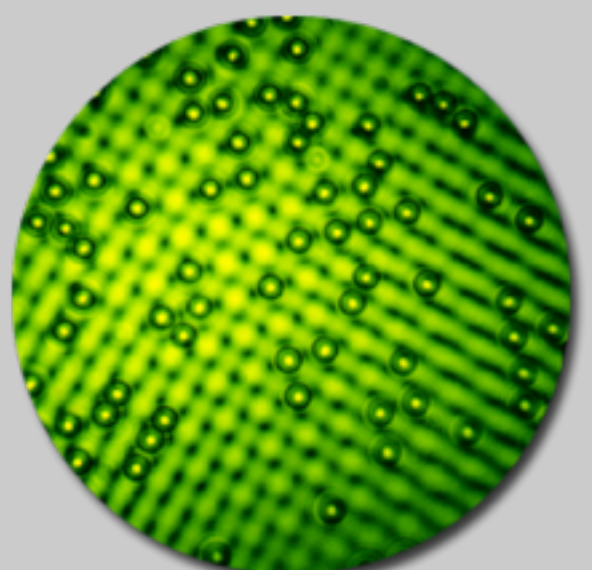
see also Ch. Salomon (ENS) and J. Thomas (Duke)

Atoms in Periodic Potentials

Optical Lattice Potential – Perfect Artificial Crystals



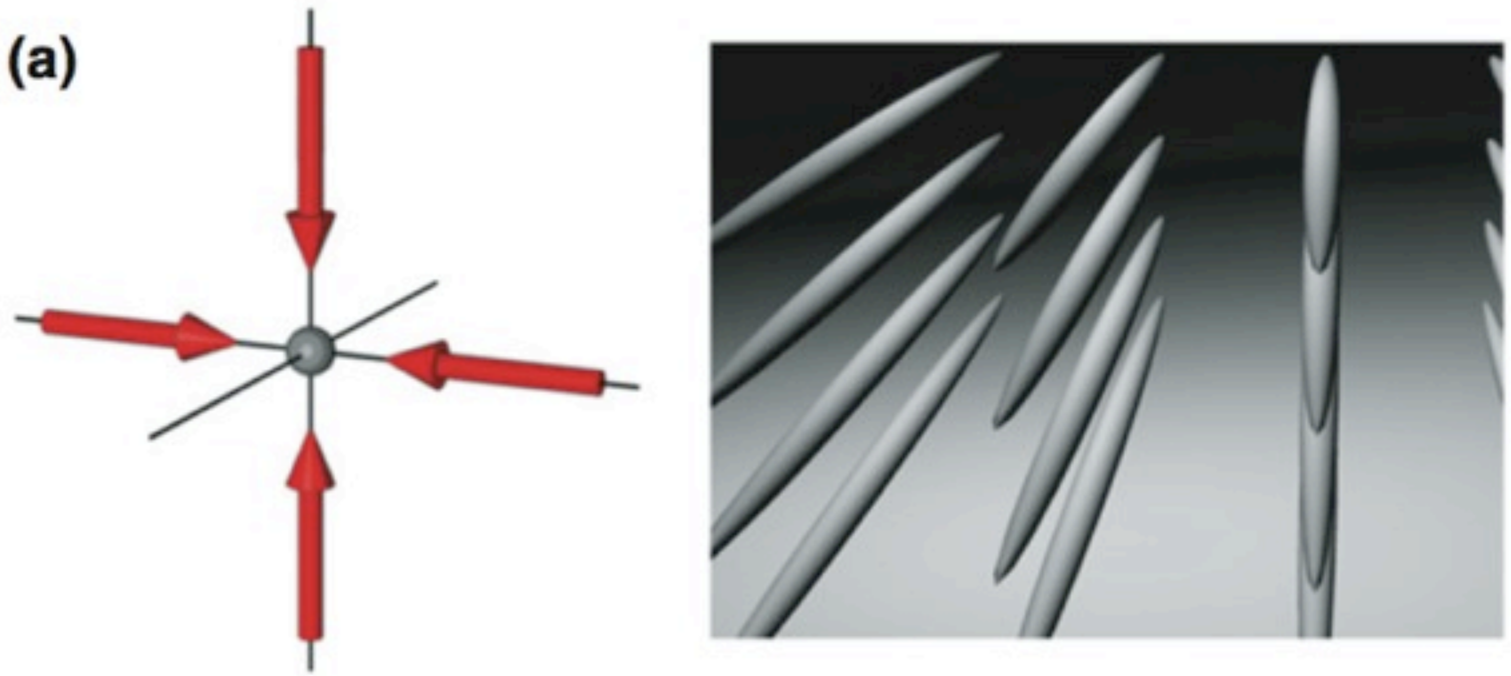




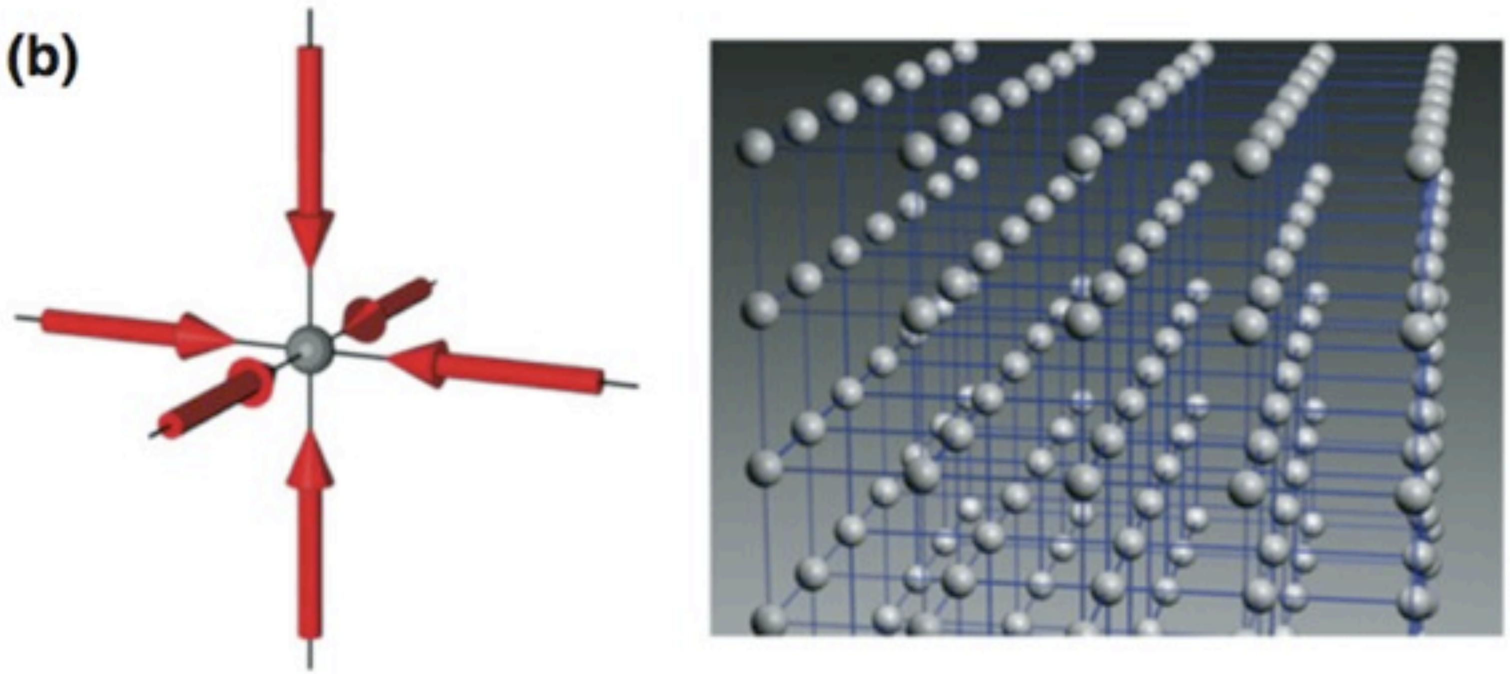
Perfect model systems** for a fundamental understanding of **quantum many body systems

Periodic intensity pattern creates 1D,2D or 3D light crystals for atoms (Here shown for small polystyrol particles).

