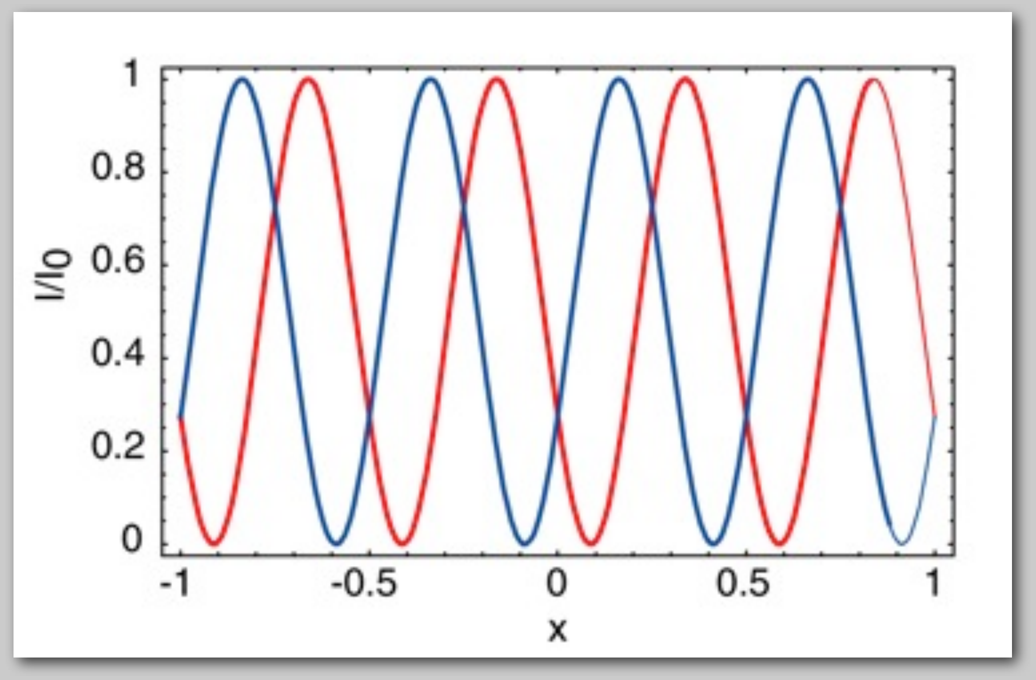
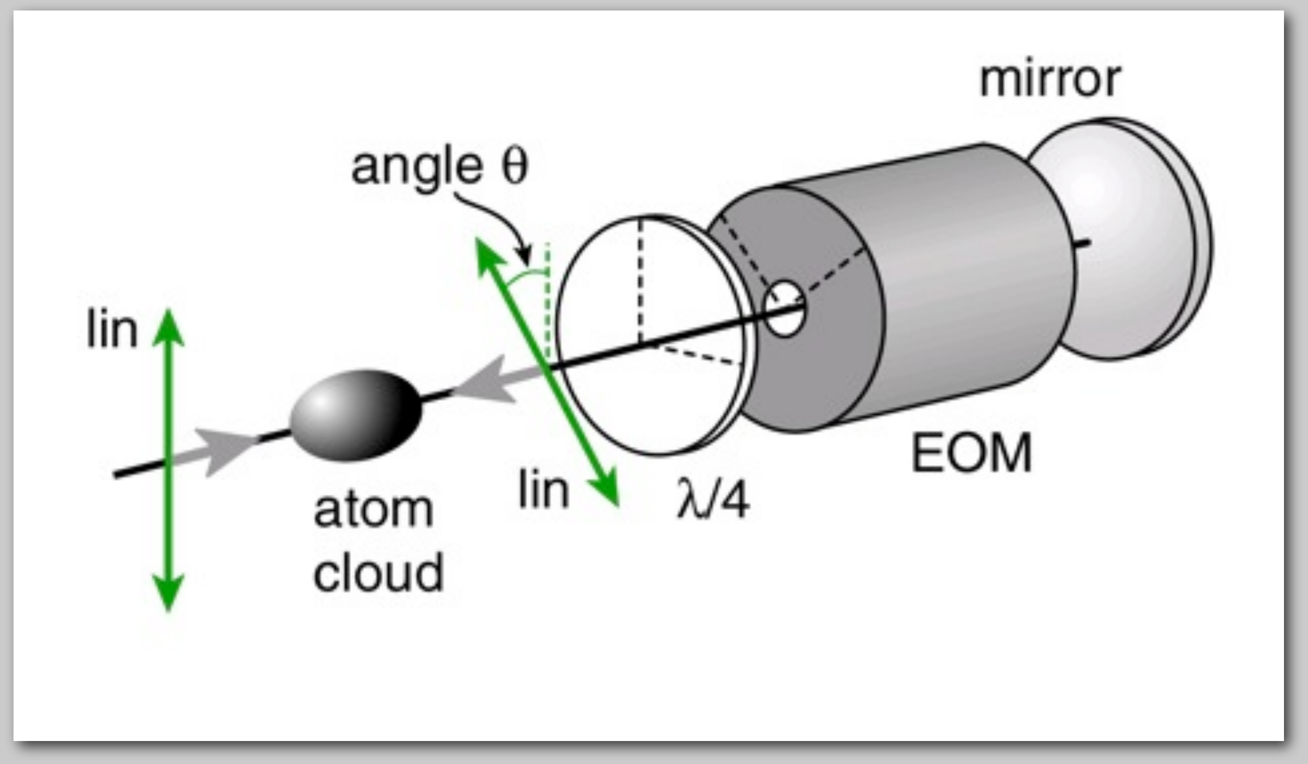
Spin Dependent Optical Lattices & Tunneling Control

Moving the Lattice Potentials

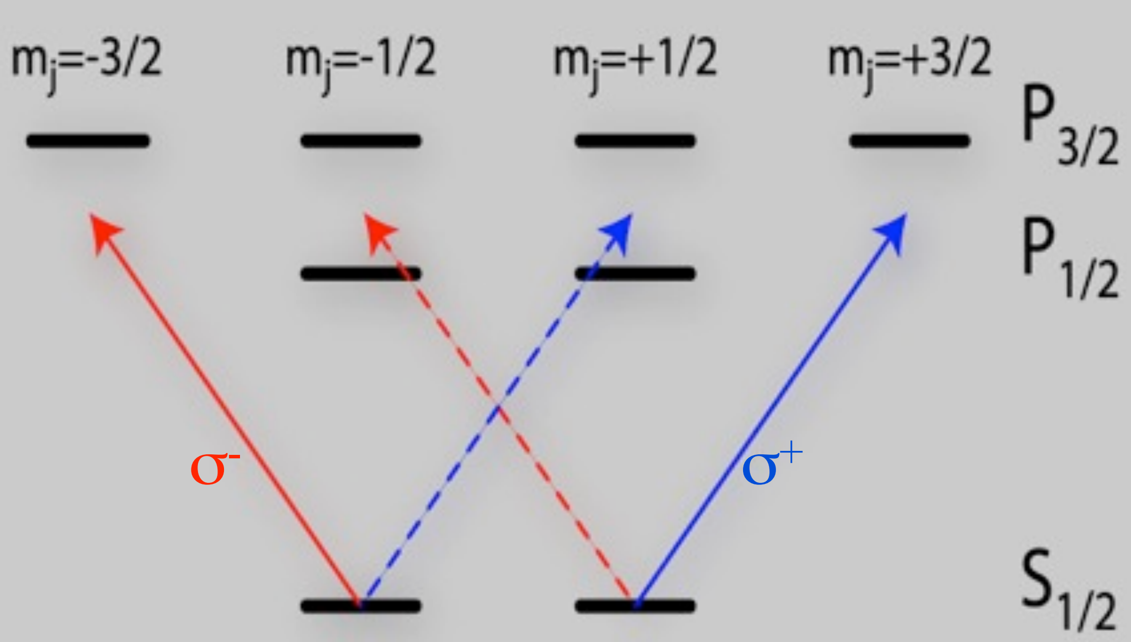
$$I_- = I_0 \sin^2(kx - \theta/2)$$
$$I_+ = I_0 \sin^2(kx + \theta/2)$$



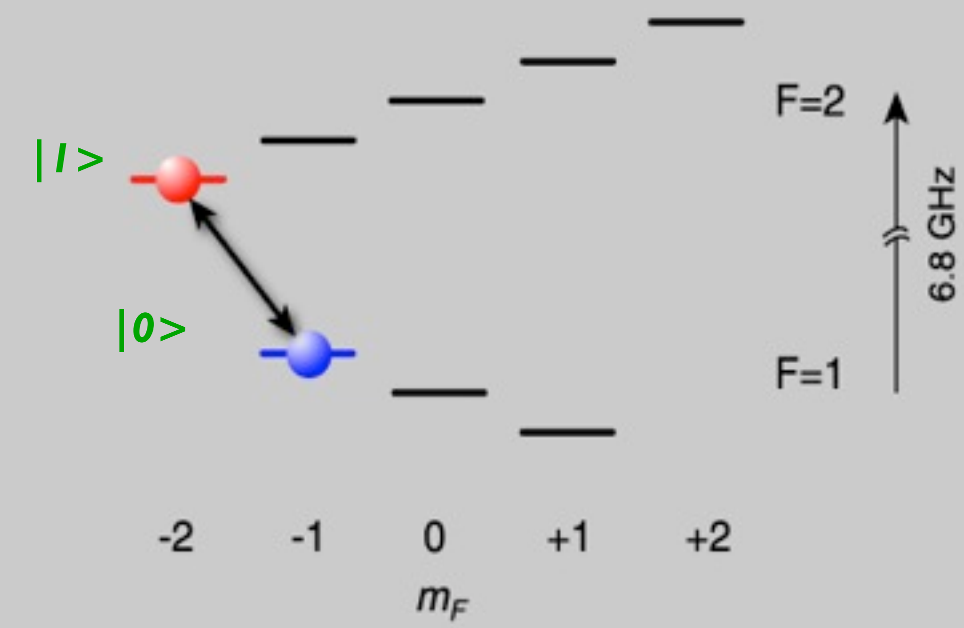


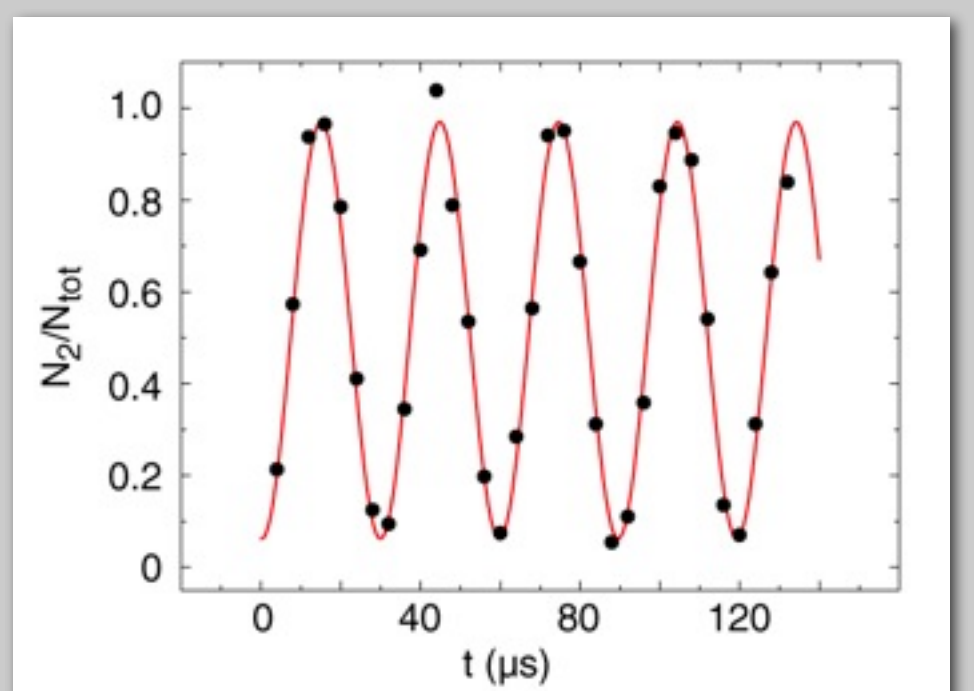
State Selective Lattice Potentials

^{87}Rb Fine- structure



Hyperfine structure





$$V_1(x,\theta) = V_-(x,\theta)$$

$$V_0(x,\theta) = \frac{1}{4} V_-(x,\theta) + \frac{3}{4} V_+(x,\theta)$$

Tensor Light Shifts

$$\hat{U} = U(\mathbf{x})\hat{1} + g_F D_{FS} \mathbf{B}_{eff}(\mathbf{x})\hat{F}$$

$$U(\mathbf{x}) = U_0 |\varepsilon(\mathbf{x})|^2$$

$$\mathbf{B}_{eff} = U_0 \frac{i}{2\hbar} \{\varepsilon^*(\mathbf{x}) \times \varepsilon(\mathbf{x})\}$$

$$U_0 = \frac{\hbar\Gamma^2}{12\Delta_{avg}} \left(\frac{I}{I_S}\right)$$

$$\Delta_{avg} = \left(\frac{1}{2\Delta_{1/2}} + \frac{1}{\Delta_{3/2}}\right)^{-1}$$

$$D_{FS} = \left(\frac{\Delta_{3/2} - \Delta_{1/2}}{\Delta_{3/2} - \Delta_{1/2}}\right)$$

Delocalization “by Hand”

TOF

↑

π/2 microwave pulse

↑

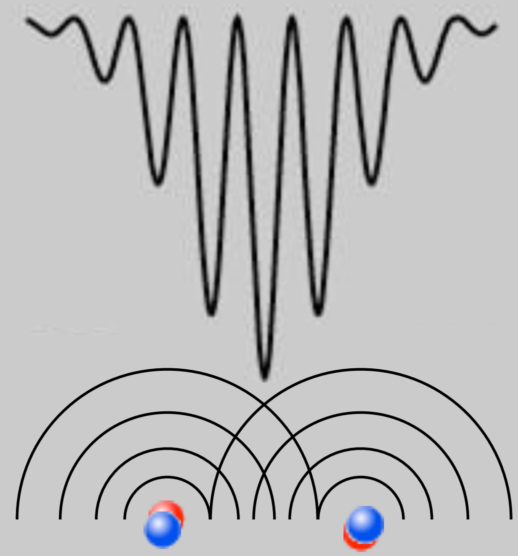
Shift

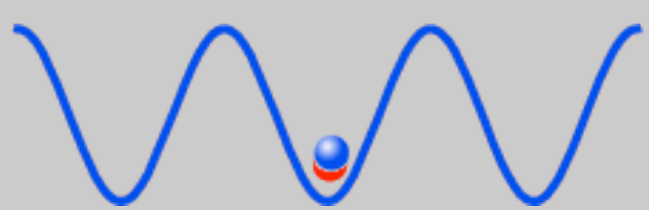
↑

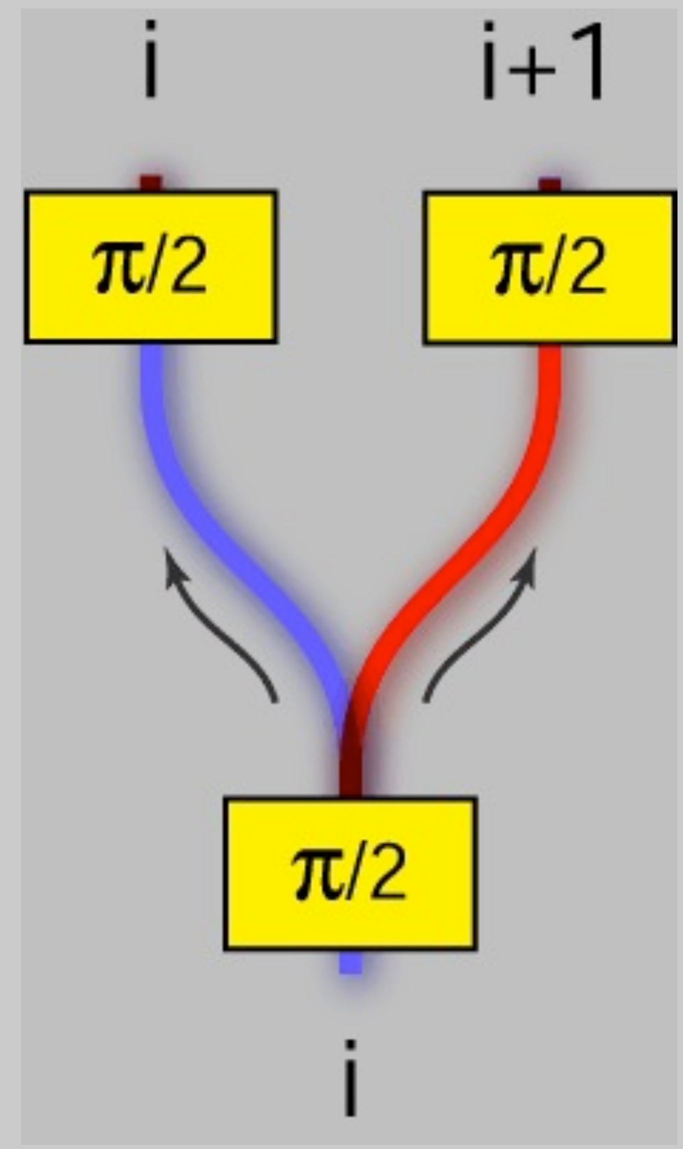
π/2 microwave pulse

↑

Initial state |0>

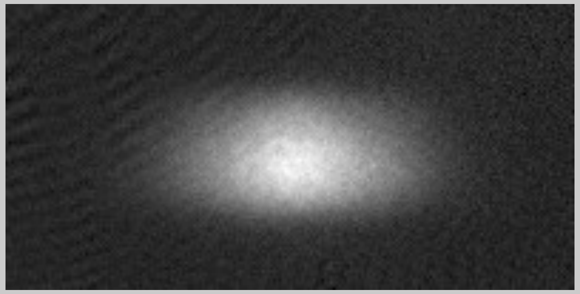






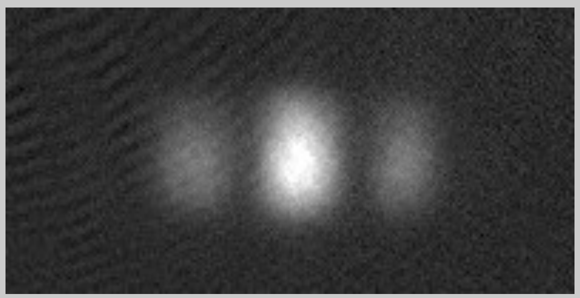
Shifting is Coherent !