1D, 2D & 3D Lattices

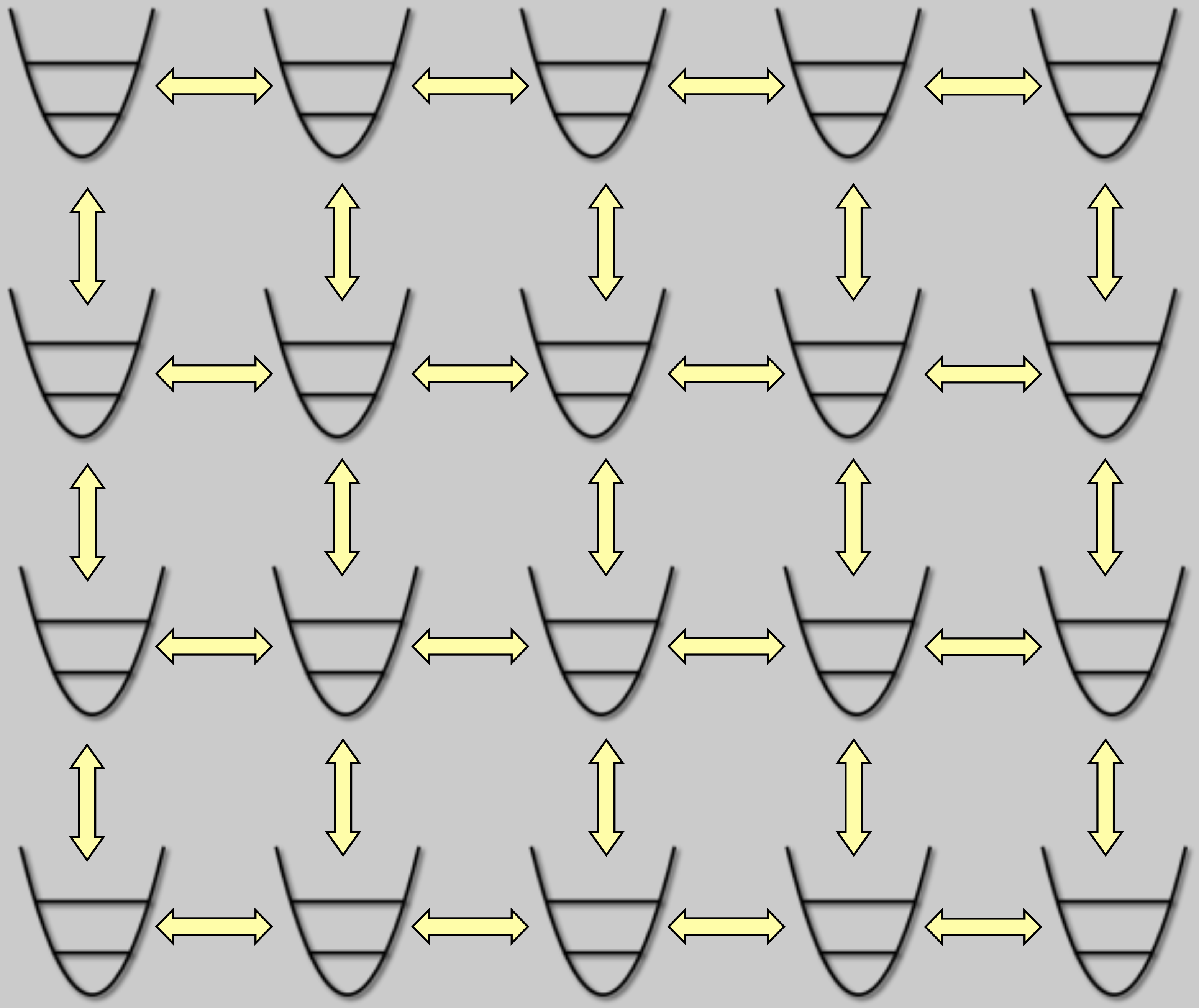


2D Lattices
Array of one-dimensional quantum systems



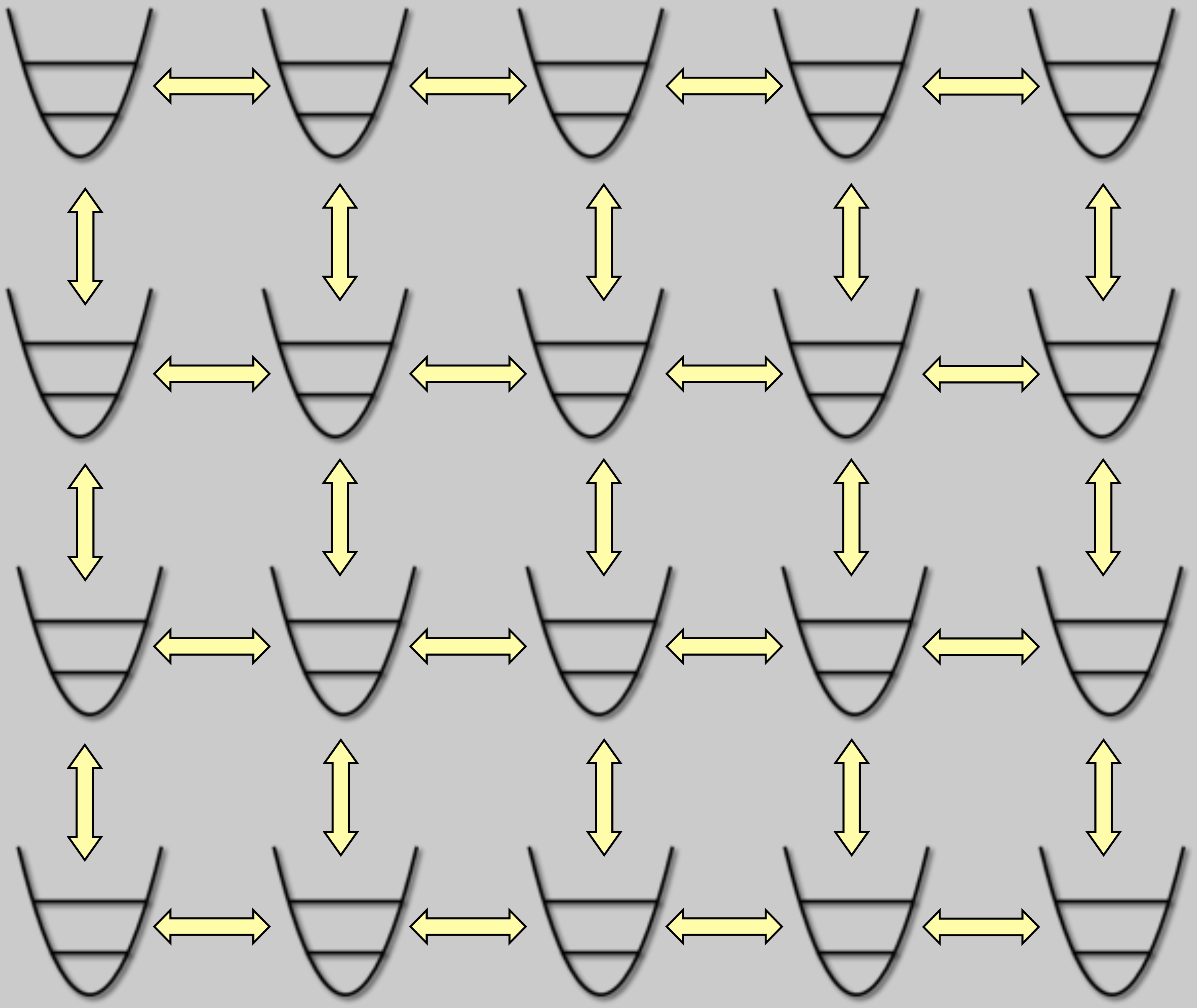
3D Lattices
Array of quantum dots

…and in Higher Dimensions



Tunnel Coupling Tunable!

…and in Higher Dimensions



Tuning the Dimensionality

Single Particle in a Periodic Potential - Band Structure (1)

$$H\phi_q^{(n)}(x) = E_q^{(n)}\phi_q^{(n)}(x) \qquad \text{with} \qquad H = \frac{1}{2m}\hat{p}^2 + V(x)$$

Solved by Bloch waves (periodic functions in lattice period)

$$\phi_q^{(n)}(x) = e^{iqx} \cdot u_q^{(n)}(x)$$

q = Crystal Momentum or Quasi-Momentum
n = Band index

Plugging this into Schrödinger Equation, gives:

$$\boxed{H_B u_q^{(n)}(x) = E_q^{(n)} u_q^{(n)}(x) \qquad \text{with} \qquad H_B = \frac{1}{2m}(\hat{p}+q)^2 + V_{lat}(x)}$$

Single Particle in a Periodic Potential - Band Structure (2)

Use Fourier expansion

$$V(x) = \sum_r V_r e^{i2rkx} \qquad \text{and} \qquad u_q^{(n)}(x) = \sum_l c_l^{(n,q)} e^{i2lkx}$$

yields for the potential energy term

$$V(x) u_q^{(n)}(x) = \sum_l \sum_r V_r e^{i2(r+l)kx} c_l^{(n,q)}$$

and the kinetic energy term

$$\frac{(\hat{p}+q)^2}{2m} u_q^{(n)}(x) = \sum_l \frac{(2\hbar kl + q)^2}{2m} c_l^{(n,q)} e^{i2lkx}.$$

In the experiment standing wave interference pattern gives

$$\boxed{V(x) = V_{lat} \sin^2(kx) = -\frac{1}{4}\left(e^{2ikx} + e^{-2ikx}\right) + \text{c.c.}}$$

Single Particle in a Periodic Potential - Band Structure (3)

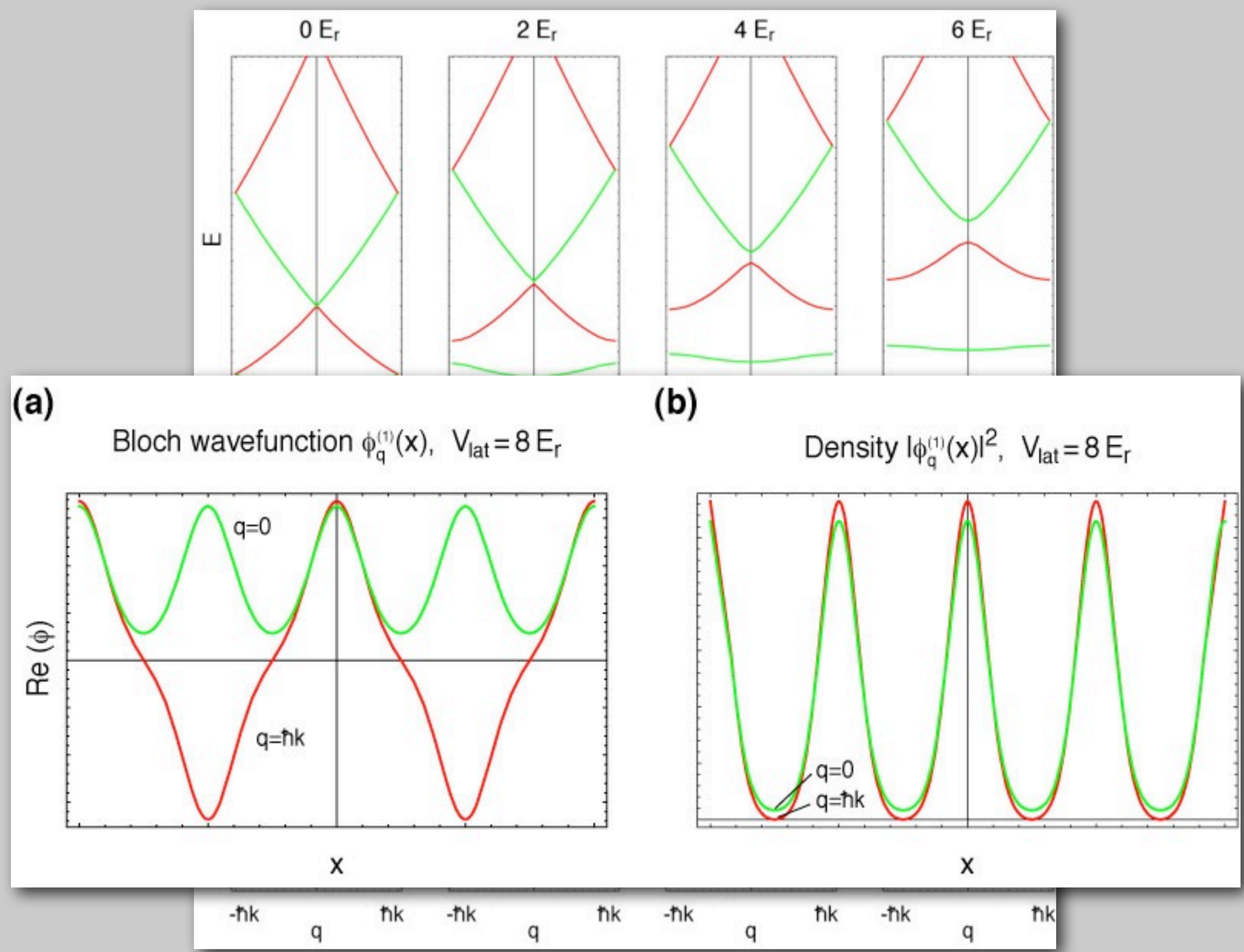
Use Fourier expansion

$$\sum_l H_{l,l'} \cdot c_l^{(n,q)} = E_q^{(n)} c_l^{(n,q)} \qquad \text{with} \qquad H_{l,l'} = \begin{cases} (2l + q/\hbar k)^2 E_r & \text{if } l = l' \\ -1/4 \cdot V_0 & \text{if } |l - l'| = 1 \\ 0 & \text{else} \end{cases}$$

$$\begin{pmatrix} (q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & 0 & \cdots \\ -\frac{1}{4}V_0 & (2+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & \\ 0 & -\frac{1}{4}V_0 & (4+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & \\ & & -\frac{1}{4}V_0 & \ddots & \\ & & & & \end{pmatrix} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix} = E_q^{(n)} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix}$$

Diagonalization gives us Eigenvalues and Eigenvectors!