Localized

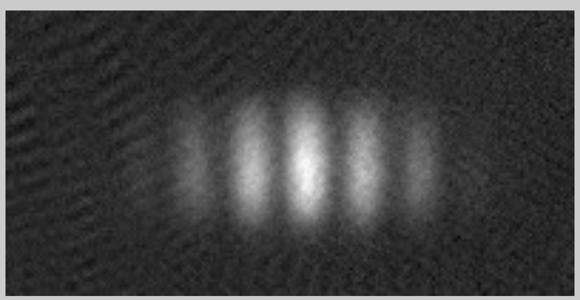


Delocalized

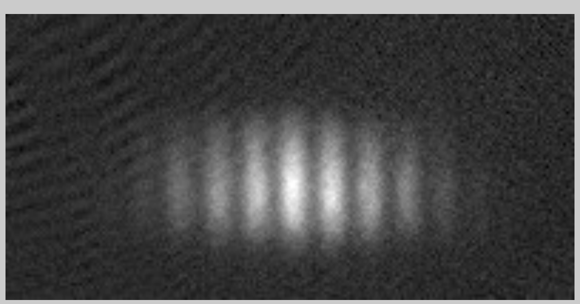
2 sites

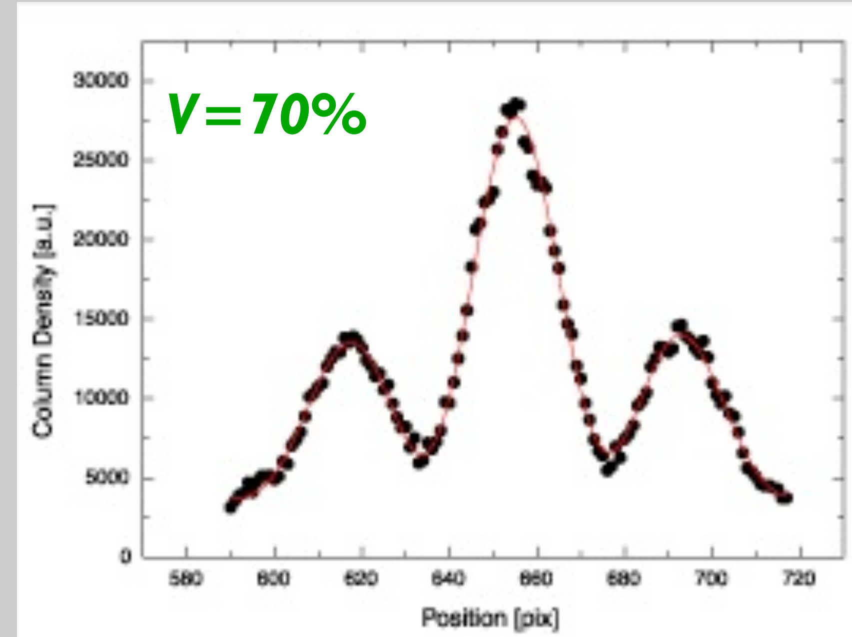


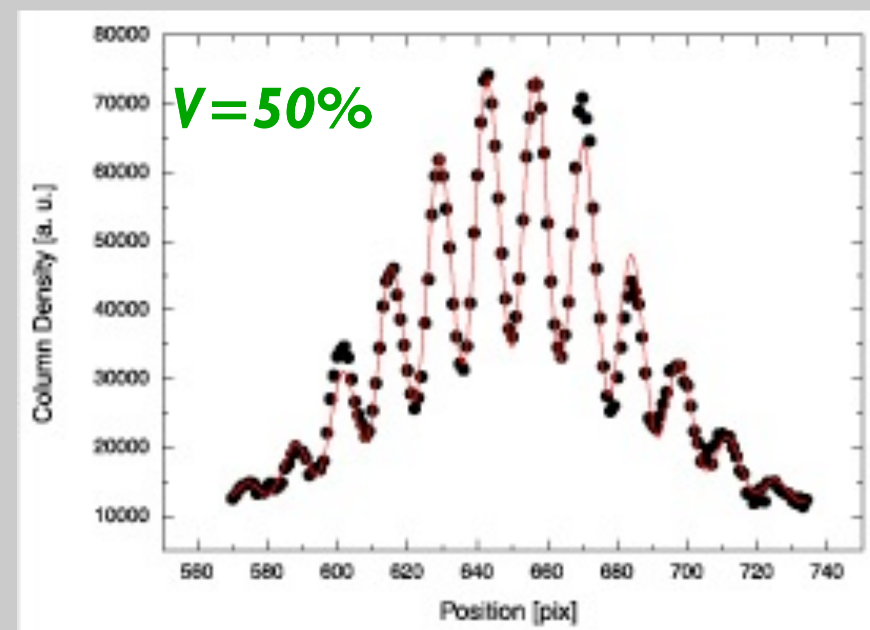
3 sites



4 sites



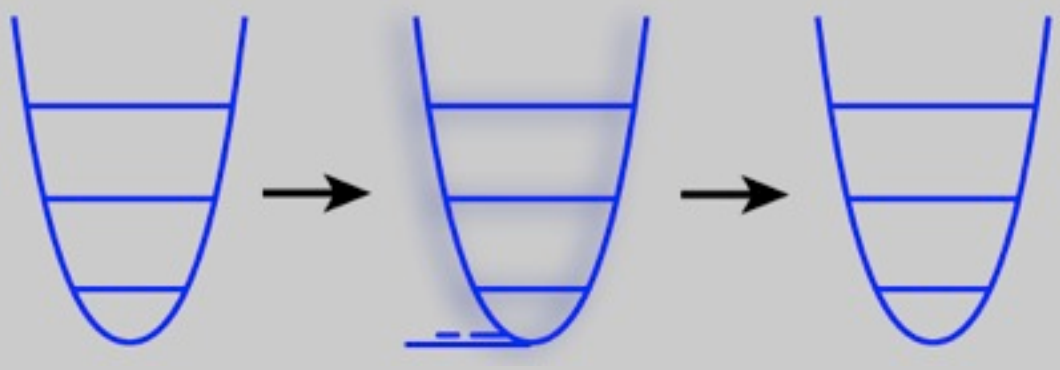


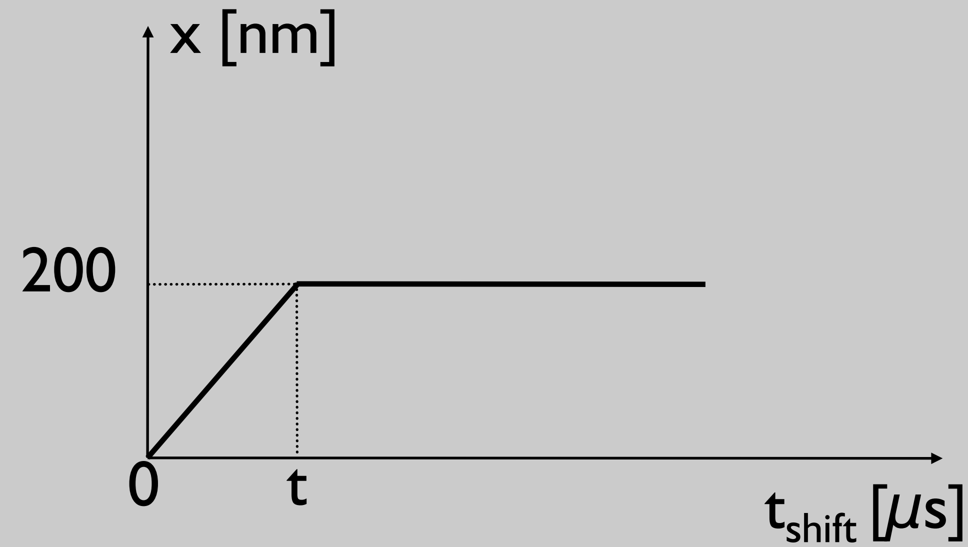


Able to delocalize atoms over up to 7 lattice sites !

O. Mandel et al., Phys. Rev. Lett. **91**, 010407 (2003)

Moving Atoms in Harmonic Potentials

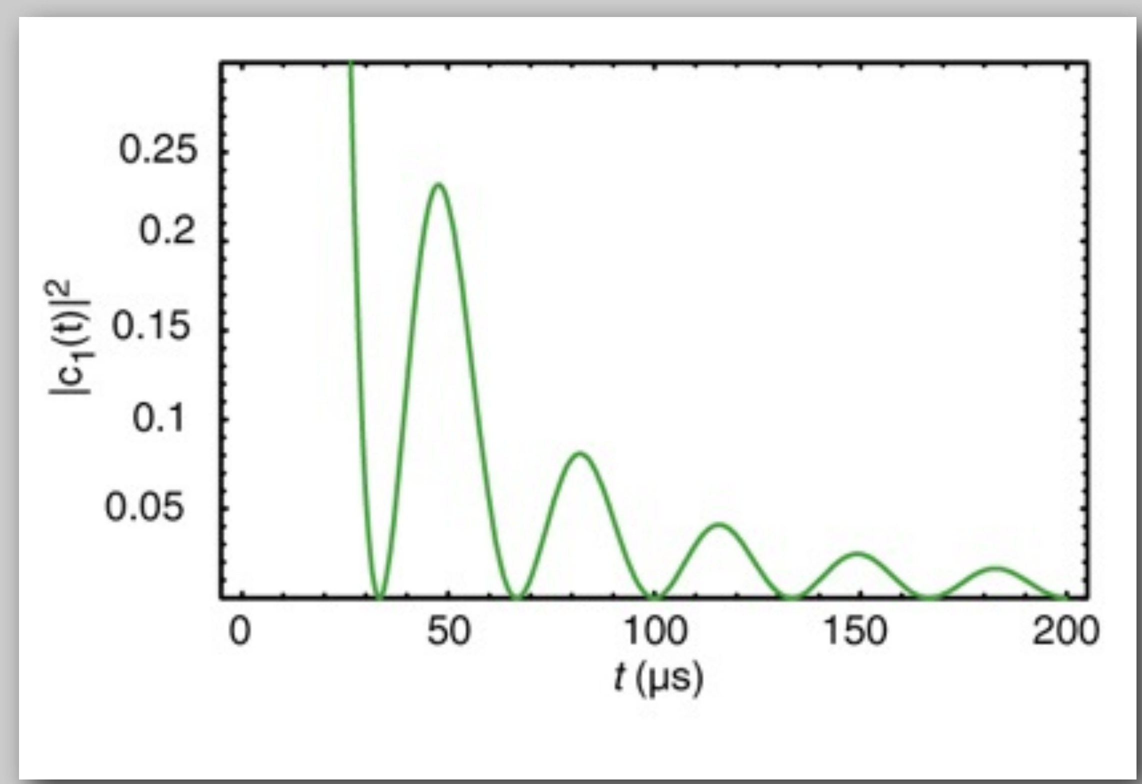




How fast can we move, in order to avoid excitations ?

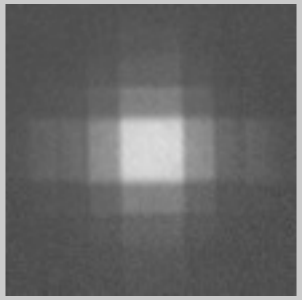
For a constant shift velocity v, first order perturbation theory yields:

$$|c_1(t)|^2 = \frac{2v^2}{(a_0\omega)^2}\sin^2(\omega t/2)$$



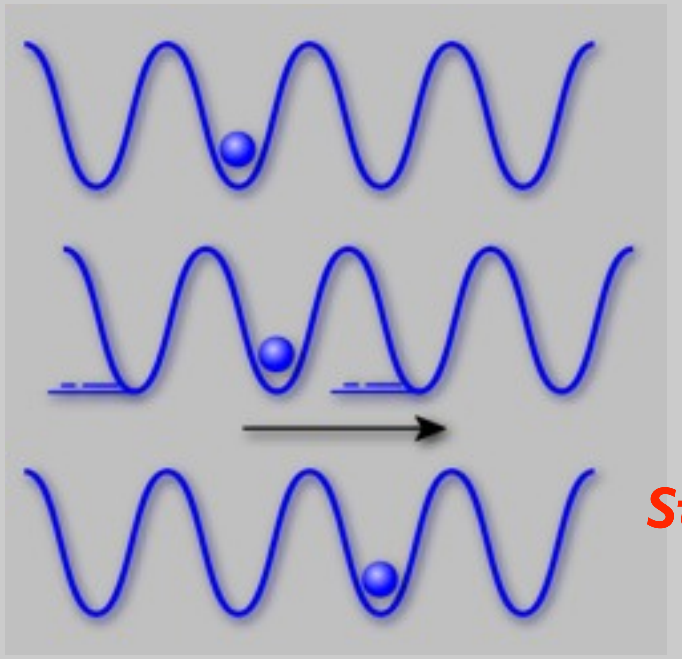
ω=2π 30 kHz, Δx = 200 nm

Measuring the Excitation Probability vs. Shift Velocity



Population of higher vibrational states (energy bands) can be mapped onto the corresponding Brillouin zones by adiabatically decreasing the lattice potential !

A. Kastberg et al. PRL (1995)
M. Greiner et al. PRL (2001)



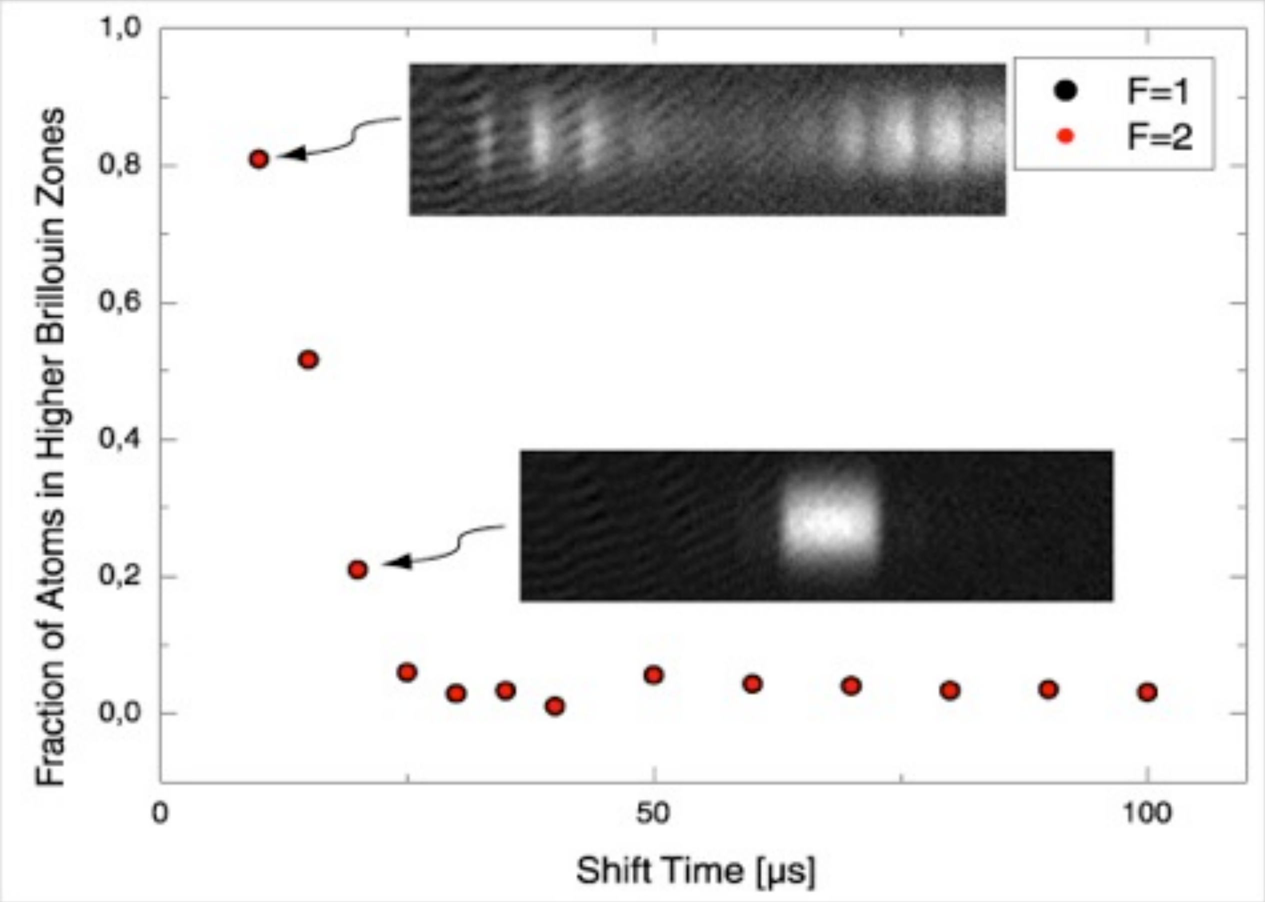
***Start** with ground state atoms*

↓

Constant Velocity

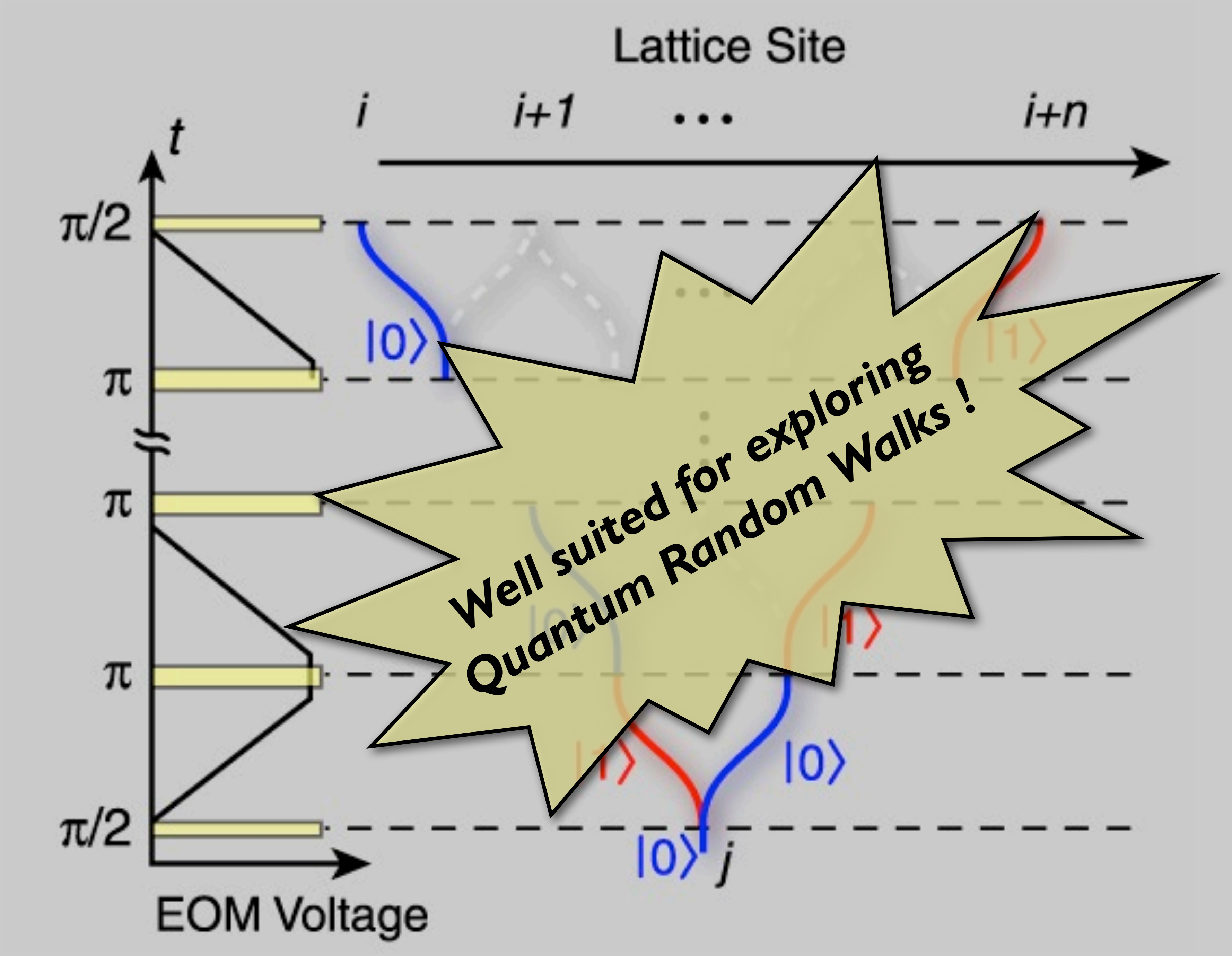
↓

***Stop**; measure remaining atoms in ground state*



Atoms moved over a distance of approx. 200 nm

Complete Sequence used in the Experiment

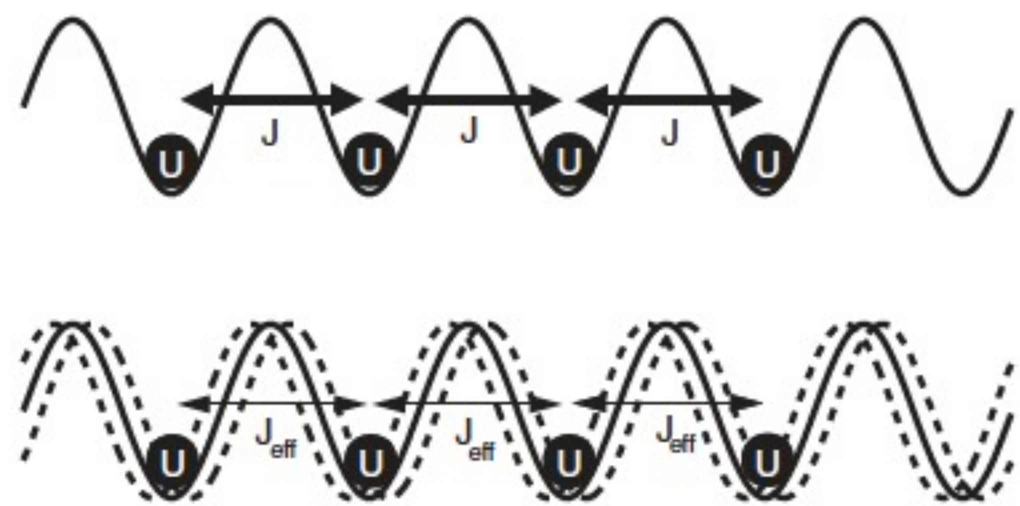


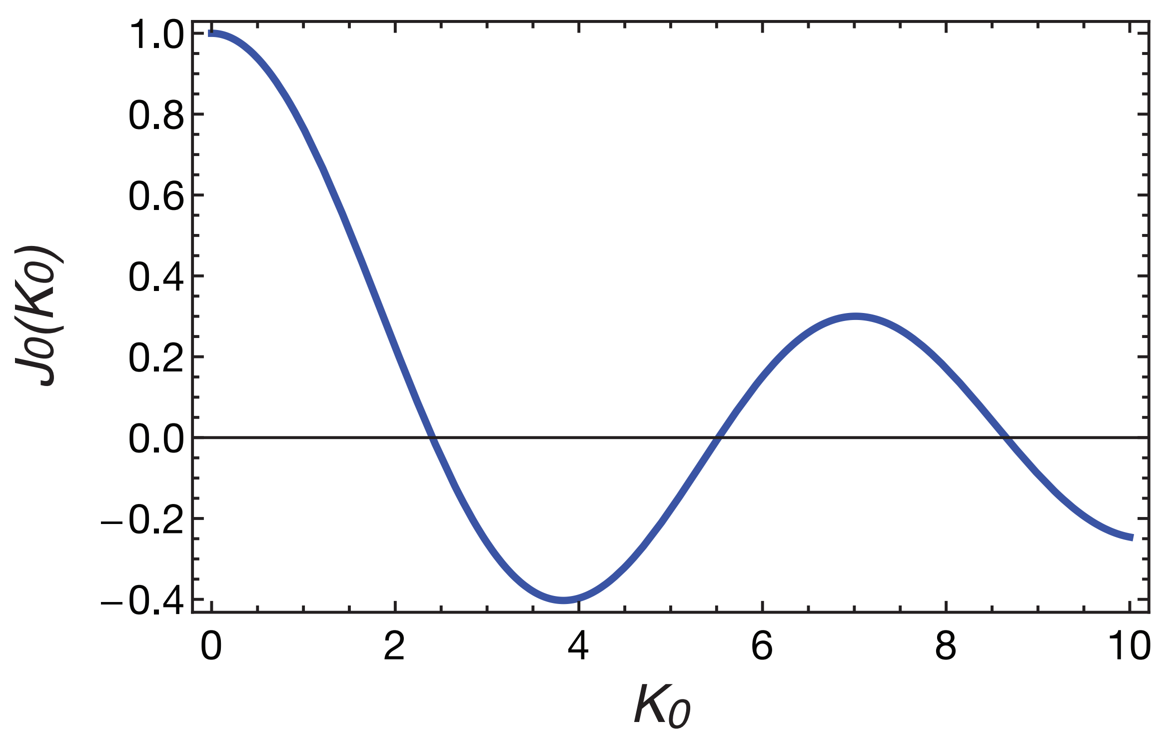
see A. Widera et al. Science (2009)

Dynamical Control of Matter Wave Tunneling

Dynamical Control of Matter Wave Tunneling

$$\hat{H}_0 = -J\sum_{\langle i,j\rangle}(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i) + \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1)$$
$$+ K\cos(\omega t)\sum_j j\hat{n}_j,$$