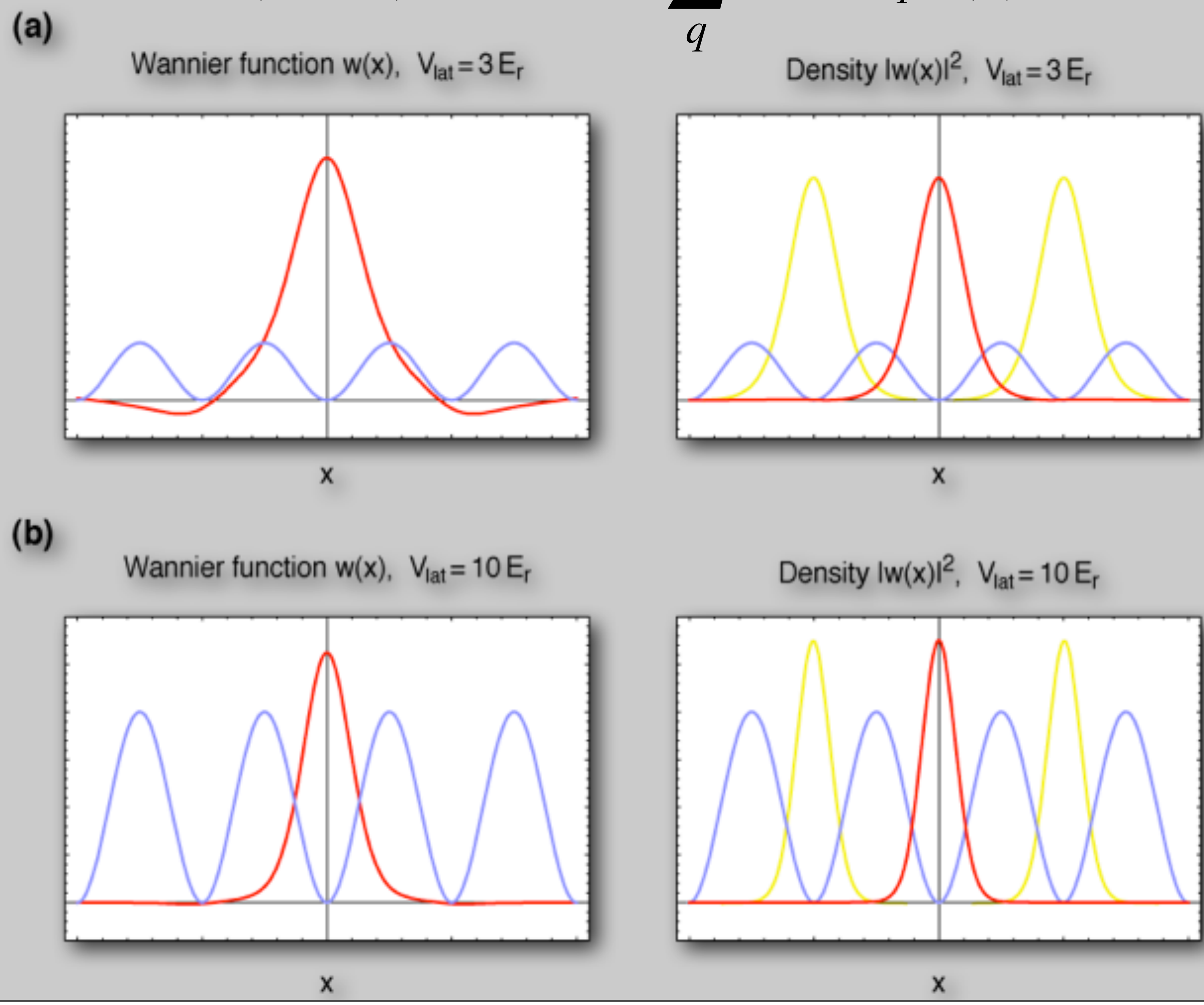
Bandstructure - Blochwaves



Wannier Functions

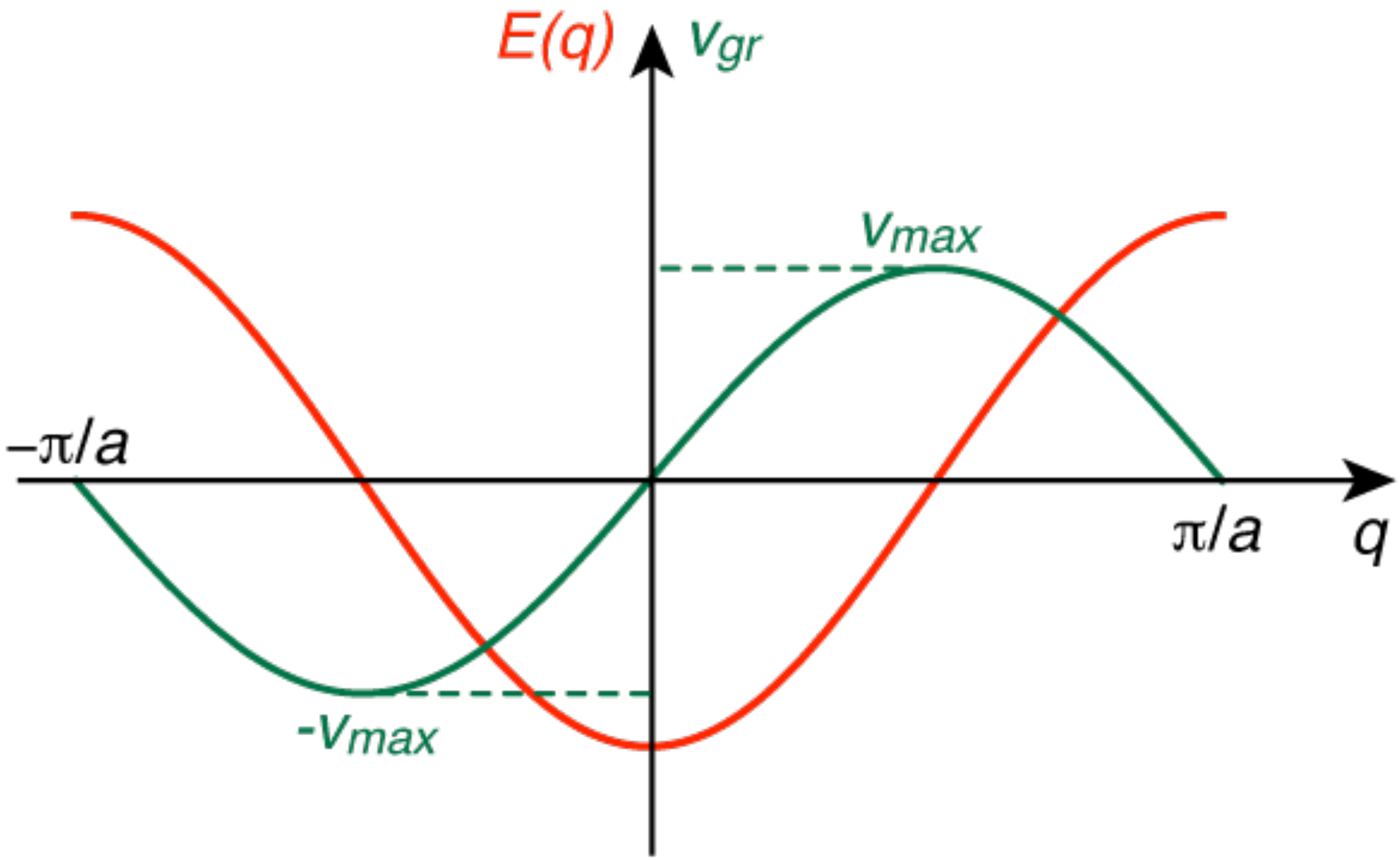
An alternative basis set to the Bloch waves can be constructed through localized wave-functions: Wannier Functions!

$$w_n(x - x_i) = \mathcal{N}^{-1/2} \sum_q e^{-iqx_i} \phi_q^{(n)}(x)$$



Dispersion Relation in a Square Lattice

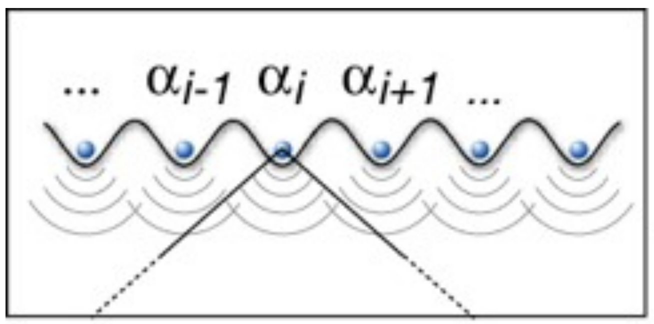
$$E(q) = -2J\cos(qa)$$



Measuring Momentum Distributions

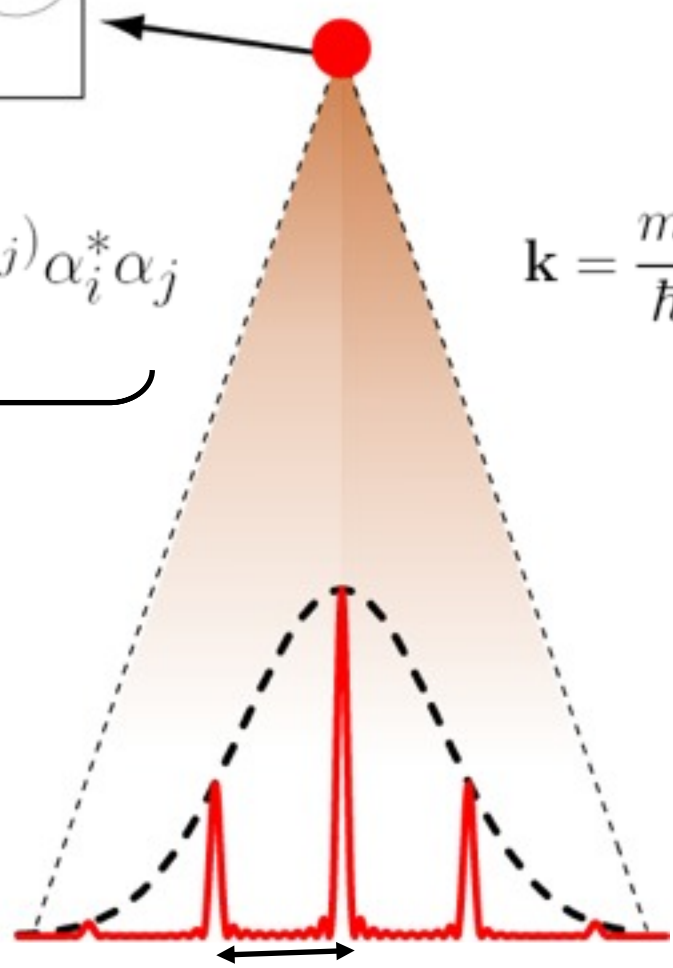
Time of flight interference pattern

- Interference between all waves coherently emitted from each lattice site

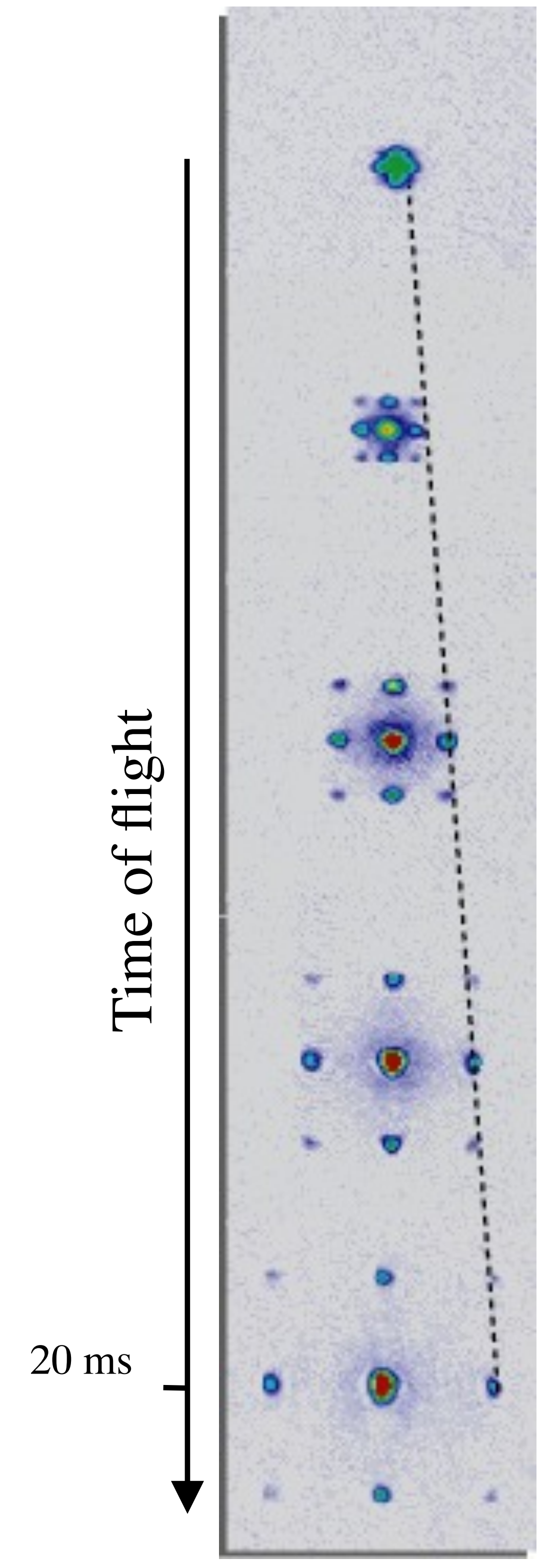


$$\tilde{n}(\mathbf{k}) = \underbrace{|\tilde{w}(\mathbf{k})|^2}_{\text{Wannier envelope}} \underbrace{\sum_{i,j} e^{i\mathbf{k}.(\mathbf{r}_i-\mathbf{r}_j)}\alpha_i^*\alpha_j}_{\text{Grating-like interference}}$$

$$\mathbf{k} = \frac{m\mathbf{r}}{\hbar t}$$



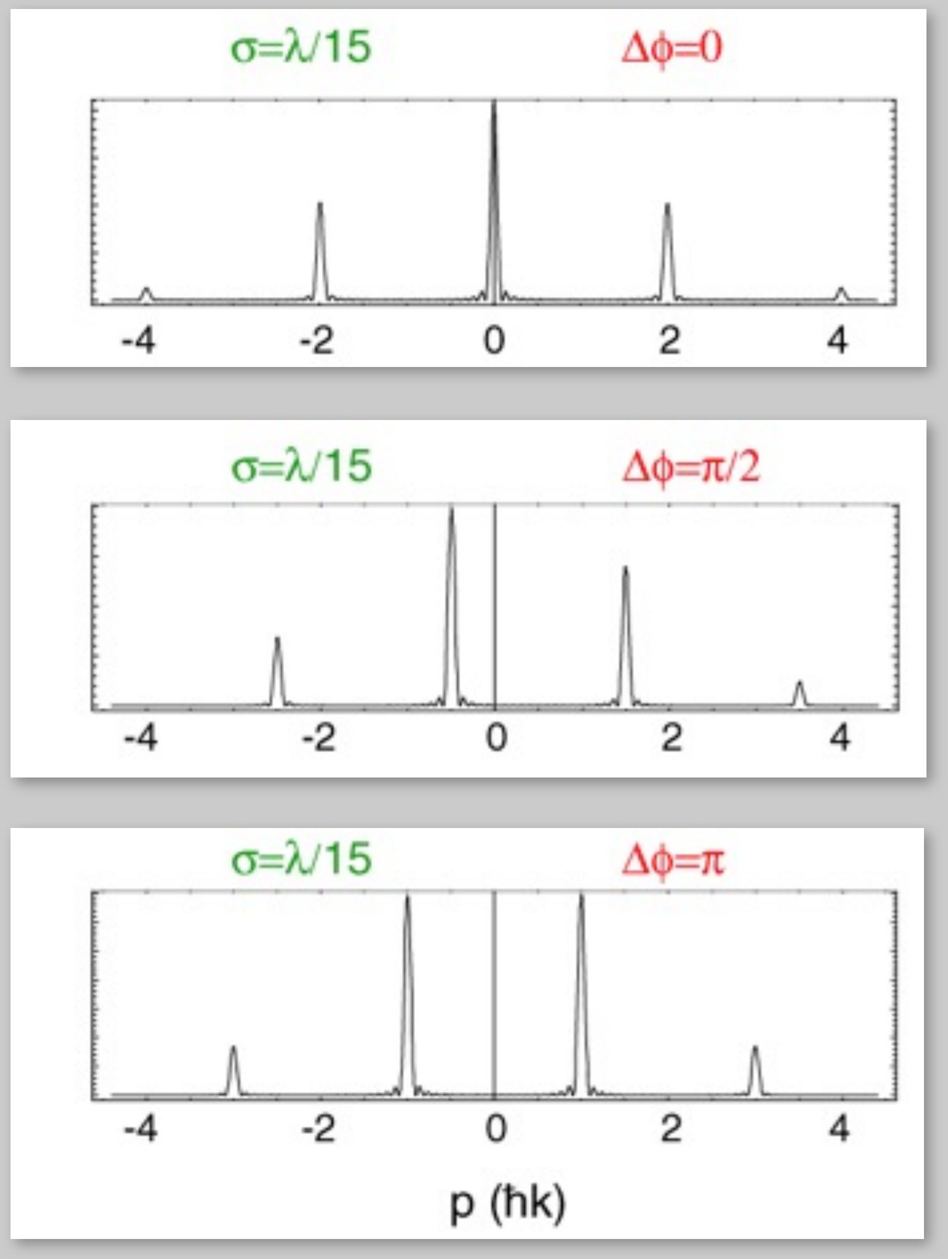
Periodicity of the reciprocal lattice $l = \dfrac{2\hbar k_{\mathrm{L}} t}{m}$