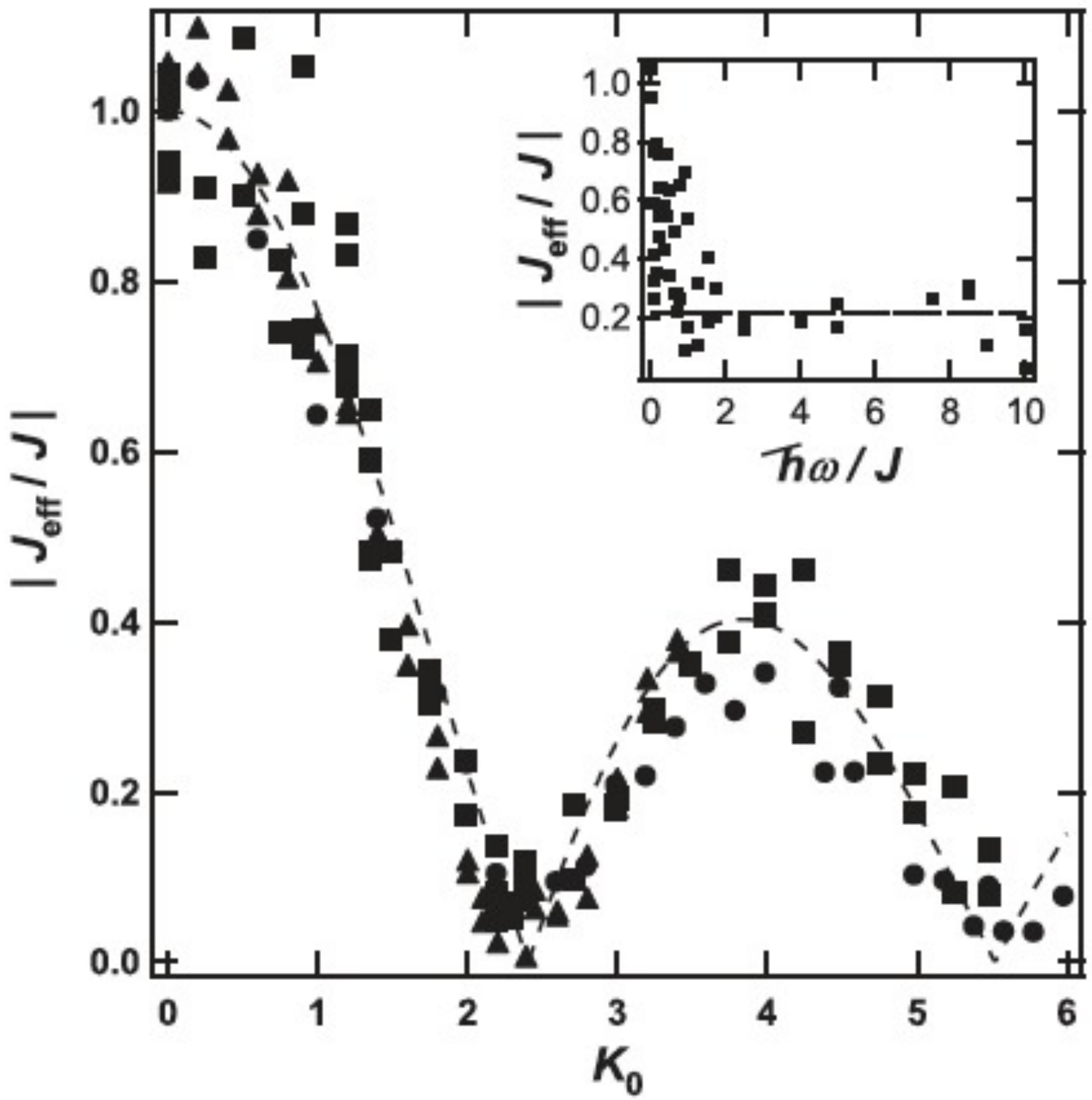


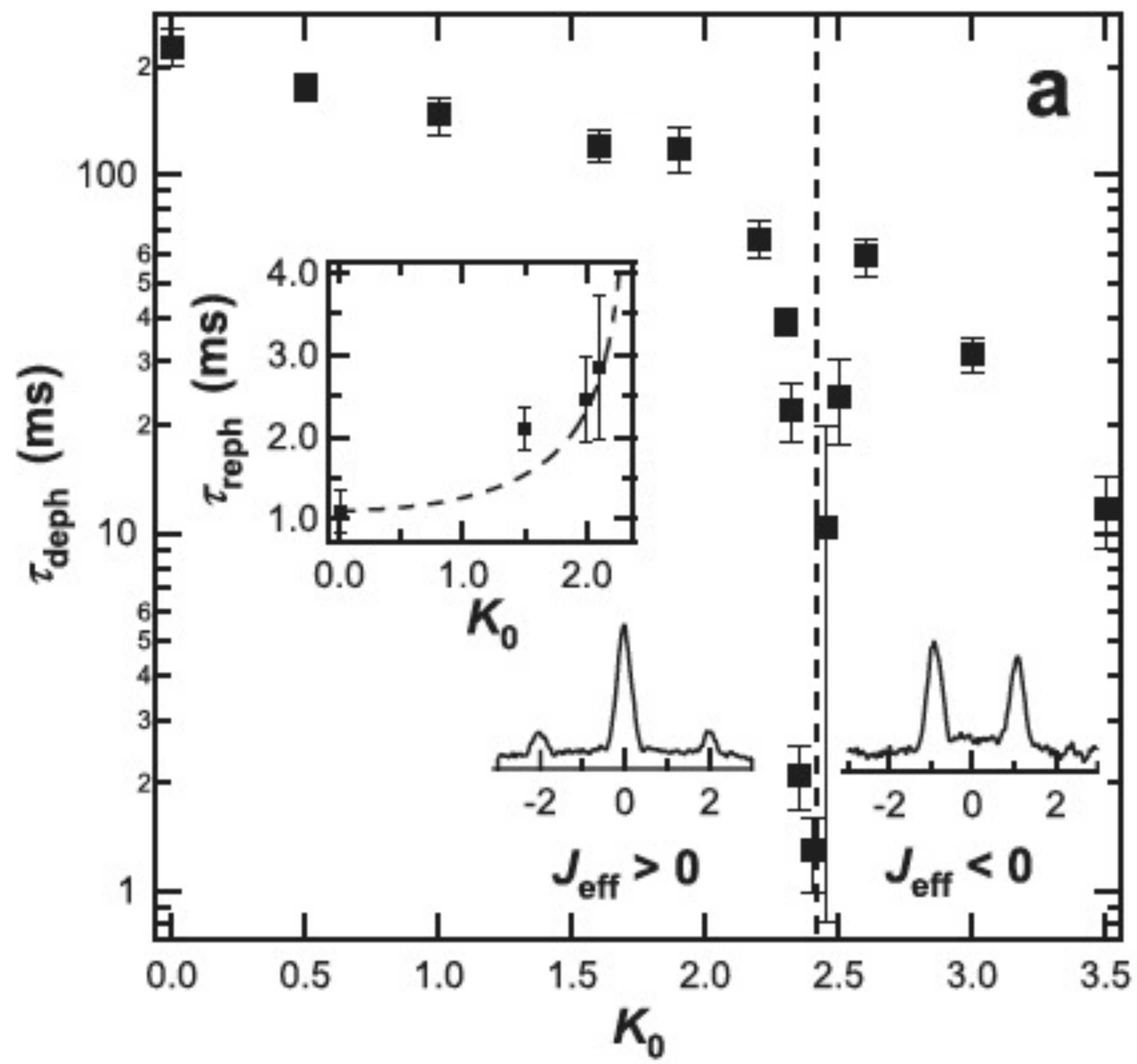
$$J_{eff} = J\,\mathscr{J}_0(K_0)$$

Dynamical Shaking of Lattice

Lignier et al., PRL (2007), original idea: M. Holthaus (Oldenburg)

Tuning Tunneling





Lignier et al., PRL **99**, 220403 (2007)

Many-Body Physics with Ultracold Atoms

Entering the Strongly Interacting Regime

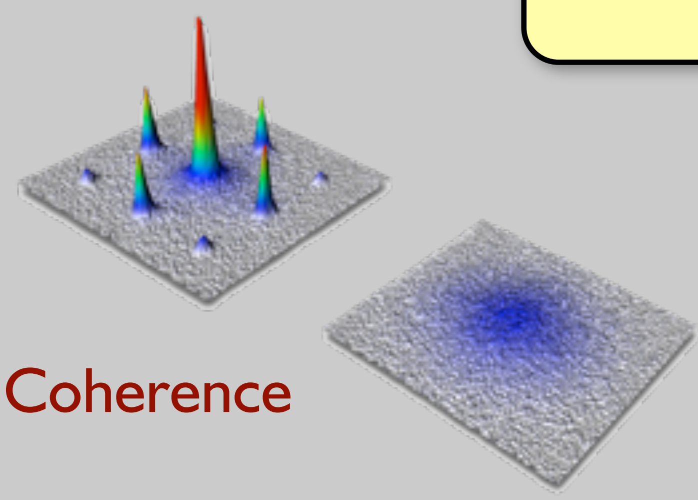
$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} >> 1$$

Use Feshbach resonances to increase ***U***

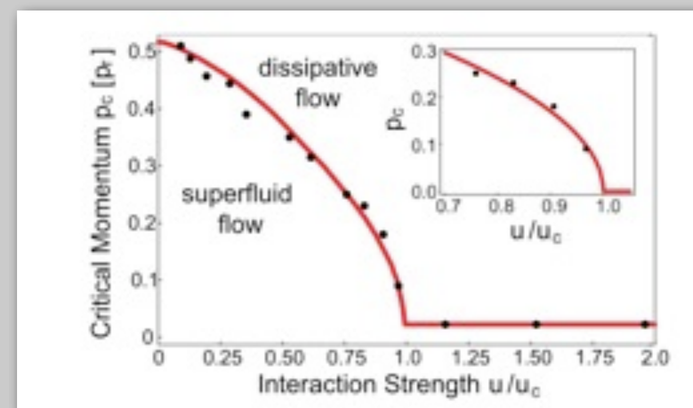
Increase lattice depth and decrease ***J***

Superfluid to Mott Insulator Transition

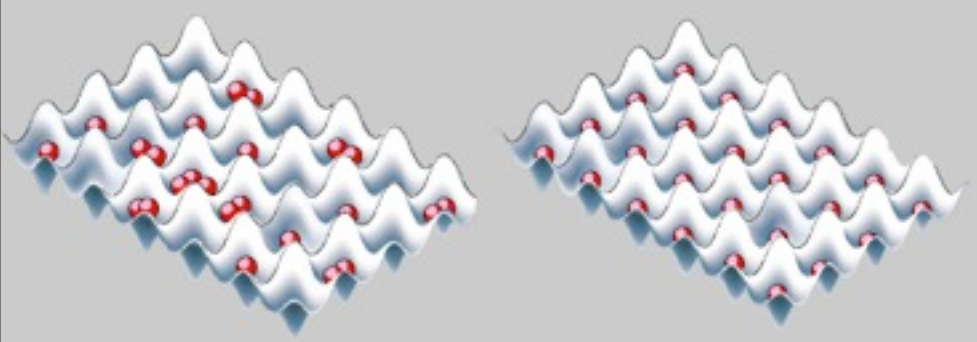
$$H = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$



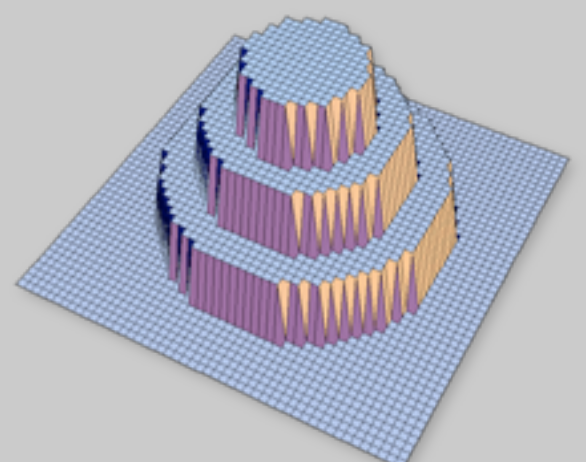
Coherence



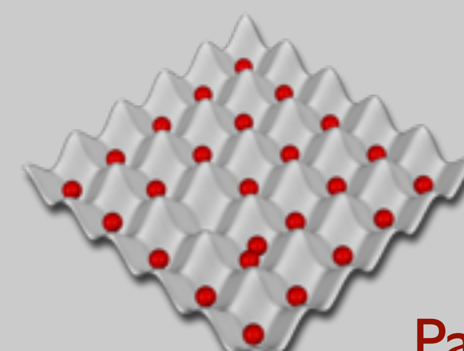
Critical momentum



Number statistics



In trap
density distribution
"shell structure"



Particle hole admixture

M.P.A. Fisher et al., PRB 40, 546 (1989); D. Jaksch et al., PRL 81, 3108 (1998)

Bosonic Mott Insulators now at: Munich,Mainz, NIST, ETHZ, MIT, Innsbruck, Florence, Garching...

Bose-Hubbard Hamiltonian

Expanding the field operator in the Wannier basis of localized wave functions on each lattice site, yields :

$$\hat{\psi}(\boldsymbol{x}) = \sum_i \hat{a}_i w(\boldsymbol{x} - \boldsymbol{x}_i)$$

Bose-Hubbard Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Tunnelmatrix element/Hopping element

$$J = -\int d^3x w(\mathbf{x} - \mathbf{x}_i) \left(-\frac{\hbar^2}{2m} \Delta + V_{lat}(\mathbf{x}) \right) w(\mathbf{x} - \mathbf{x}_j)$$

Onsite interaction matrix element

$$U = \frac{4\pi\hbar^2 a}{m} \int d^3x |w(\mathbf{x})|^4$$

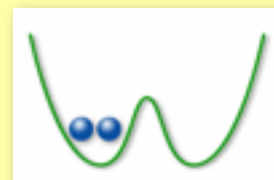
M.P.A. Fisher et al., PRB 40, 546 (1989); D. Jaksch et al., PRL 81, 3108 (1998)

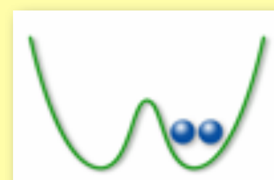
Mott Insulators now at: NIST, ETHZ, MIT, Innsbruck, Florence, Garching...

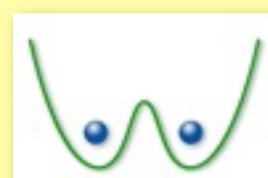
The Simplest Possible “Lattice“: 2 Atoms in a Double Well

Superfluid State

$$\frac{1}{\sqrt{2}}(\phi_l+\phi_r)\otimes\frac{1}{\sqrt{2}}(\phi_l+\phi_r)$$

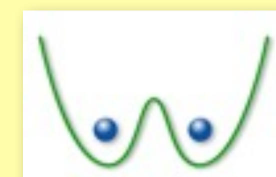
0.25 x  +

0.25 x  +

0.5 x 

MI State

$$\frac{1}{\sqrt{2}}\phi_l\otimes\phi_r+\frac{1}{\sqrt{2}}\phi_r\otimes\phi_l$$



	Superfluid State	MI State
Average atom number per site:	<*n*> = 1	<*n*> = 1
Average onsite Interaction per site:	$\langle E_{int}\rangle = \frac{1}{2}U$	$\langle E_{int}\rangle = 0$

Superfluid Limit

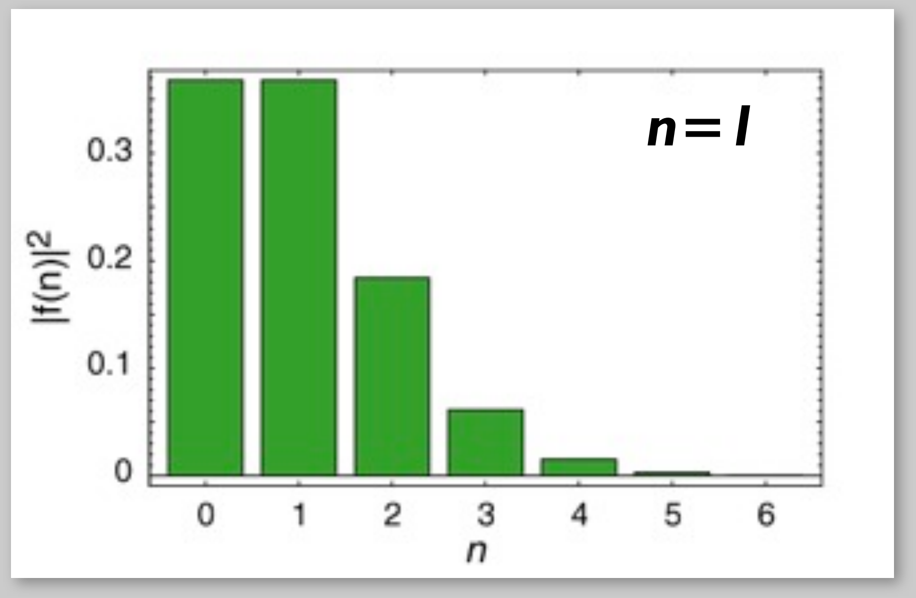
$$H = -J\sum_{i,j} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Atoms are delocalized over the entire lattice !
Macroscopic wave function describes this state very well.

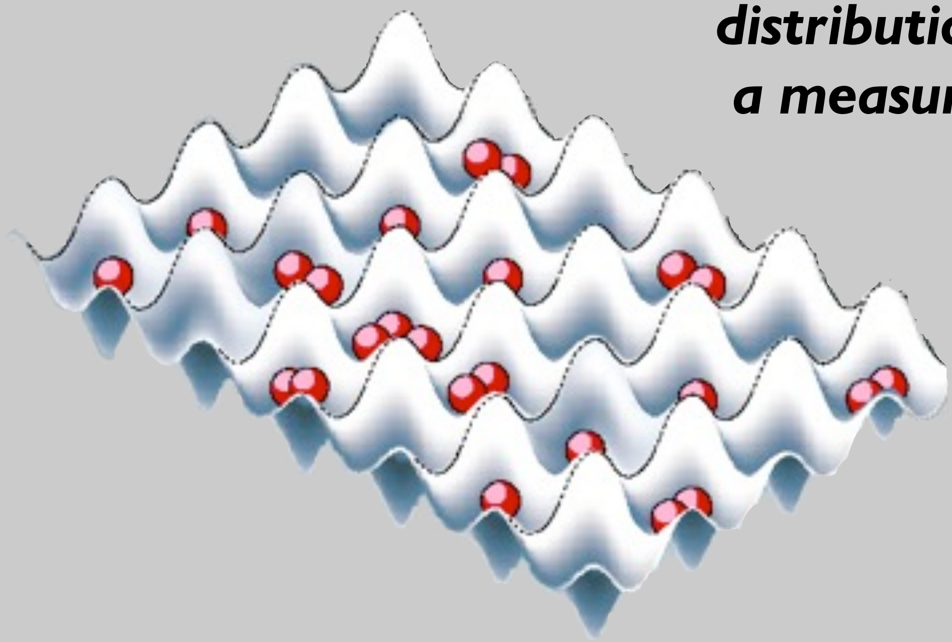
$$|\Psi_{SF}\rangle_{U=0} = \left(\sum_{i=1}^{M} \hat{a}_i^\dagger\right)^N |0\rangle$$

$$\langle \hat{a}_i \rangle_i \neq 0$$

Poissonian atom number distribution per lattice site



Atom number distribution after a measurement



"Atomic Limit" of a Mott-Insulator

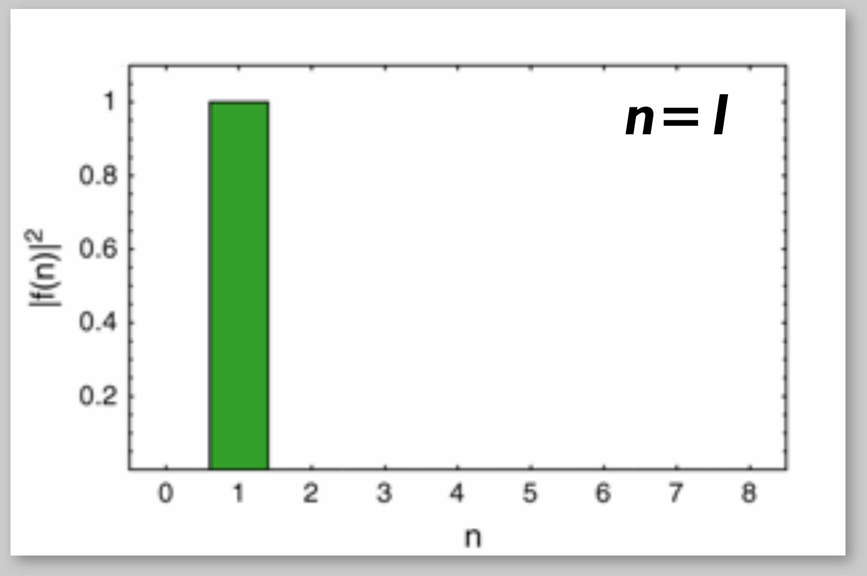
$$H = -J\sum_{i,j} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Atoms are completely localized to lattice sites !