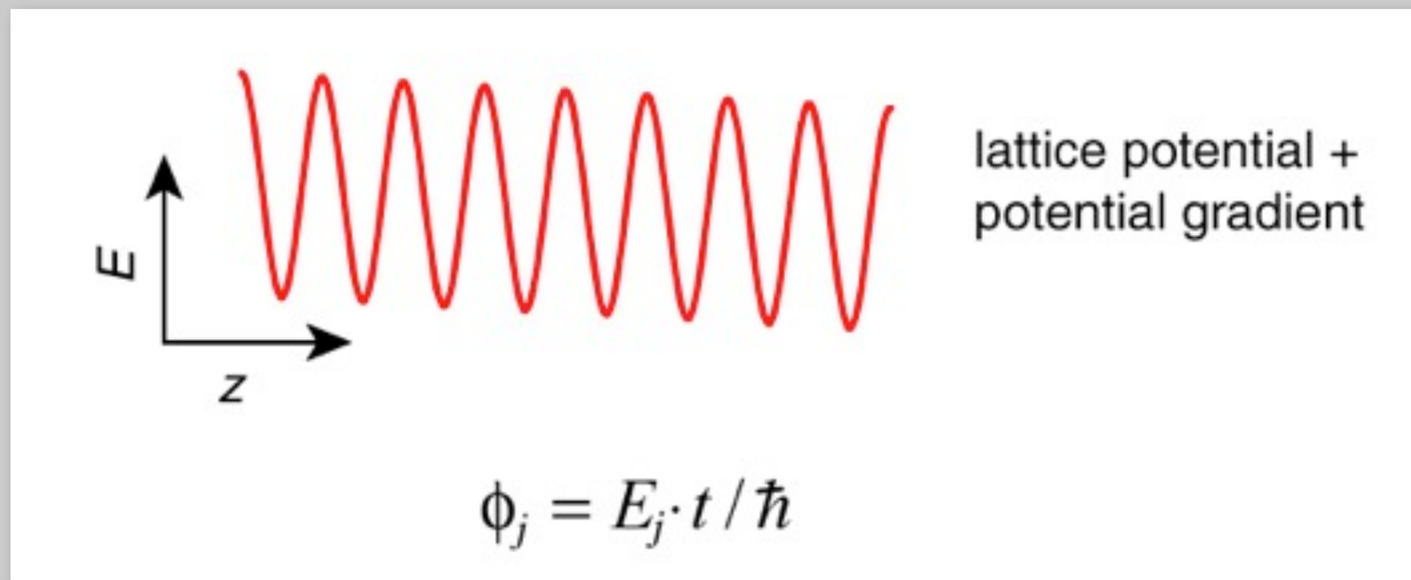
Momentum Distributions – 1D

Momentum distribution can be obtained by Fourier transformation of the macroscopic wave function.

$$\Psi(x) = \sum_i A(x_j) \cdot w(x - x_j) \cdot e^{i\phi(x_j)}$$



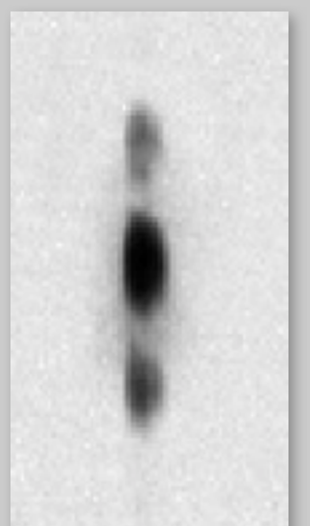
Preparing Arbitrary Phase Differences Between Neighbouring Lattice Sites



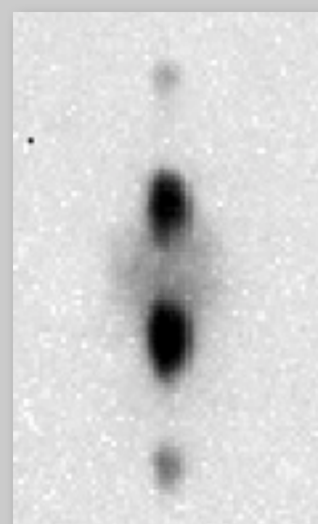
Phase difference between neighboring lattice sites

$$\Delta\phi_j = (V'\lambda/2)\,\Delta t$$

(cp. Bloch-Oscillations)



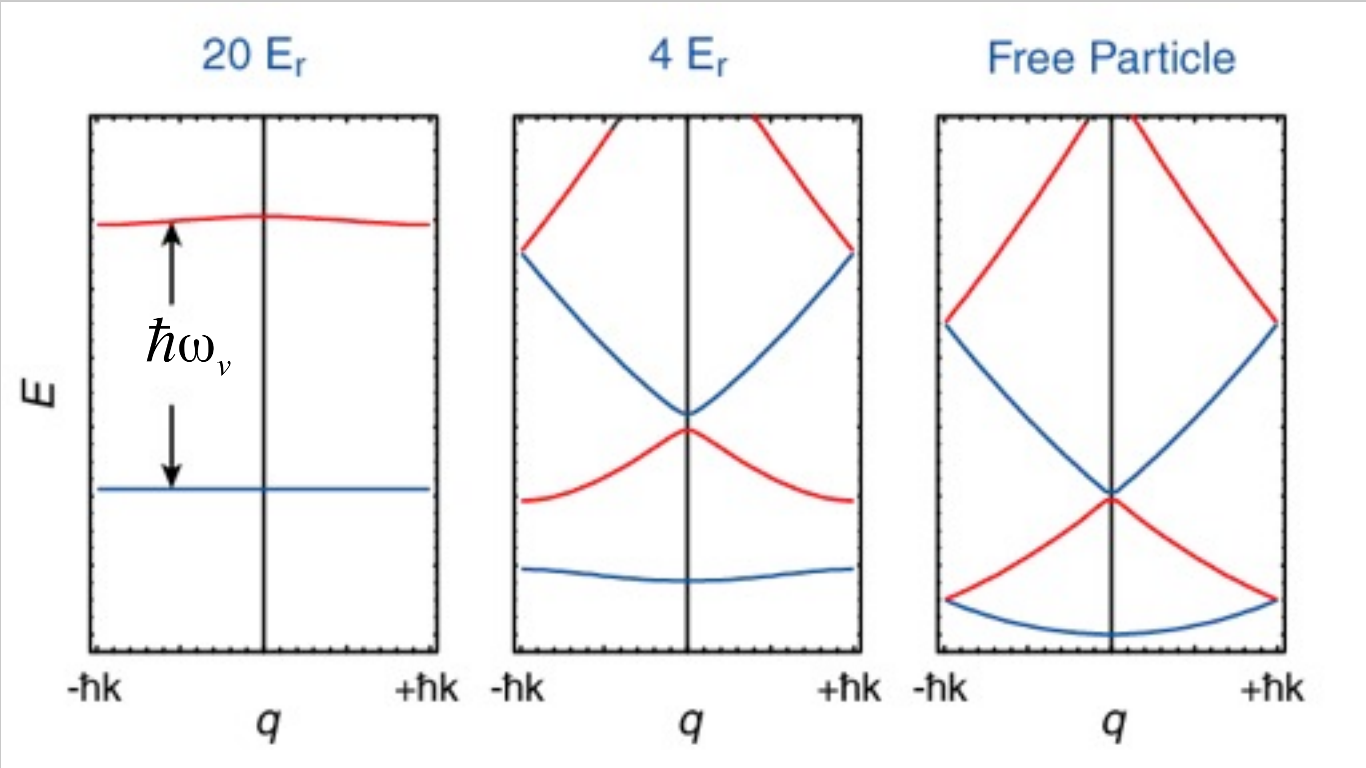
$\Delta\phi = 0$



$\Delta\phi = \pi$

But: dephasing if gradient is left on for long times !