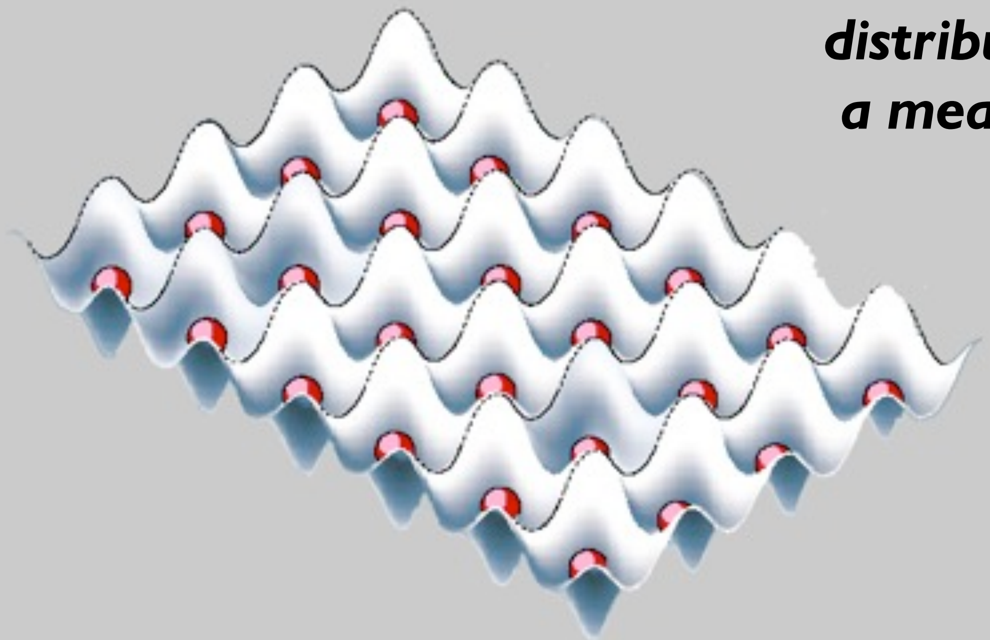
$$|\Psi_{Mott}\rangle_{J=0} = \prod_{i=1}^{M} \left(\hat{a}_i^\dagger\right)^n |0\rangle$$

$$\langle \hat{a}_i \rangle_i = 0$$

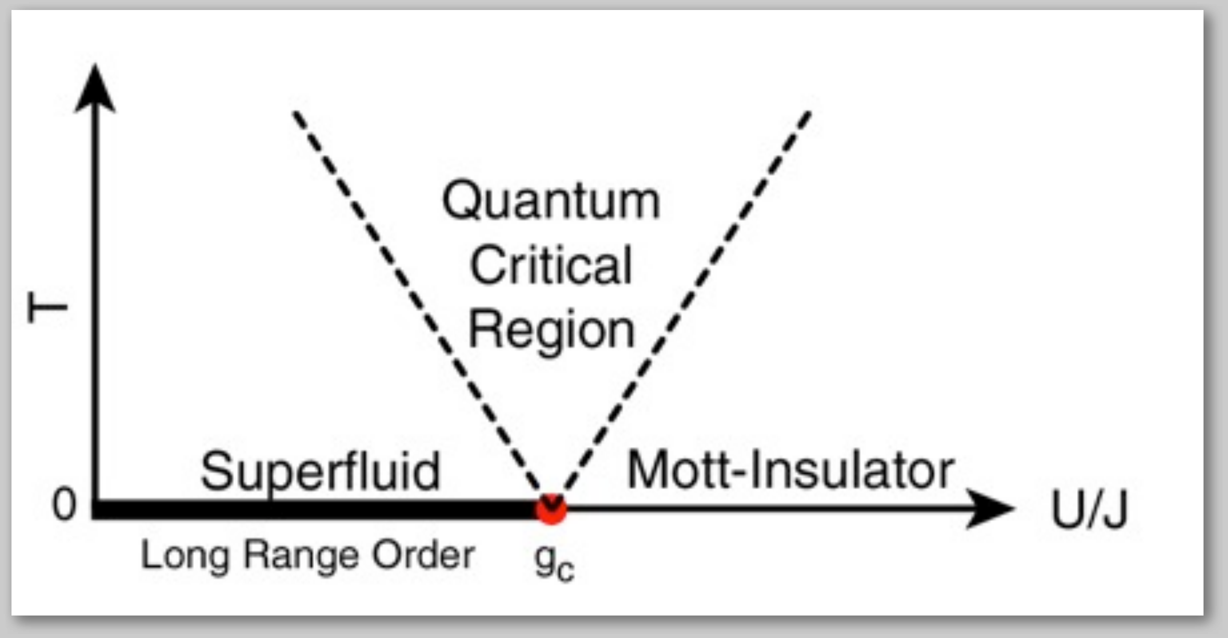
Fock states with a vanishing atom number fluctuation are formed.



Atom number distribution after a measurement



Quantum Phase Transition (QPT) from a Superfluid to a Mott-Insulator



At the critical point g_c the system will undergo a phase transition from a superfluid to an insulator !

This phase transition occurs even at T=0 and is driven by quantum fluctuations !

<u>Characteristic for a QPT</u>

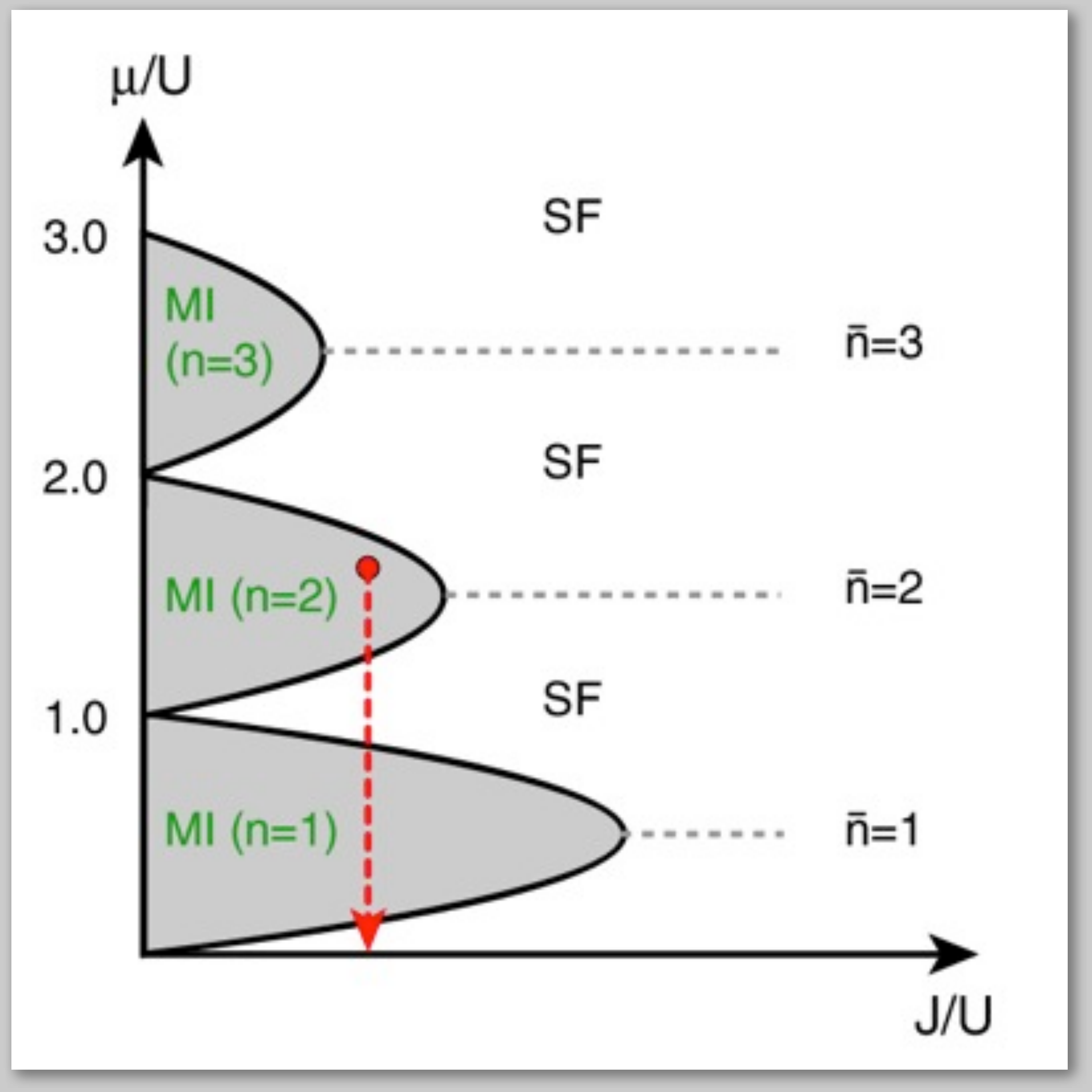
- ***Excitation spectrum is dramatically modified at the critical point.***
- ***$U/J < g_c$ (Superfluid regime)***
  Excitation spectrum is gapless
- ***$U/J > g_c$ (Mott-Insulator regime)***
  Excitation spectrum is gapped

Critical ratio for:

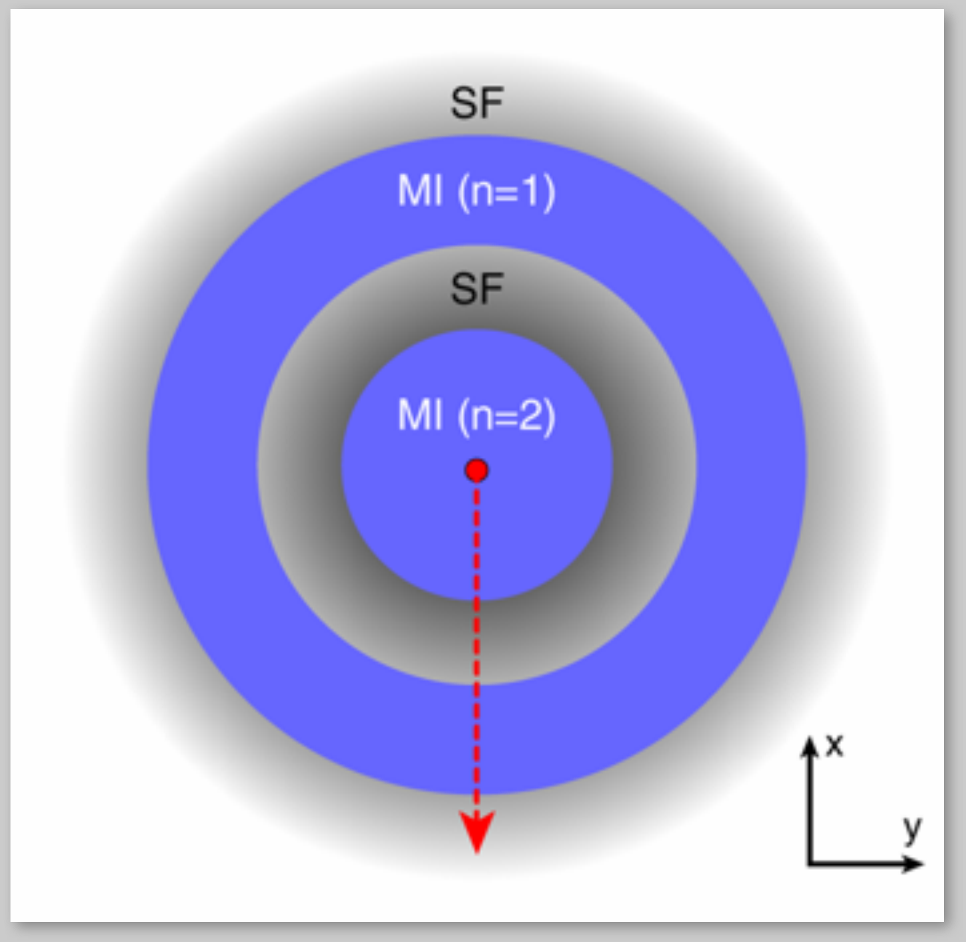
$U/J = z\ 5.8$

see Subir Sachdev, Quantum Phase Transitions, Cambridge University Press

Superfluid – Mott-Insulator Phase Diagram



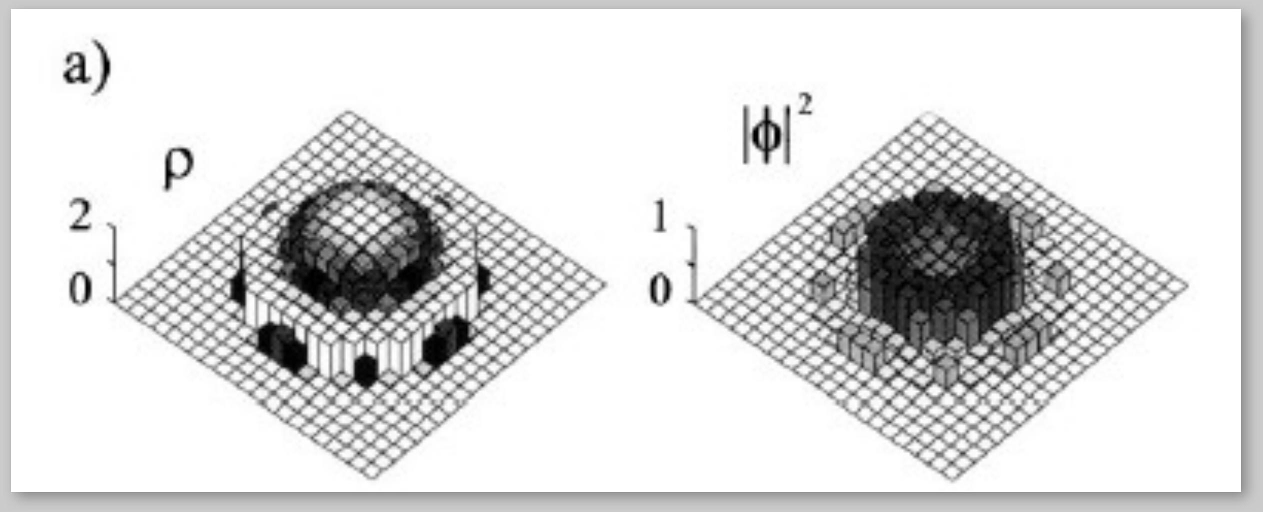
Jaksch et al. PRL 81, 3108 (1998)



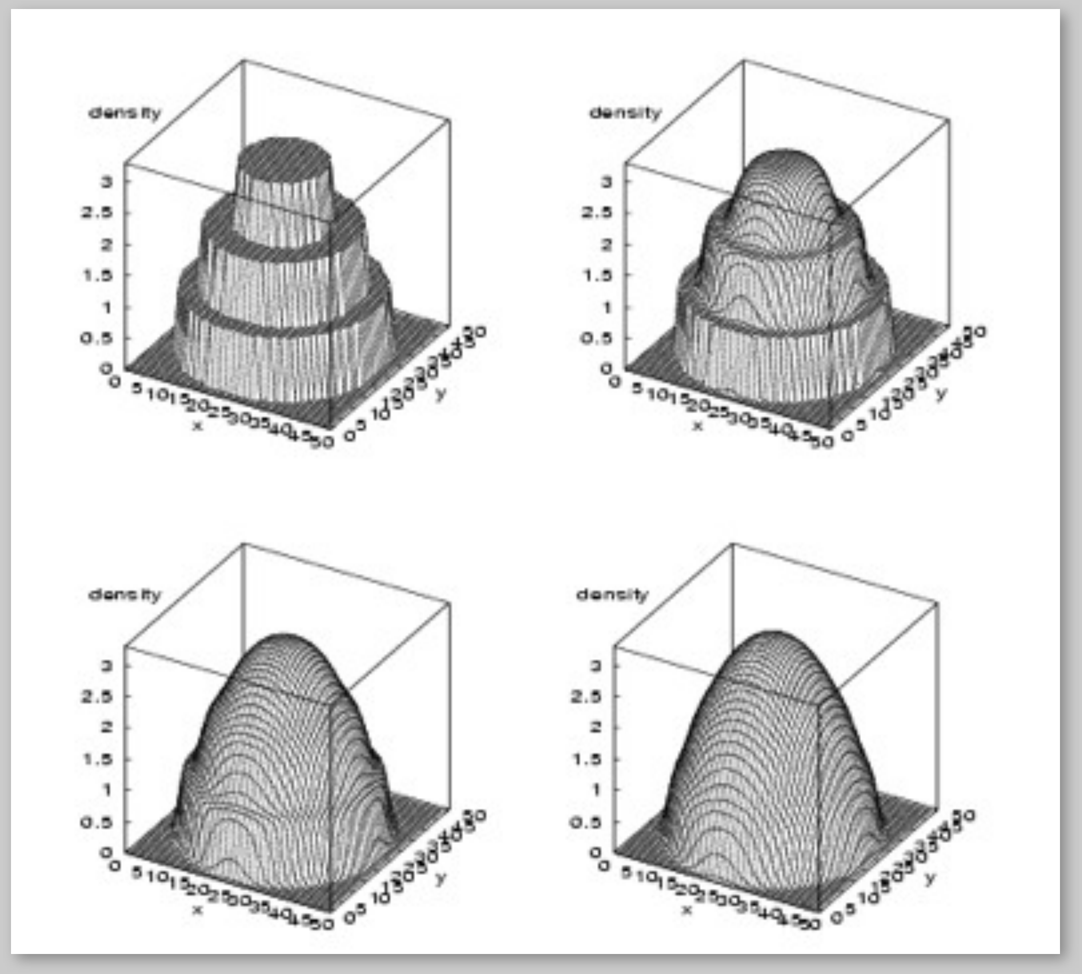
For an inhomogeneous system an effective local chemical potential can be introduced

$$\mu_{loc} = \mu - \varepsilon_i$$

Ground State of an Inhomogeneous System



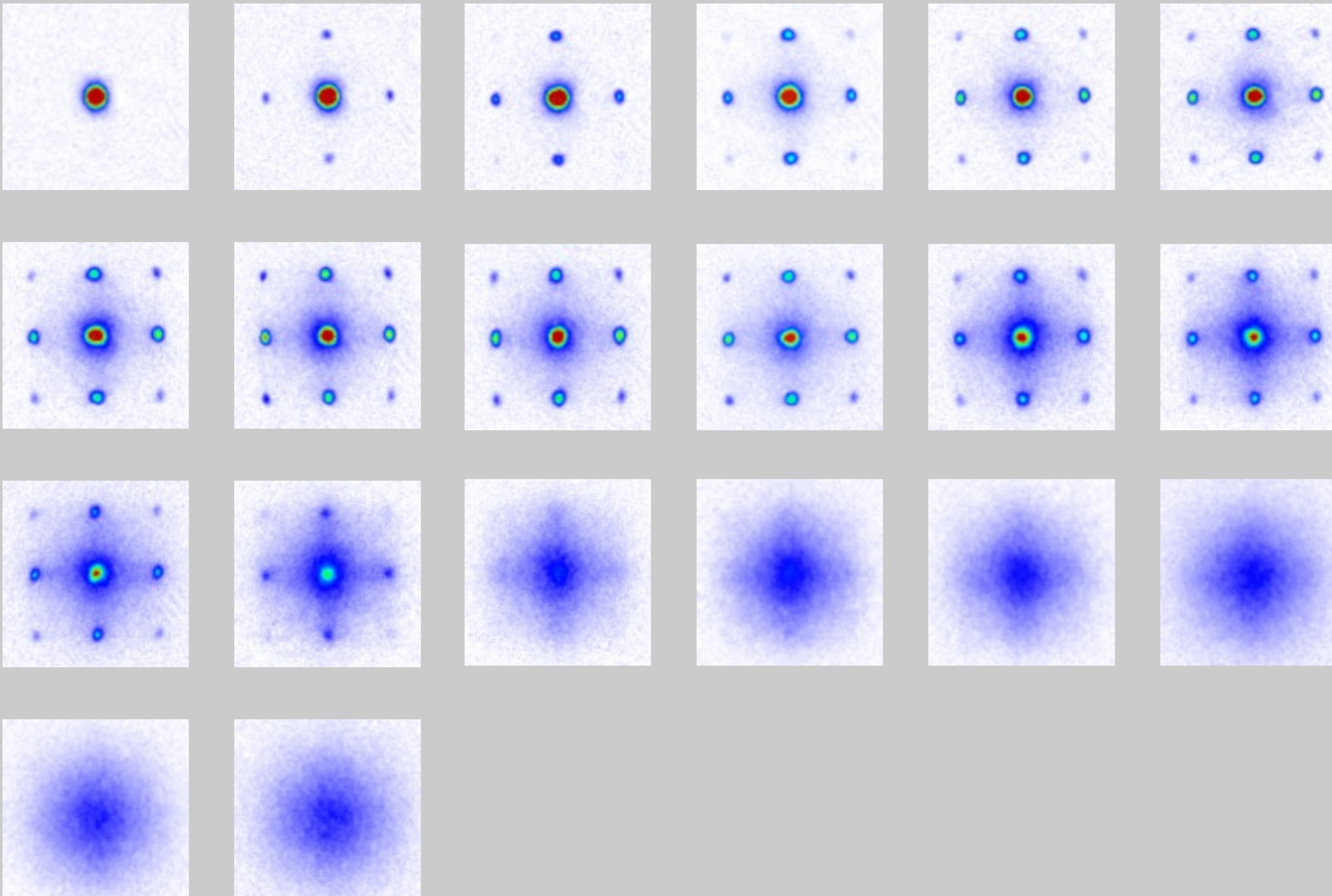
From Jaksch et al. PRL 81, 3108 (1998)



From M. Niemeyer and H. Monien (private communication)