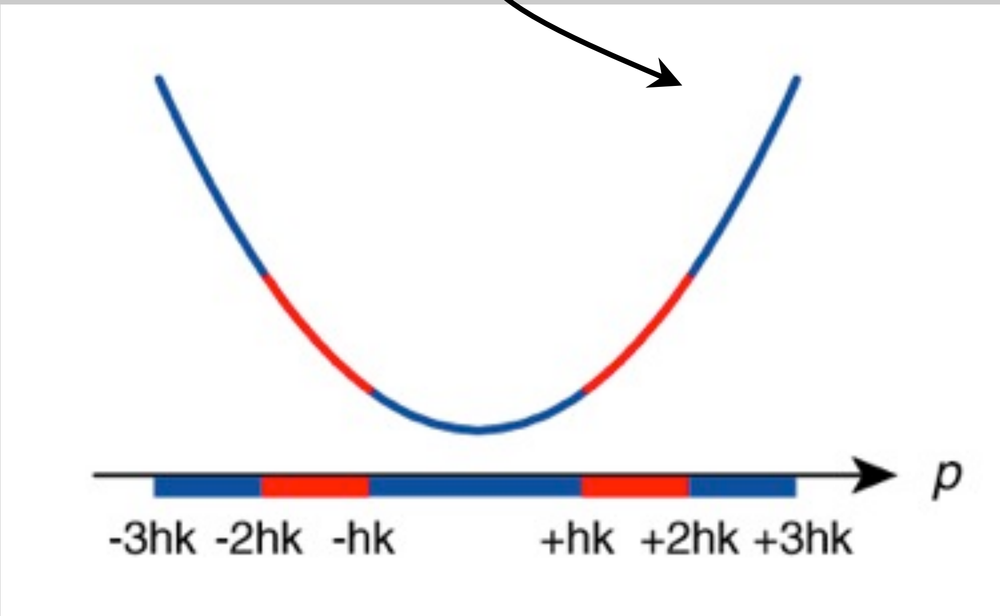
Mapping the Population of the Energy Bands onto the Brillouin Zones



Crystal momentum

Crystal momentum is conserved while lowering the lattice depth adiabatically !

Population of nth band is mapped onto nth Brillouin zone !

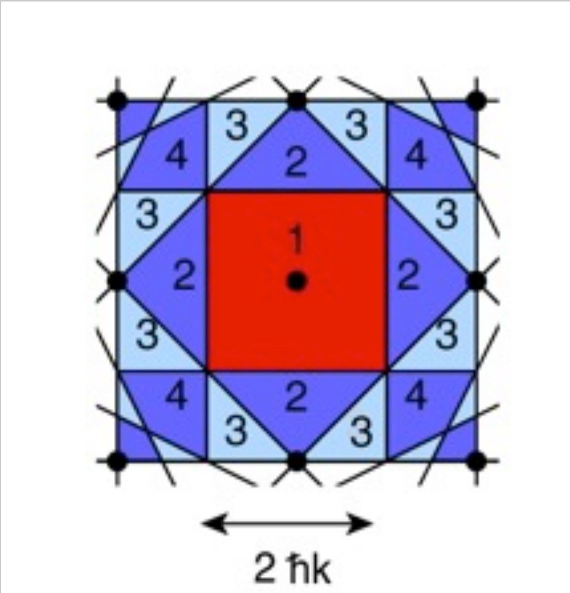


Free particle momentum

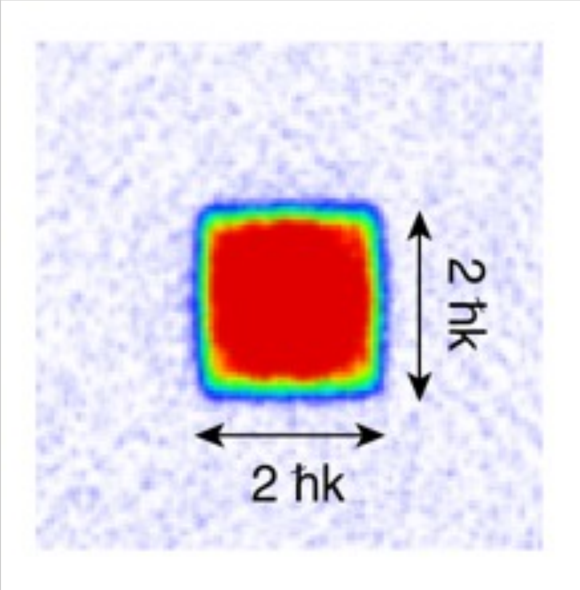
A. Kastberg et al. PRL 74, 1542 (1995)
M. Greiner et al. PRL 87, 160405 (2001)

Experimental Results

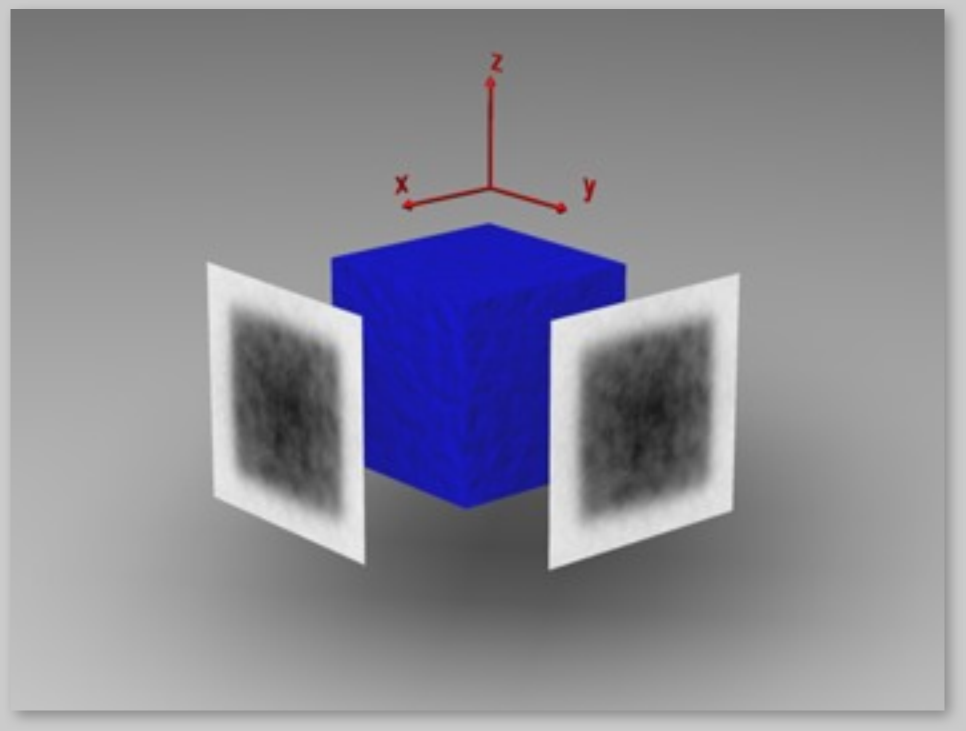
Brillouin Zones in 2D



Momentum distribution of a dephased condensate after turning off the lattice potential adiabtically