Momentum Distribution for Different Potential Depths

$0\ E_{recoil}$



$22\ E_{recoil}$

Spin Changing Collisions

Spin Changing Collisions

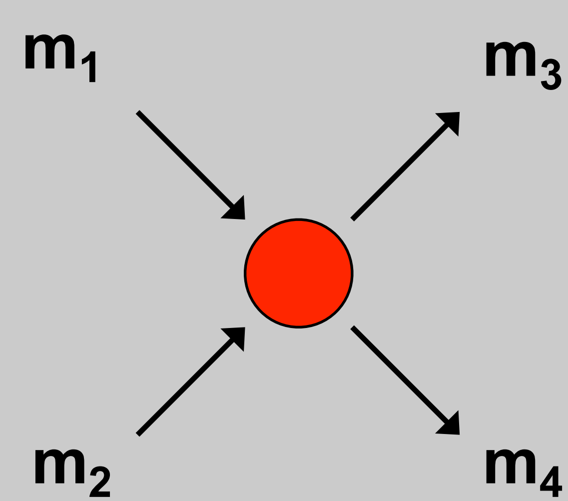
Spin-independent case

s-wave collisions

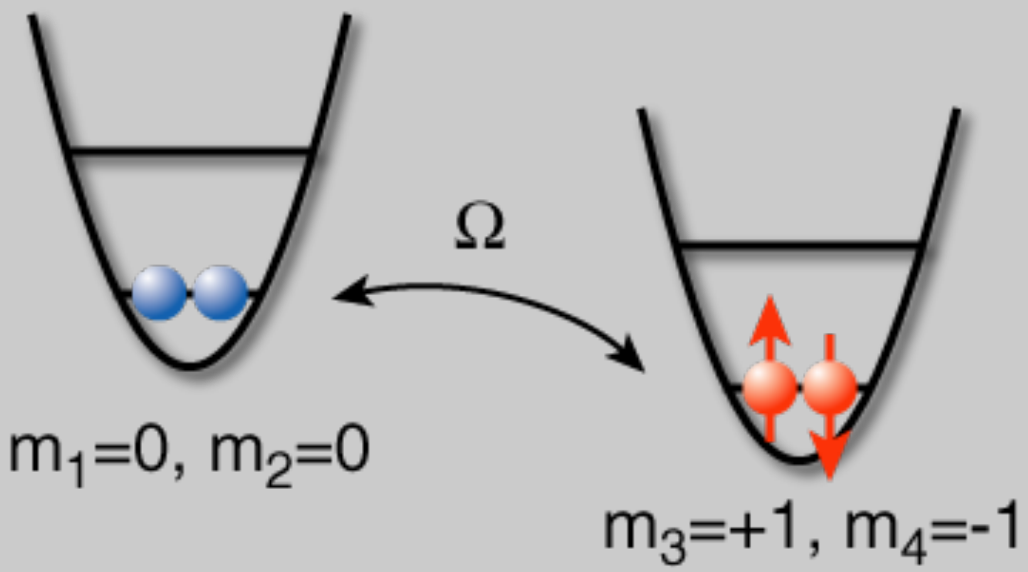
Spin-dependent case

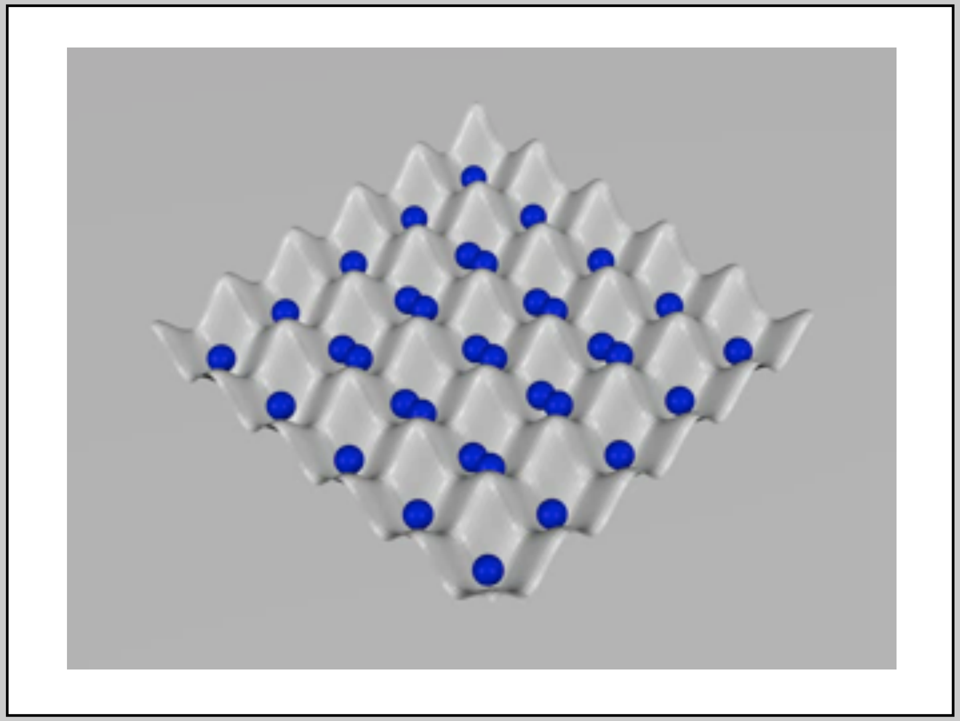
$$V(\vec{r}-\vec{r}\,')=\frac{4\pi\hbar^2}{M}\times\Delta a_{m3,m4}^{m1,m2}\times\delta(\vec{r}-\vec{r}\,')$$

Spin-dependent interaction strength



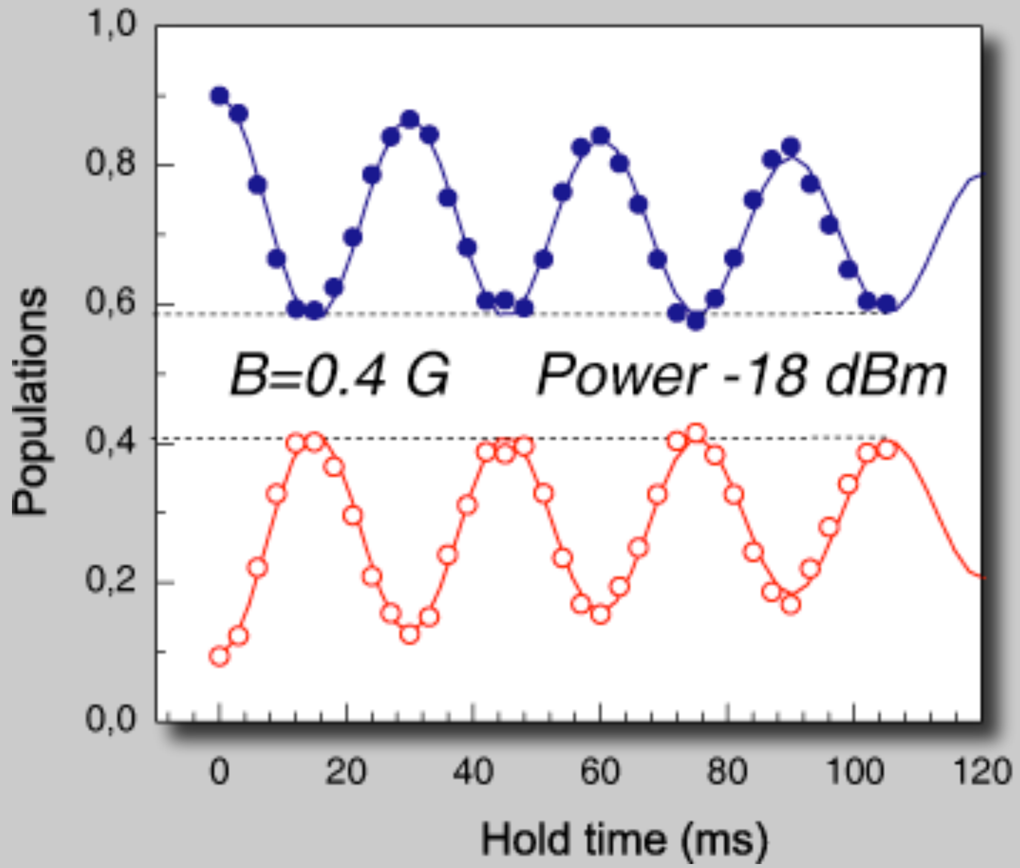
Spin Changing Collisions in an Optical Lattice





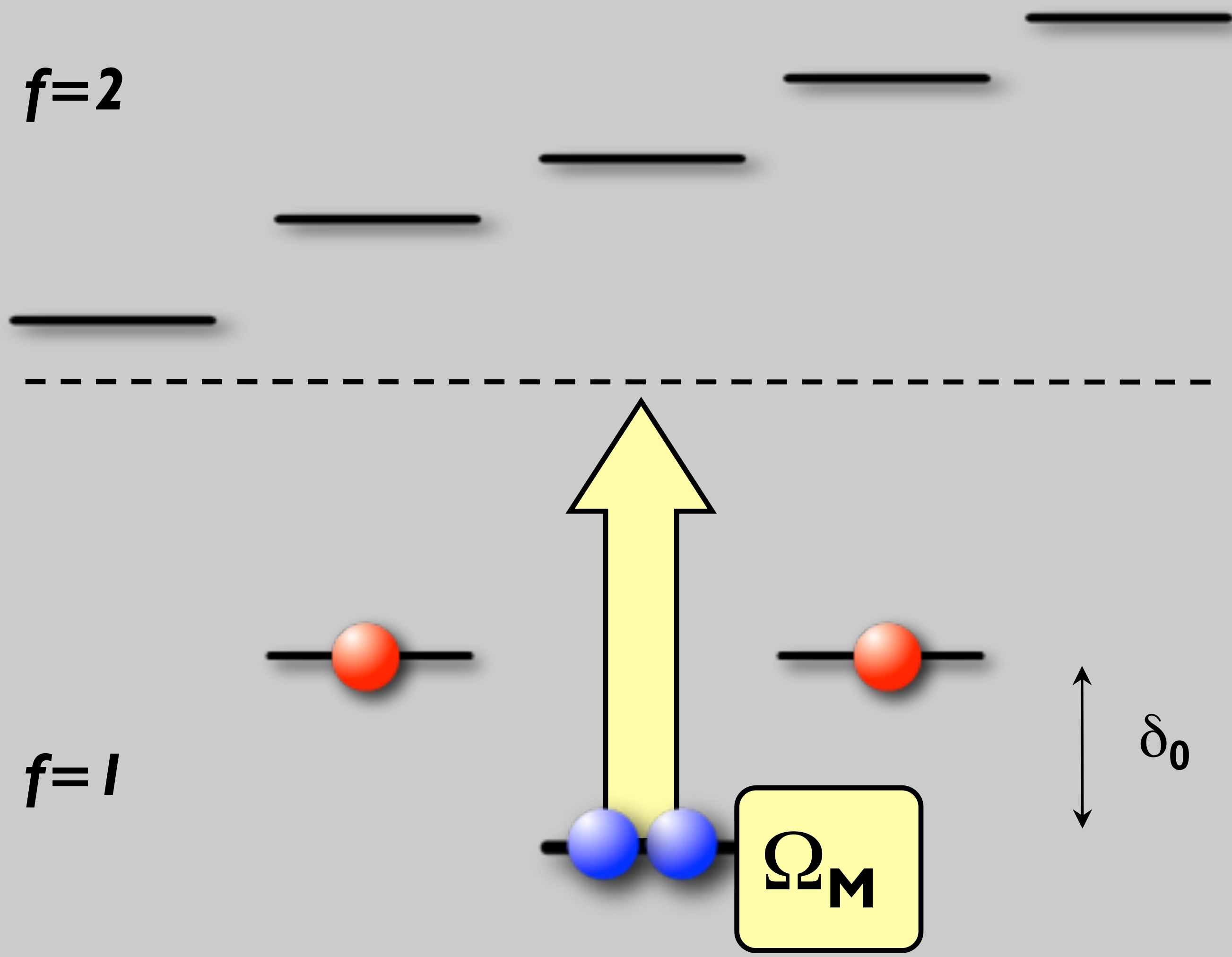
Collisionally induced
„Rabi-Type" Oscillations

$$|0,0\rangle \leftrightarrow \left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)/\sqrt{2}$$



AC-“Stark“ shift control of the resonance frequency

Spin-1 two-level system at zero magnetic field



Detuning δ_0 is present even at zero magnetic field

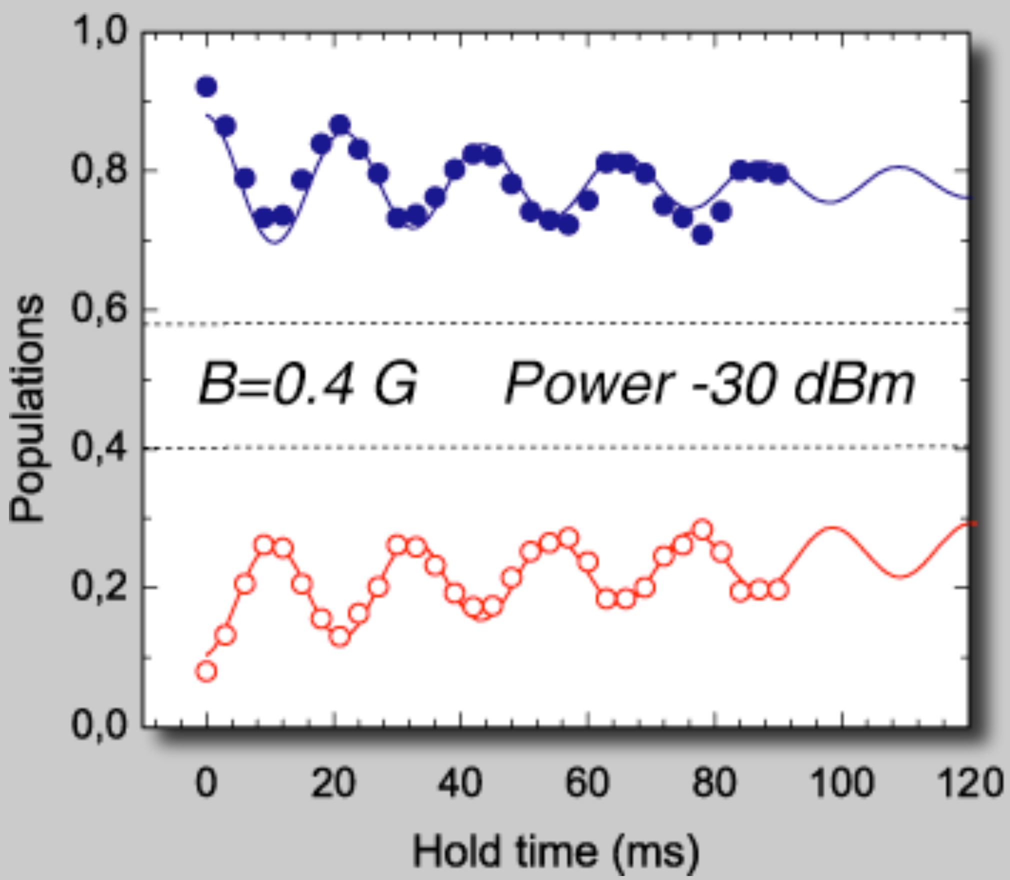
Energy shift due to microwave field can bring levels into resonance.

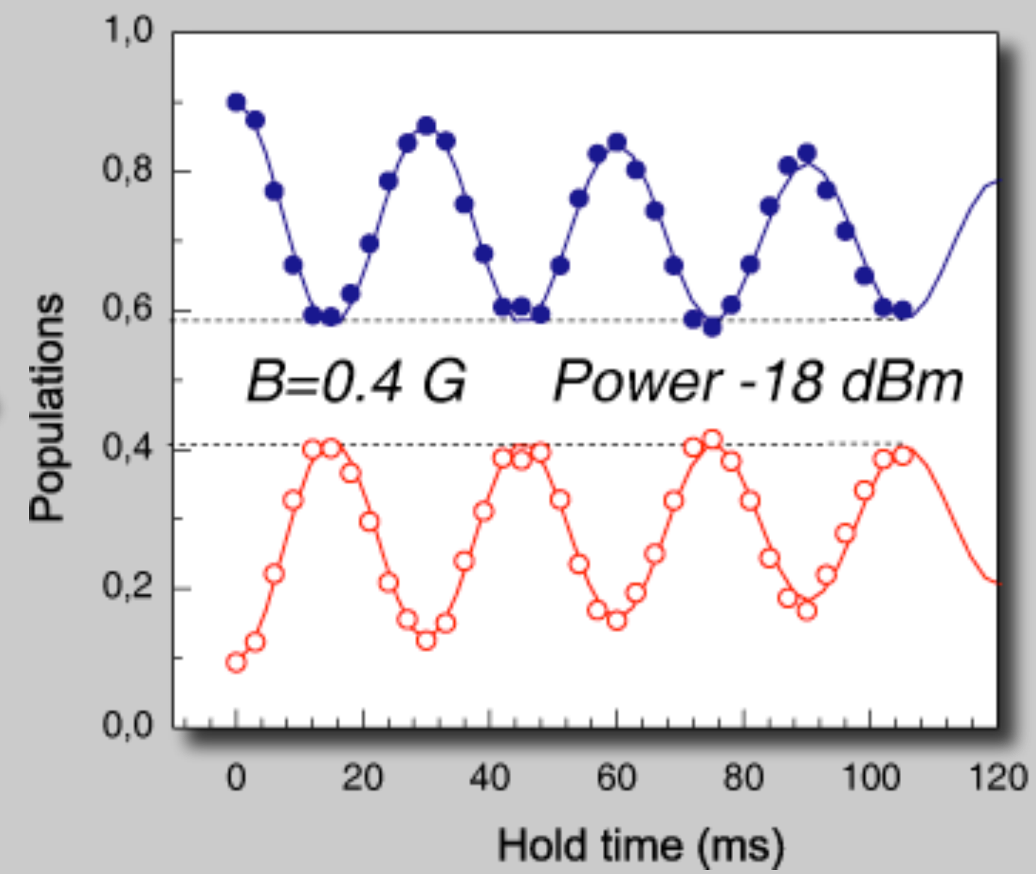
H. Pu and P. Meystre PRL 2000 and
Duan, Sorensen, Cirac, Zoller PRL 2000

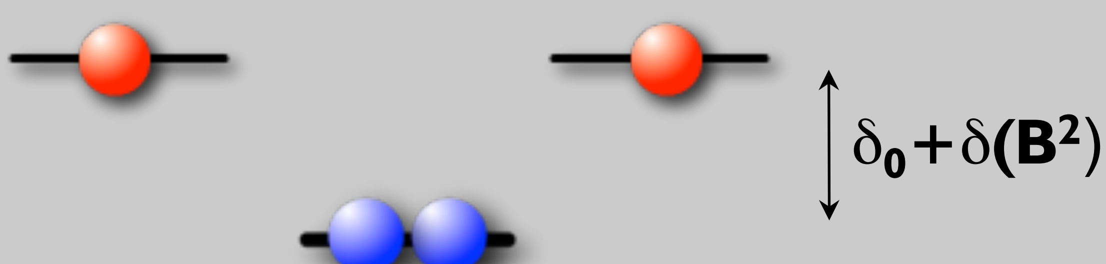
AC-Stark shift control of the resonance frequency

Energy shift can be tuned by power of the microwave and detuning

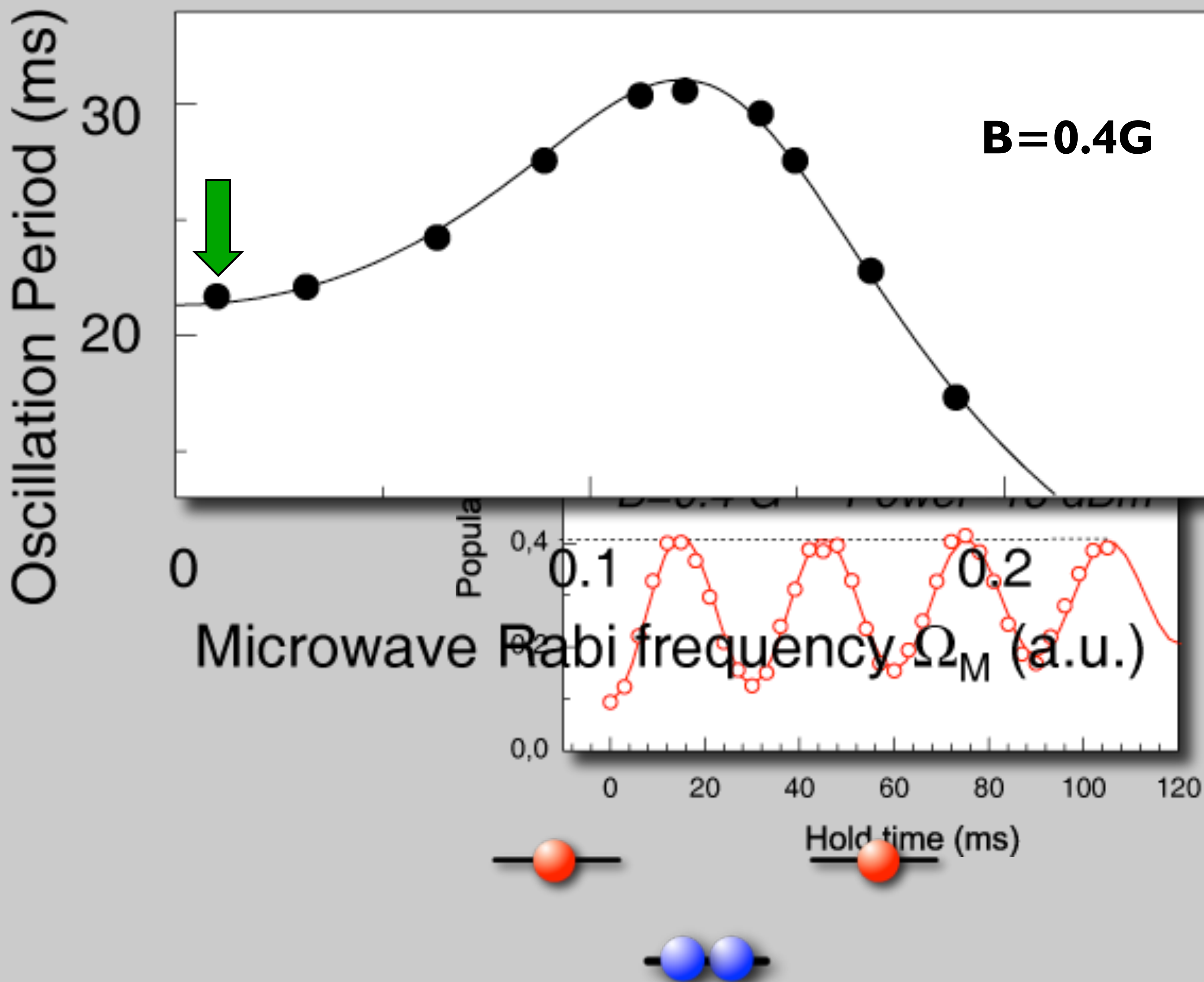
$$\Delta E \propto \frac{\Omega_M^2}{4\Delta}$$







AC-Stark shift control of the resonance frequency

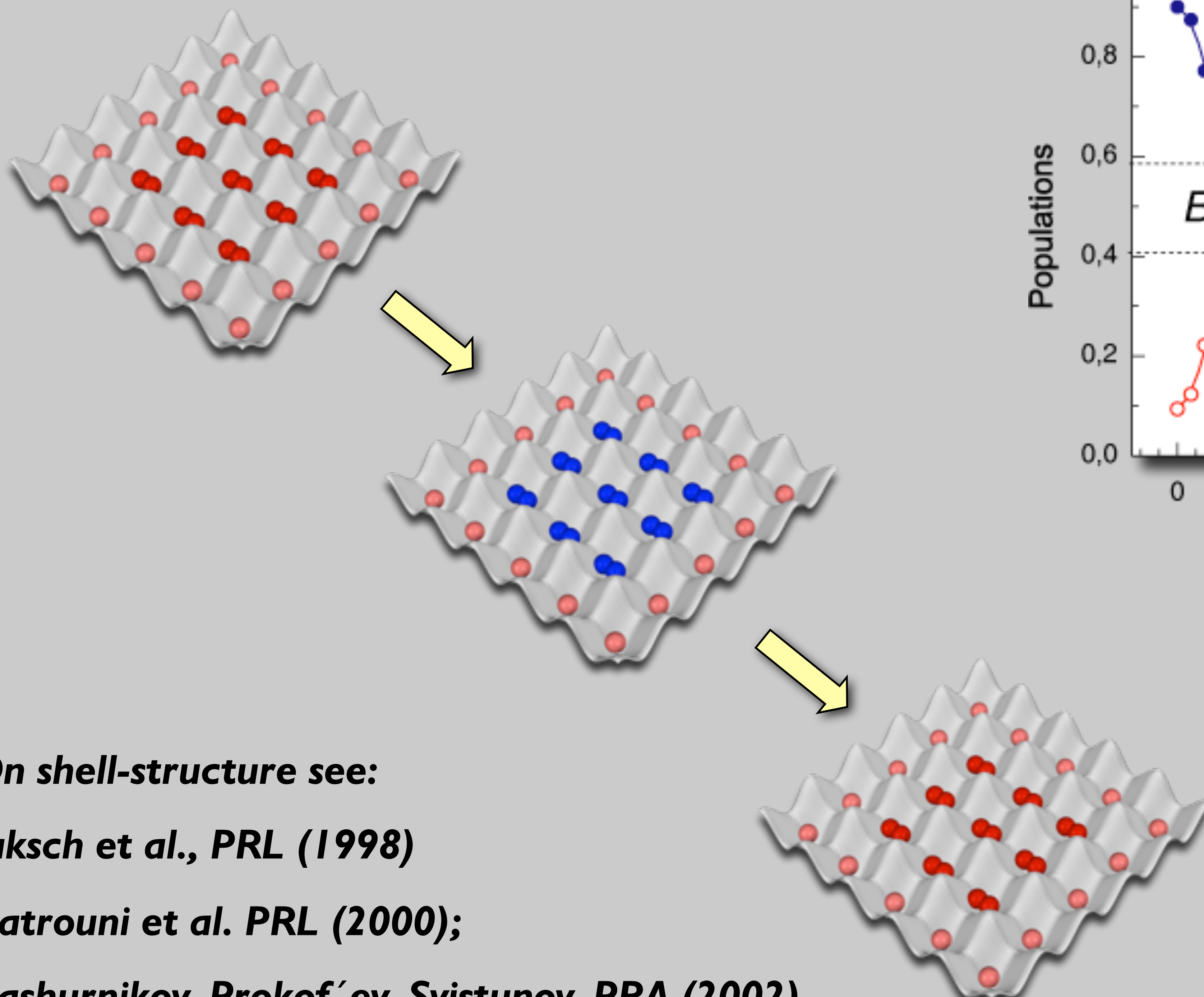


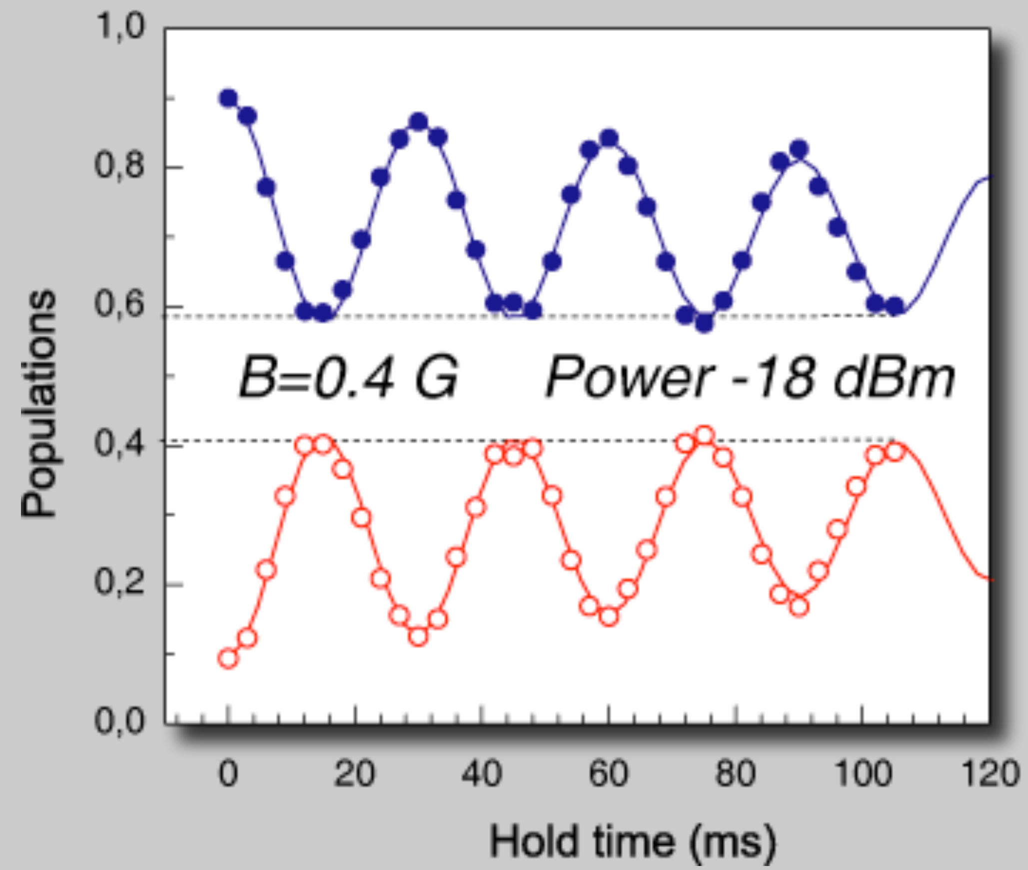
$$\frac{N_{+1}+N_{-1}}{N_{tot}} \propto \left(\frac{\Omega_0}{\Omega'}\right)^2$$

F. Gerbier et al., PRA **73**, 041602 (2006)

$\delta=\delta_0+O(B^2)$

Amplitude decrease due to single site spectators





Sensitive and non-destructive detector for doubly occupied lattice sites

On shell-structure see:

Jaksch et al., PRL (1998)

Batrouni et al. PRL (2000);

Kashurnikov, Prokof´ev, Svistunov, PRA (2002)

Alet et al., PRA (2004), recent work P. Denteneer

Quantum Spin Oscillations as Non-Destructive Probe of Atom Number Statistics

***Classical field** (mean field) limit (continuous frequencies)*

$$\Omega(n) \square c_2 n$$

***Quantum limit** (discrete frequencies)*

$$\Omega_{N_{at}} = \Omega_0 \sqrt{N_{at}\left(N_{at}-1\right)}$$

$$\Omega_2 = \sqrt{2}\Omega_0 \quad \Omega_3 = \sqrt{6}\Omega_0 \quad \Omega_4 = \sqrt{12}\Omega_0 \quad \ldots$$

Leads to quantum dynamics beyond mean field!
Collapse & Revivals, Cat states, etc.

Cf. Work of L. You, J. Ho,…

Amplitude of Spin-Changing Oscillations at Freq Ω_{Nat} ⟺ ***Number of sites with N_{at} atoms***

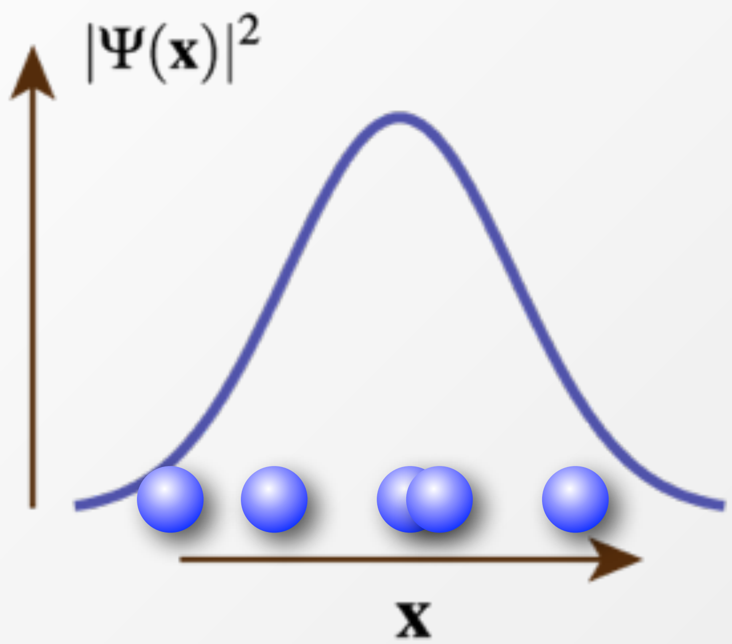
Resembles exp. in Cavity QED to reveal photon number statistics (Haroche, Walther)
see also work of G. Campbell et al. (MIT)

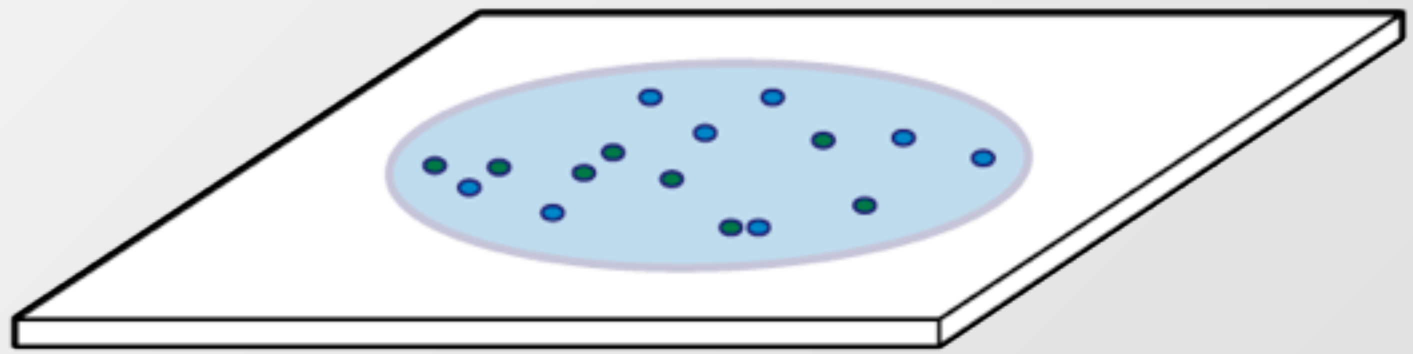
Single Atom Detection in a Lattice

S. Kuhr, J. Sherson, Ch. Weitenberg, M. Endres, M. Cheneau, T. Fukuhara, P. Schauss

Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

Measuring a Quantum System





Single Particle

Correlated 2D Quantum Liquid

$\Psi(\mathbf{x})$ wave function

$|\Psi(\mathbf{x})|^2$ probability distribution

$\Psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_N)$

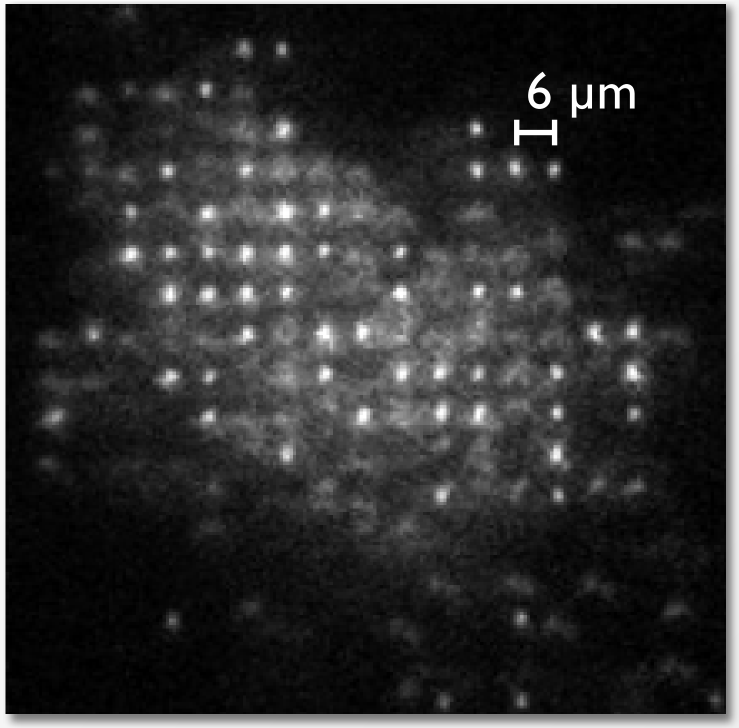
$|\Psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_N)|^2$

averaging over single-particle measurements, we obtain $|\Psi(\mathbf{x})|^2$

For many-body system: need access to single snapshots of the many-particle system!

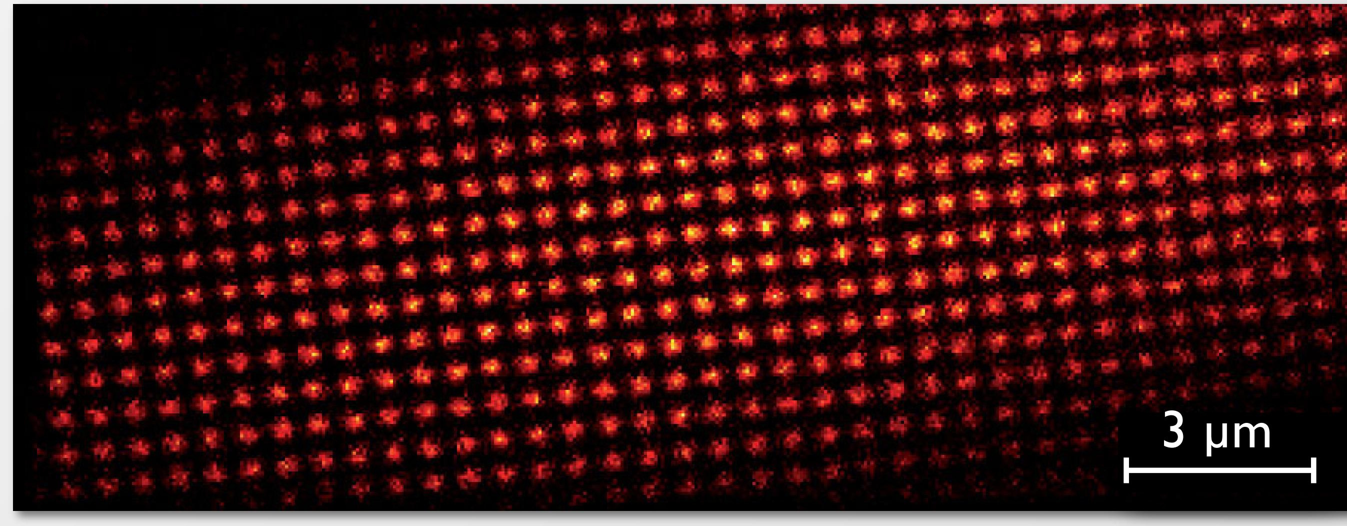
State of the Art

David Weiss, Pennsylvania State University
Nature Physics 3, 556 (2007)



fluorescence imaging

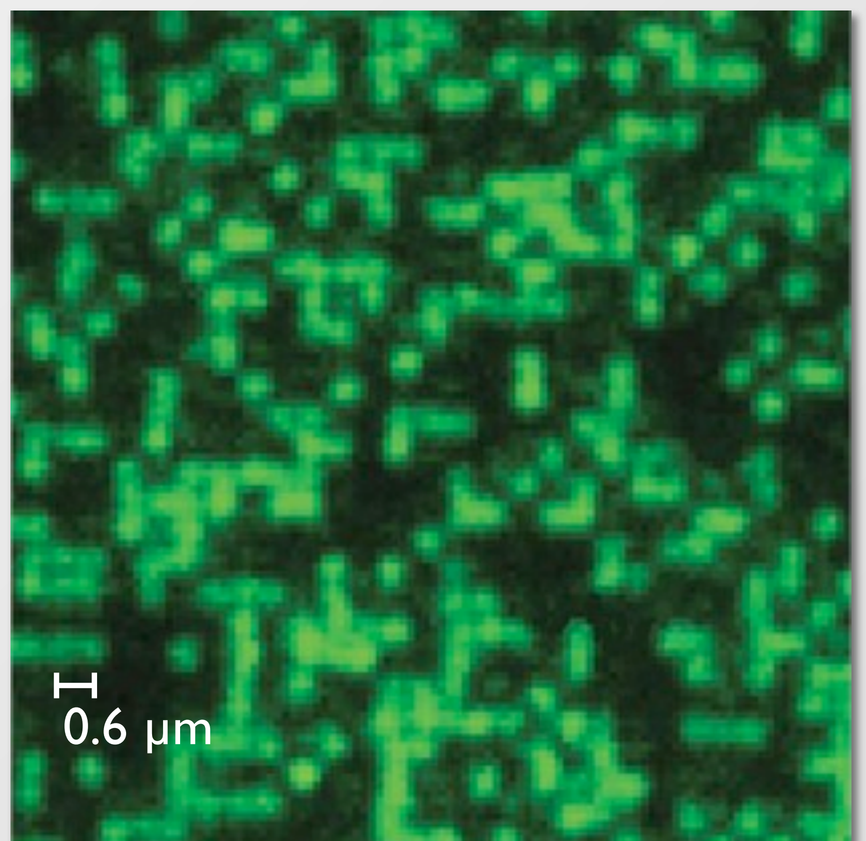
Herwig Ott, Mainz University
Phys. Rev. Lett. 103, 080404 (2009)



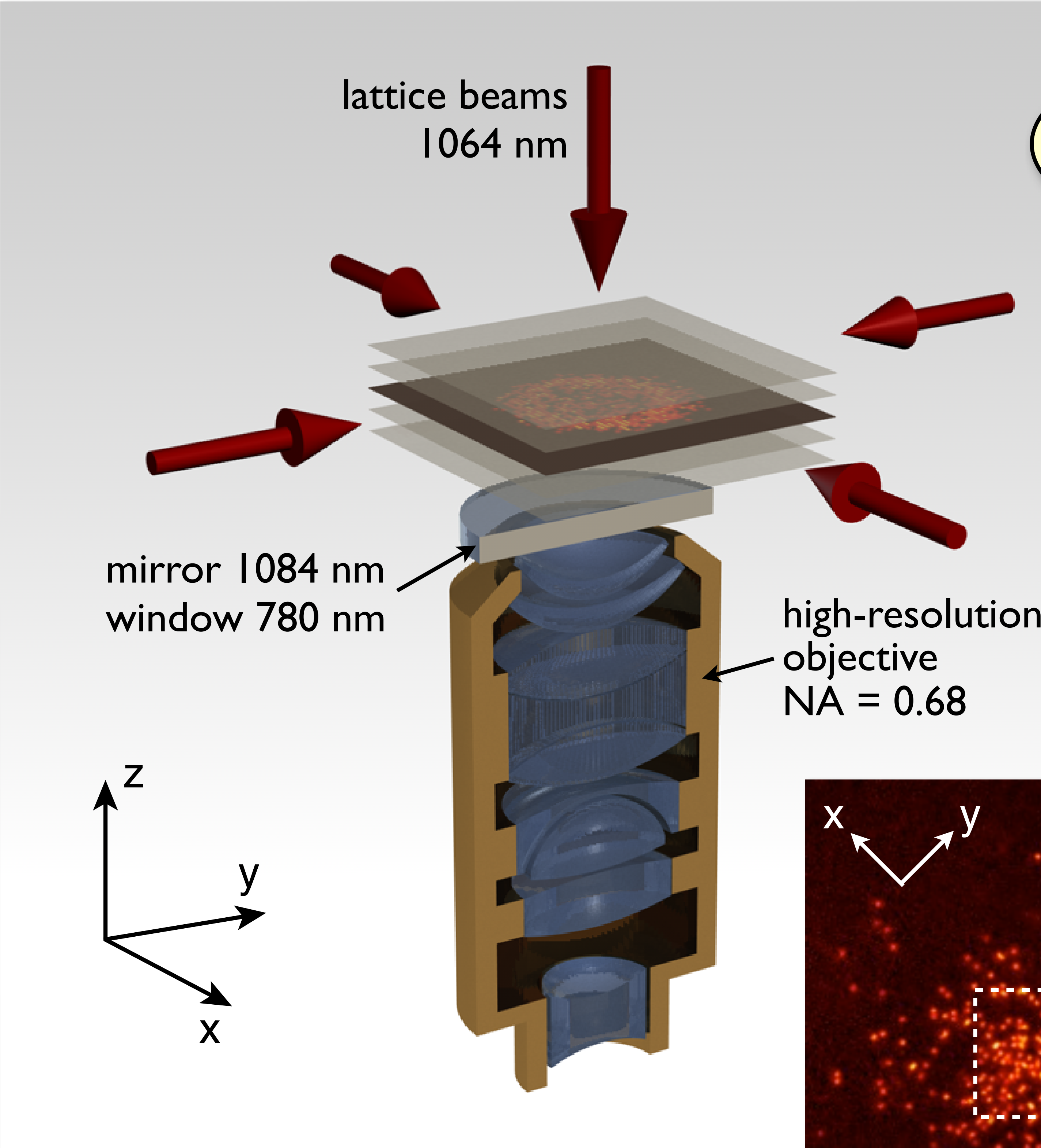
electron microscopy

Markus Greiner, Harvard
Nature 462, 74 (2009)