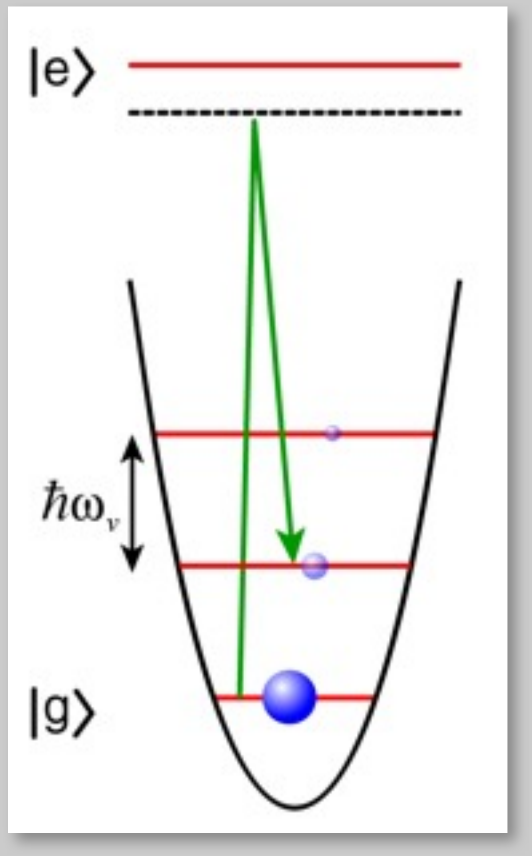
2D



3D

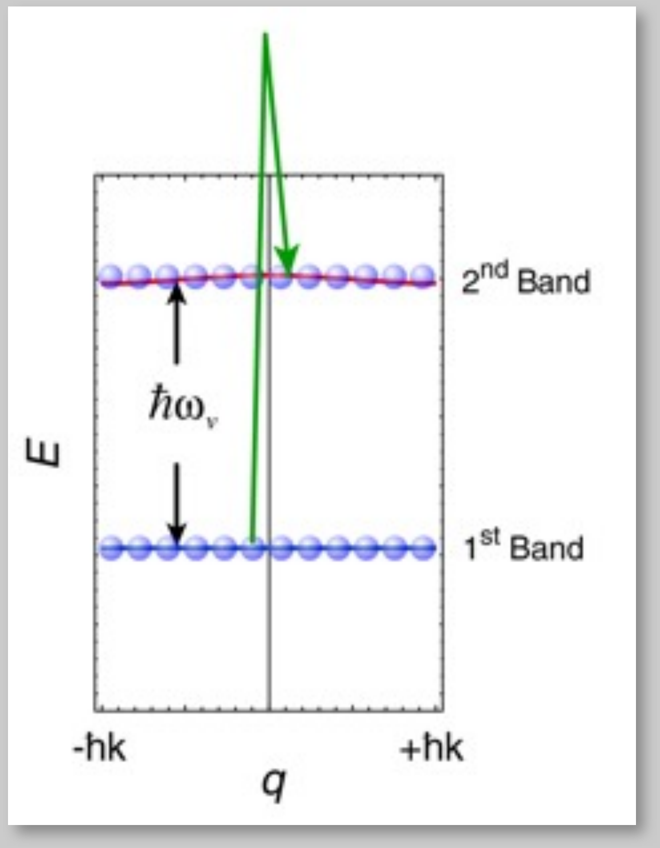
Populating Higher Energy Bands

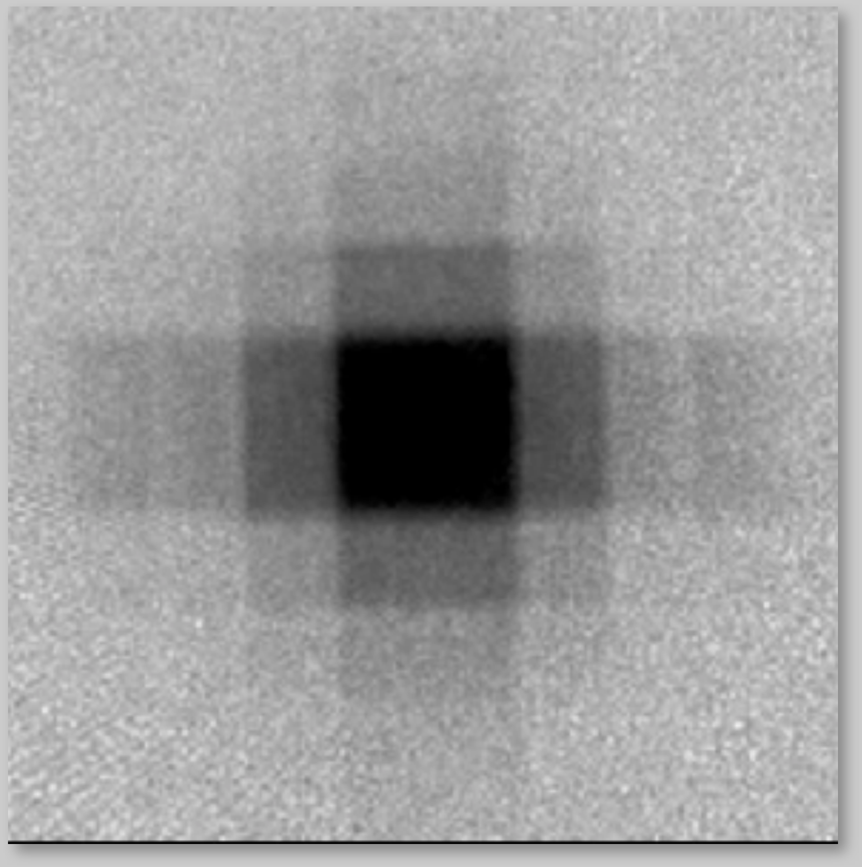
Single lattice site



Stimulated Raman transitions between vibrational levels are used to populate higher energy bands.

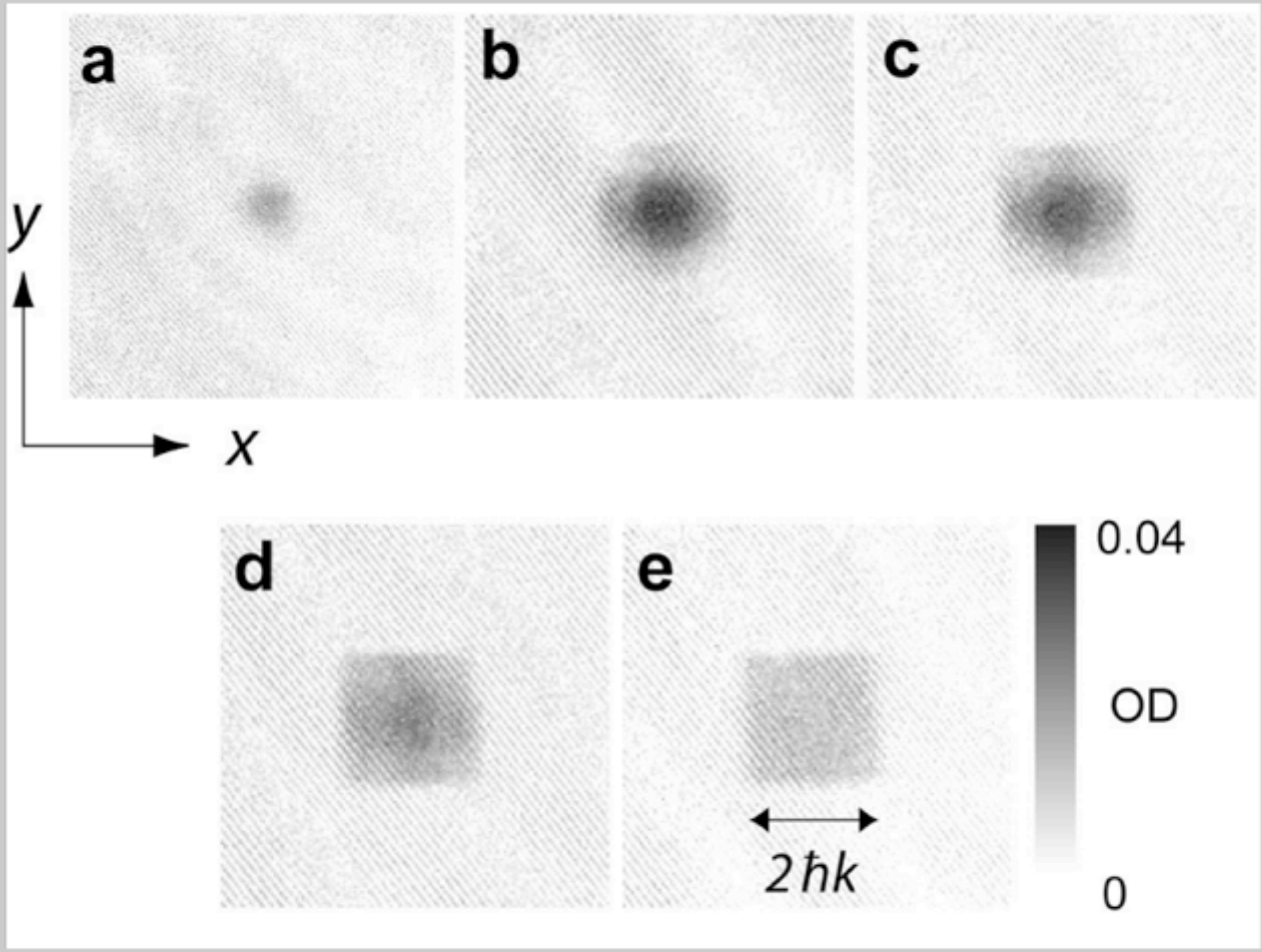
Energy bands





Measured Momentum Distribution !

From a Conductor to a Band Insulator



Fermi Surfaces become directly visible!

M. Köhl et al. Physical Review Letters (2005)