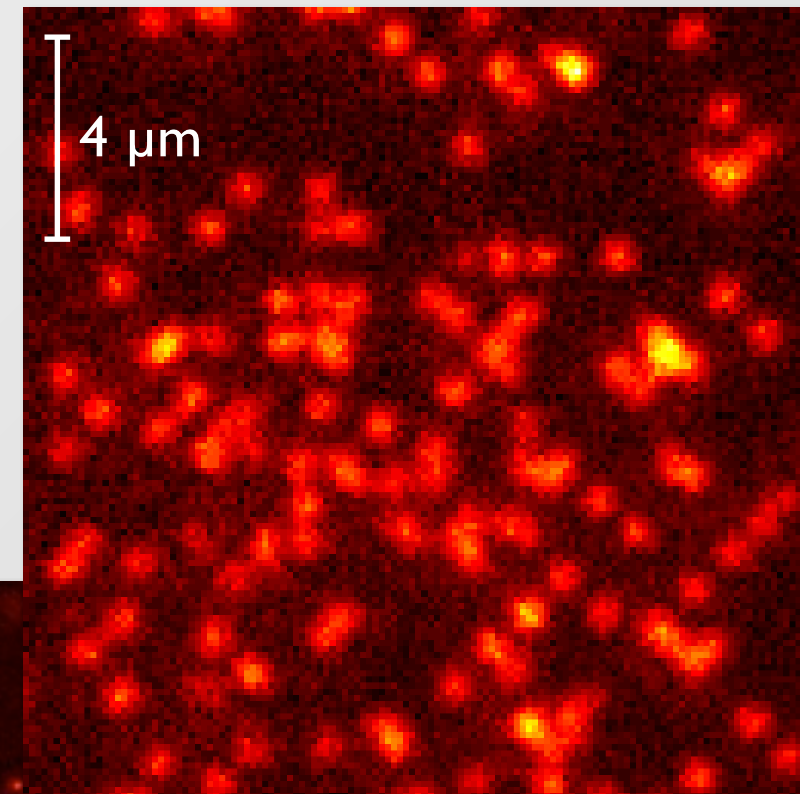
fluorescence imaging

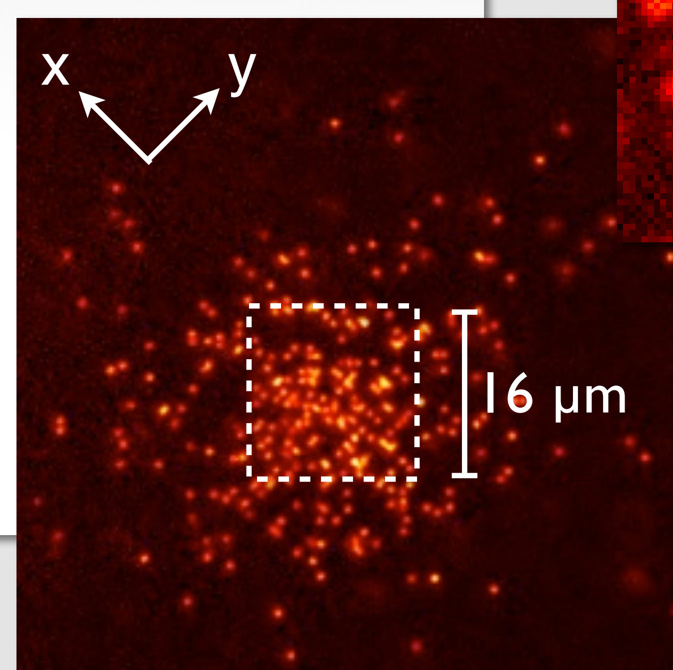


Experimental Setup



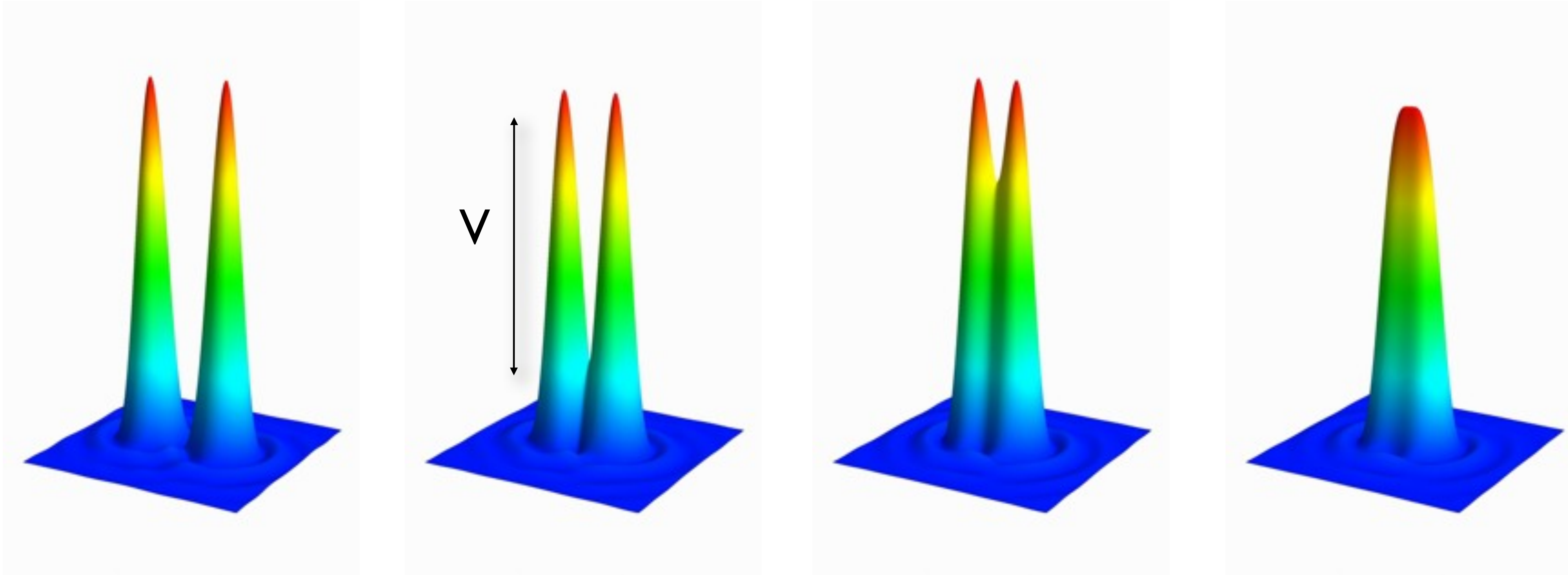
single 2D degenerate gas
~ 1000 ^{87}Rb atoms (bosons)





resolution of the imaging system: ~700 nm

Imaging Resolution: Definitions

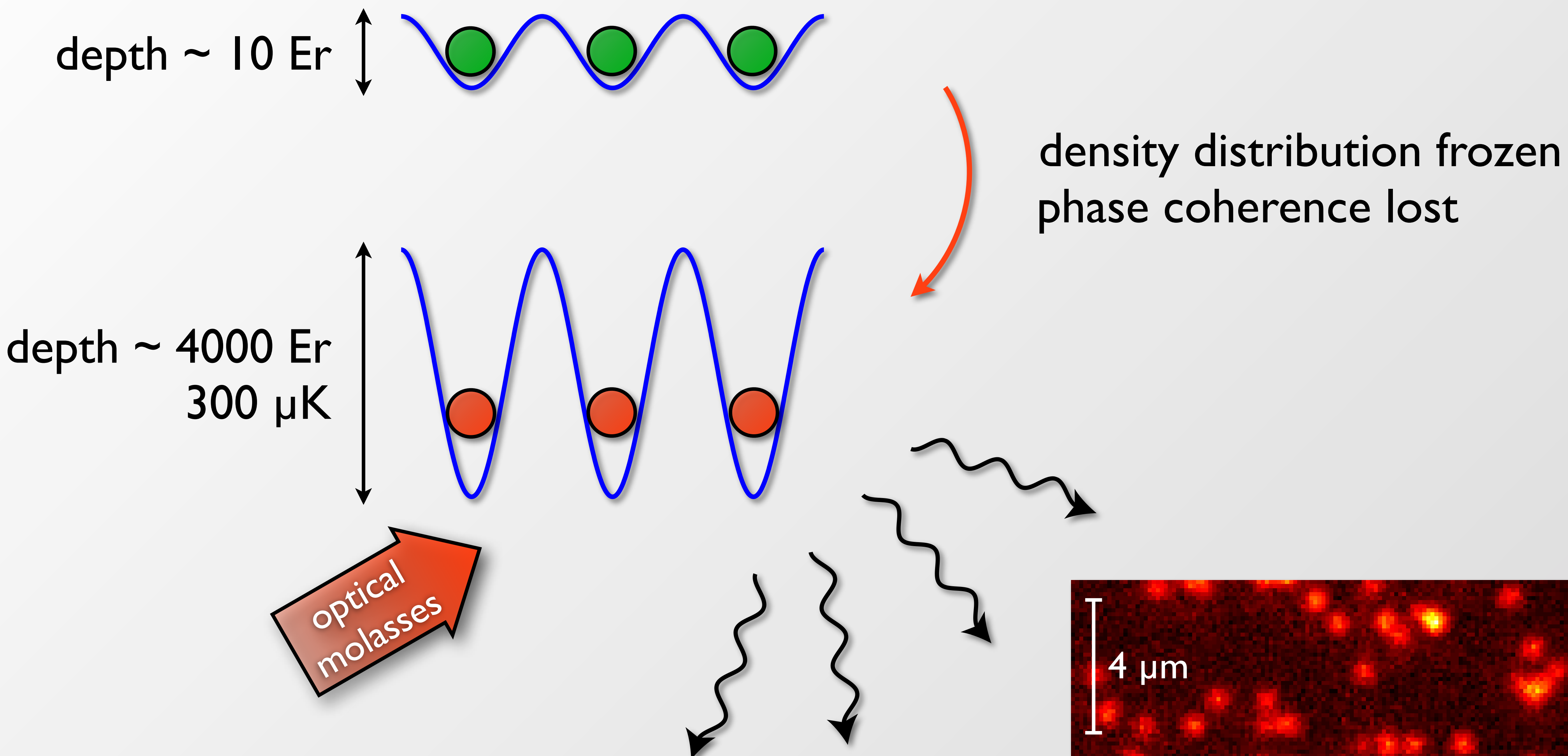


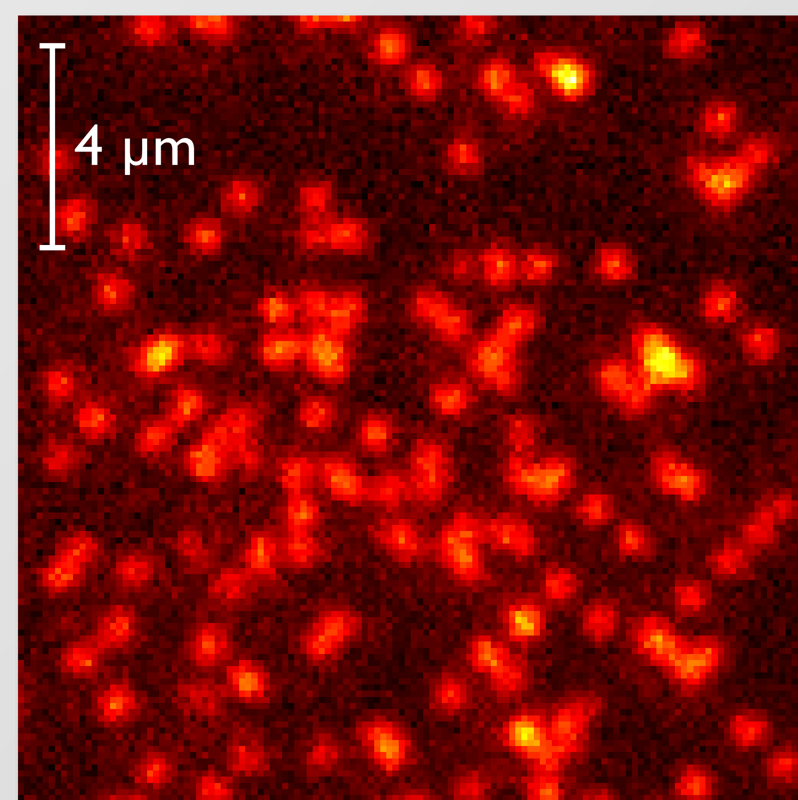
total resolution	visible resolution	Rayleigh criterion	Sparrow criterion
(minima of PSF overlap)	(contrast 0.59)	(maximum and zero overlap)	(curvature vanishes)
V=1	V=0.59	V=0.15	V=0
1400 nm	**953 nm**	**700 nm**	**544 nm**

⇨ Resolution by the criteria for NA=0.68 and λ = 780 nm. Lattice spacing is **532 nm**

But: we know the lattice structure! Only need to reconstruct configuration on lattice.

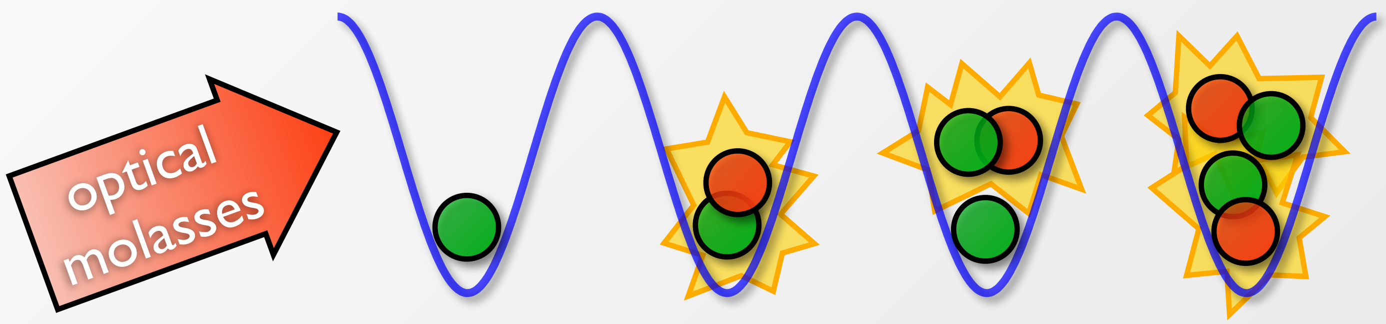
Fluorescence imaging





fluorescence rate / atom: 60 kHz
~ 5000 photons / atom collected in 900ms

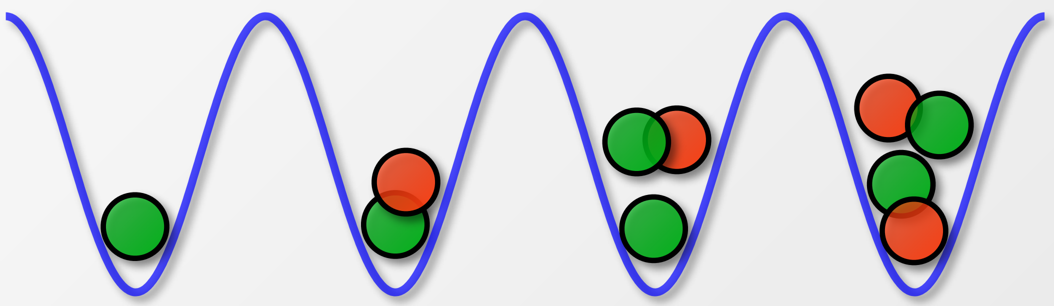
Parity projection



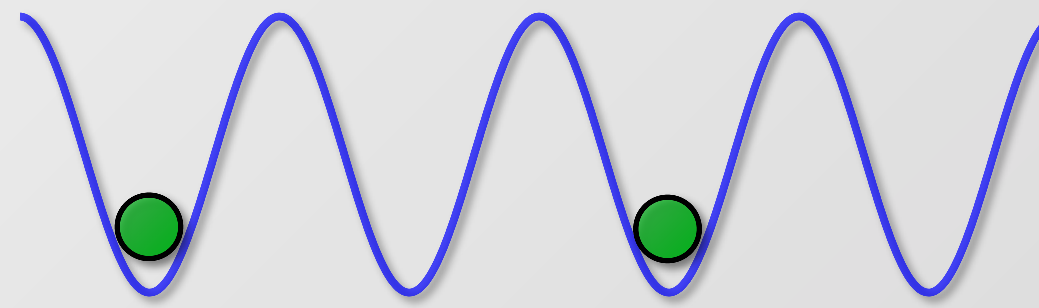
dePue *et al.*, PRL **82**, 2262 (1999)

Light-induced collisions

initial density distribution



measured density distribution



measured occupation: $n_{\mathrm{det}} = \mathrm{mod}_2 n$

measured variance: $\sigma^2_{\mathrm{det}} = \langle n^2_{\mathrm{det}} \rangle - \langle n_{\mathrm{det}} \rangle^2$

parity projection $\Rightarrow$ $\langle n^2_{\mathrm{det}} \rangle = \langle n_{\mathrm{det}} \rangle$

see also E. Kapit & E. Mueller, Phys. Rev. A **82**, 013644 (2010)

Parity projection

Weakly interacting BEC

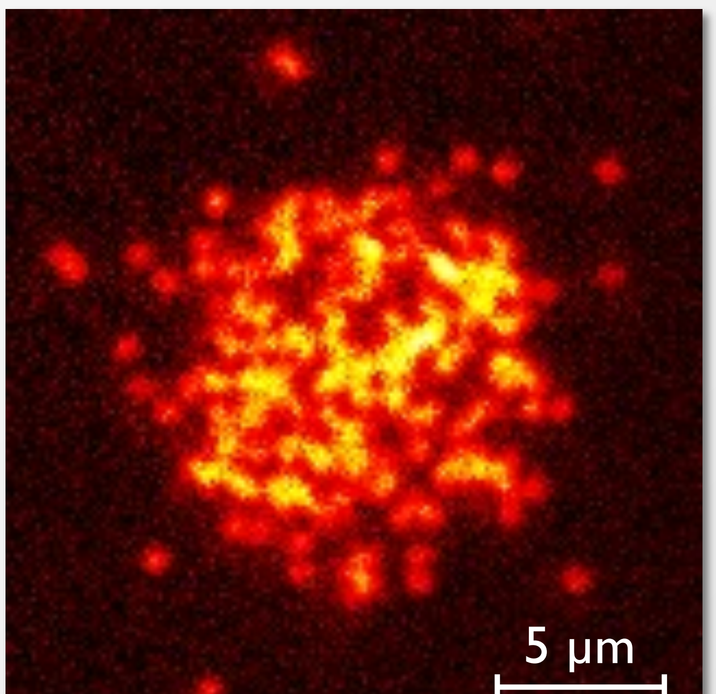
$$|\alpha\rangle = e^{-|\alpha^2|/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

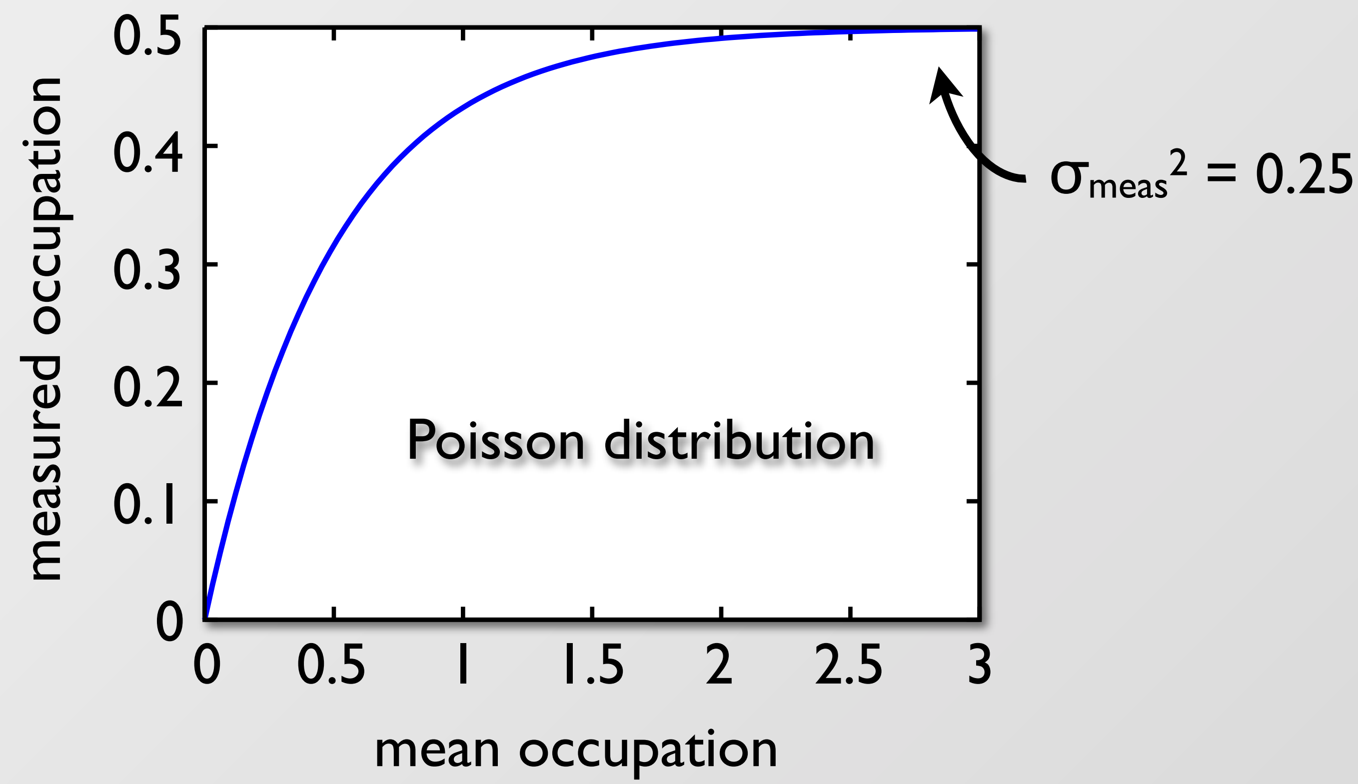
Coherent State

$$p_n = \frac{e^{-\bar{n}} \bar{n}^n}{n!}$$

On-Site Poissonian
atom number distribution

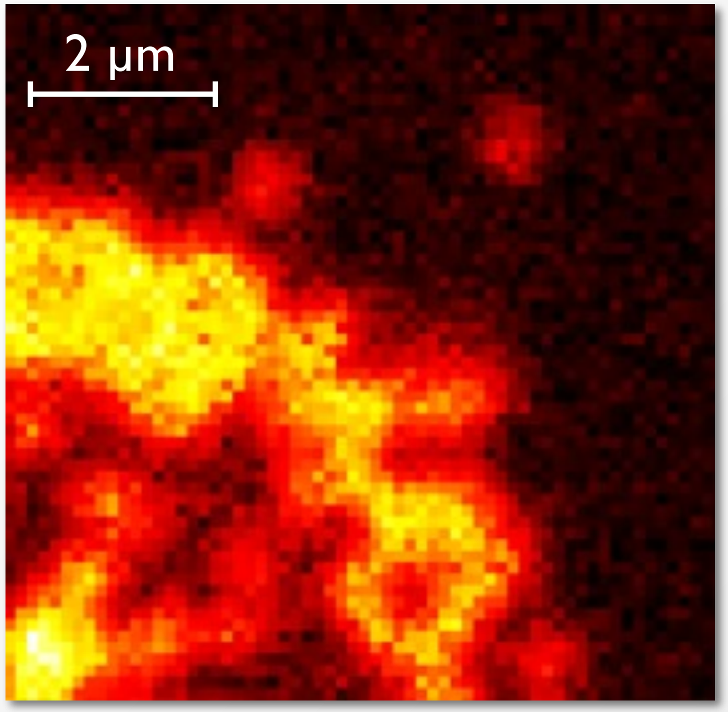
Measured density of a quasi-BEC



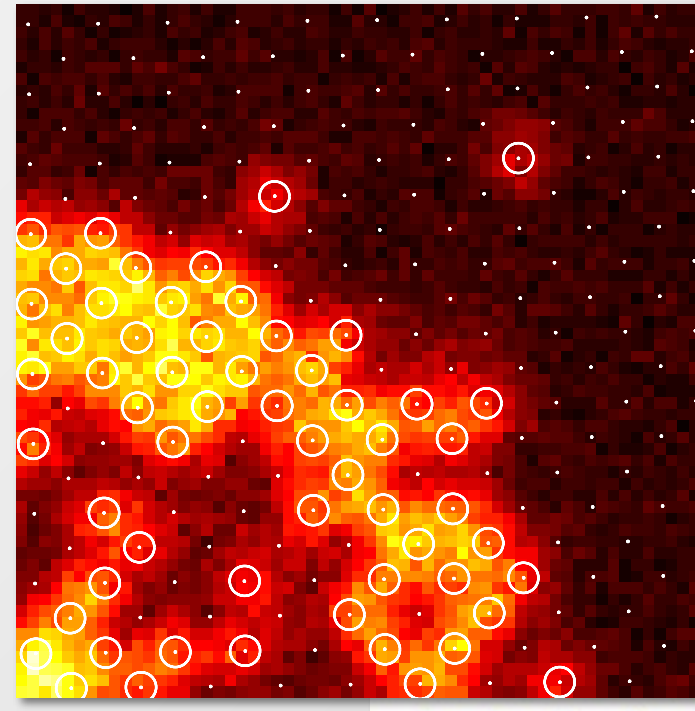


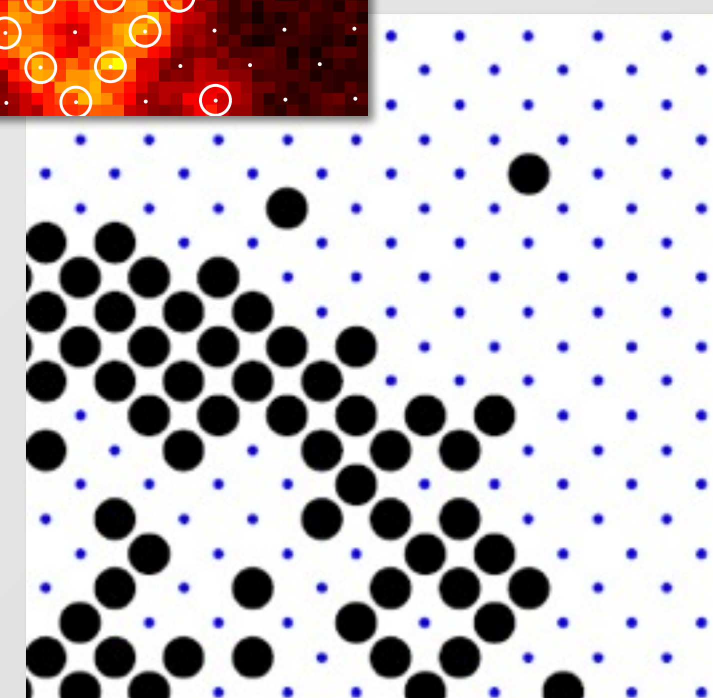
$$\bar{n}_{\text{det}} = \sum_n p_n \,\text{mod}_2 n = \sum_{n\,\text{odd}} p_n = \frac{1}{2}\left(1 - e^{-2\bar{n}}\right)$$

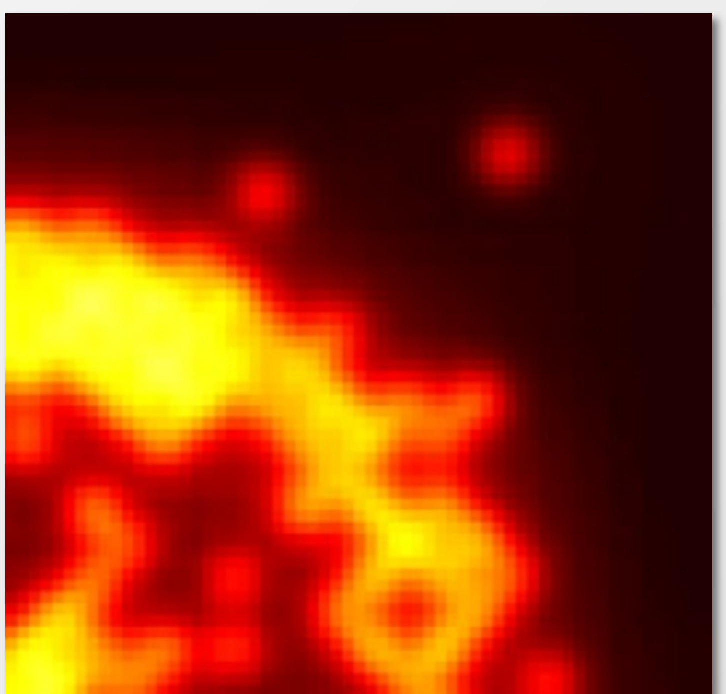
Reconstruction of site occupation



Reconstruction algorithm







Digitized image convoluted with point-spread function

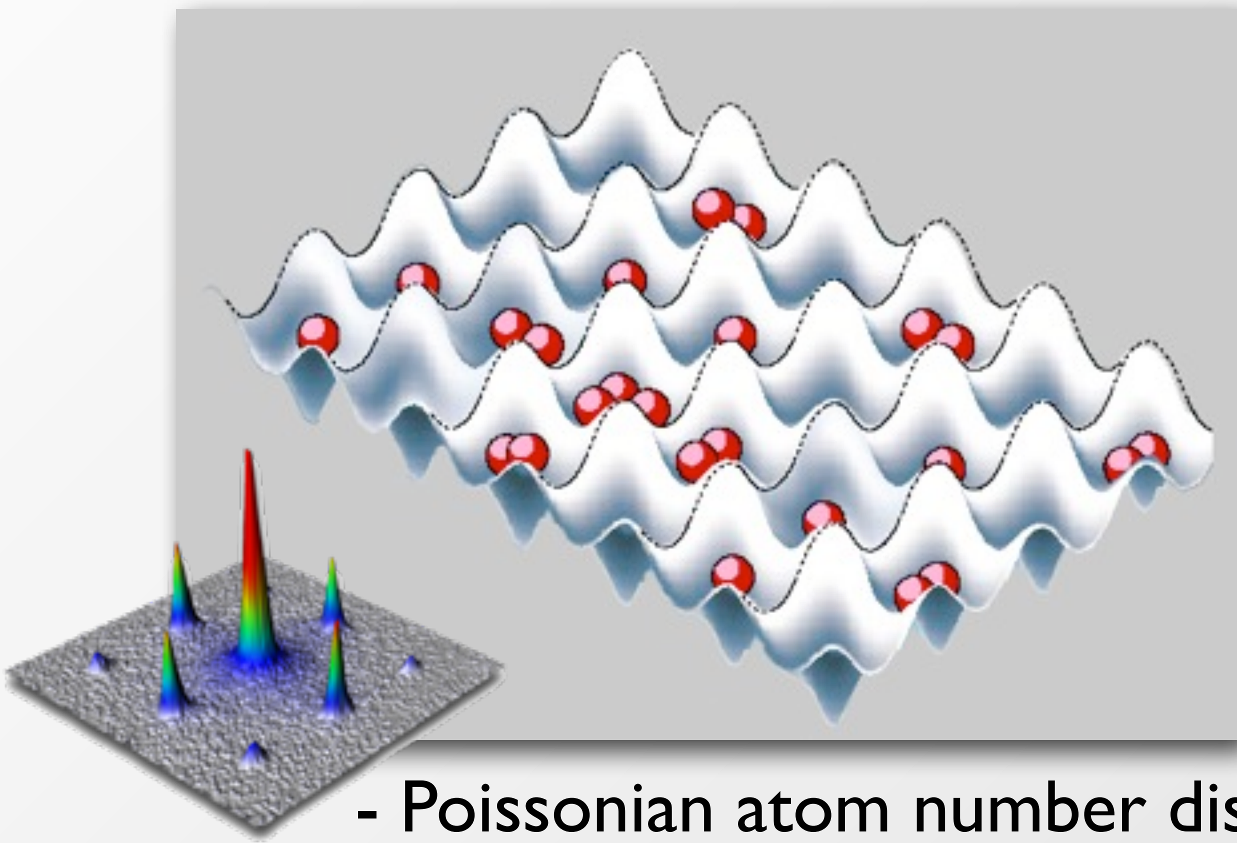
digitized image
no experimental noise

In-Situ Imaging of a Mott Insulator

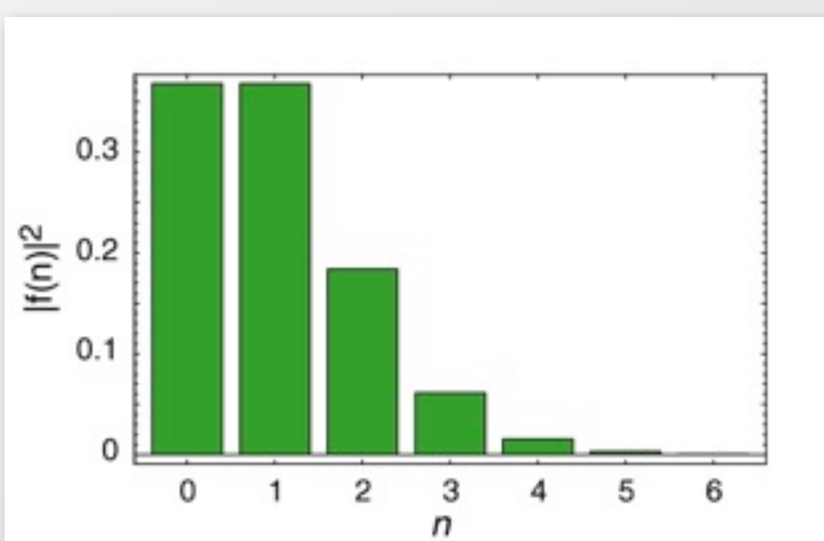
Sherson et al. Nature **467**, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

Mott Insulators

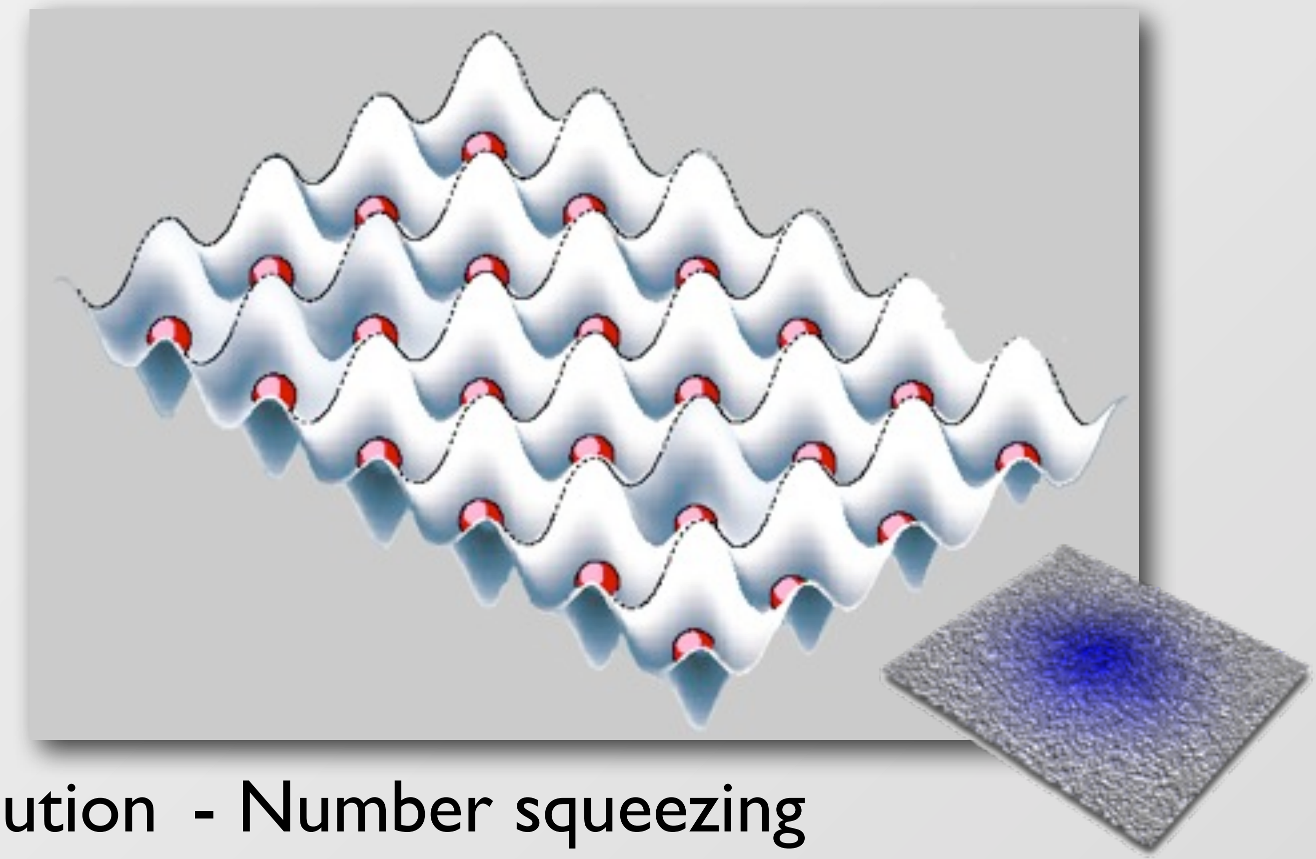
Superfluid



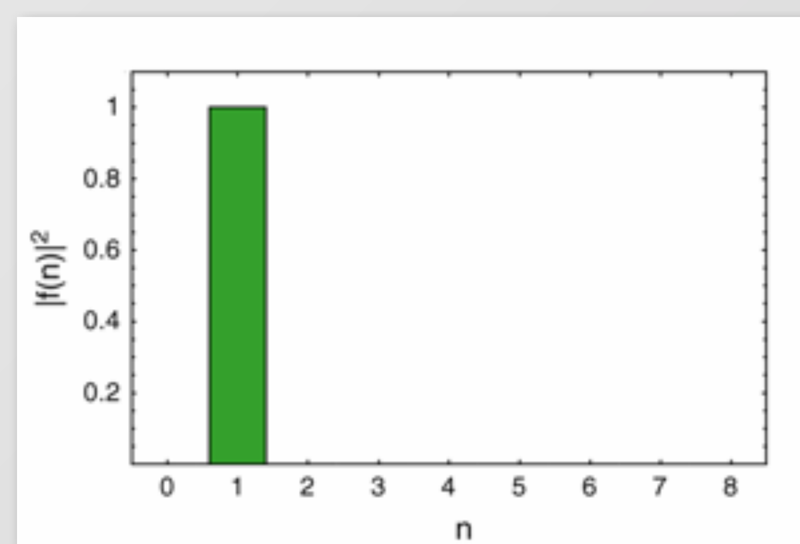
- Poissonian atom number distribution
- Long range phase coherence



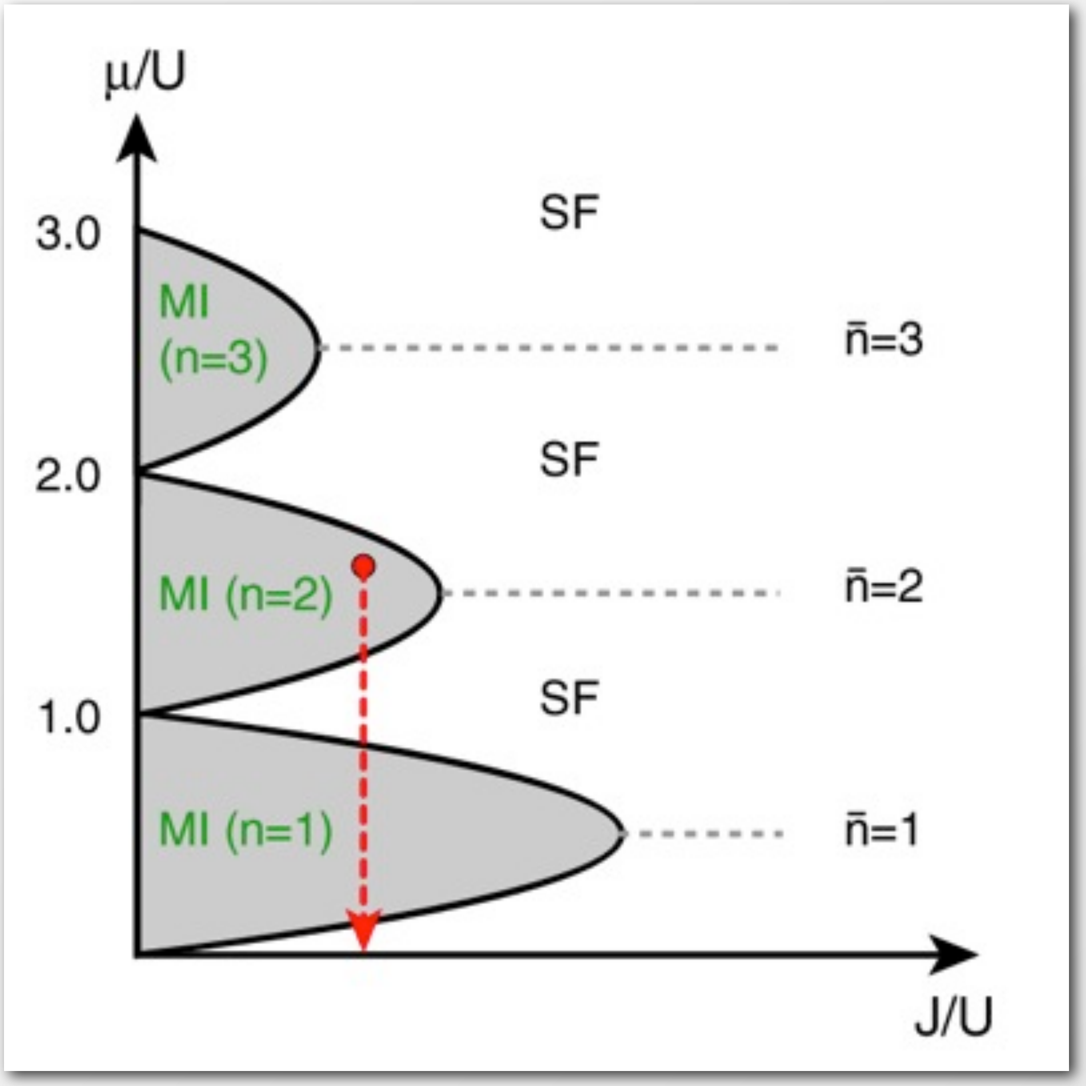
Mott-Insulator



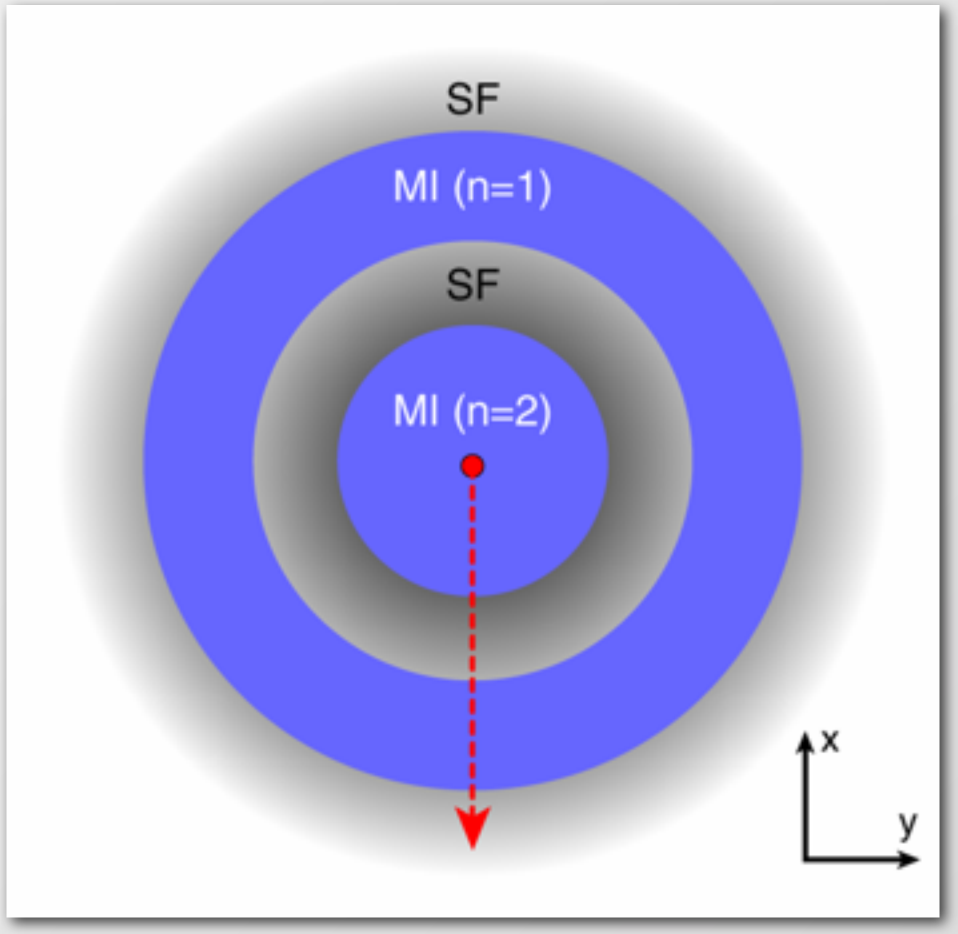
- Number squeezing
- No phase coherence



Bose-Hubbard Phase Diagram



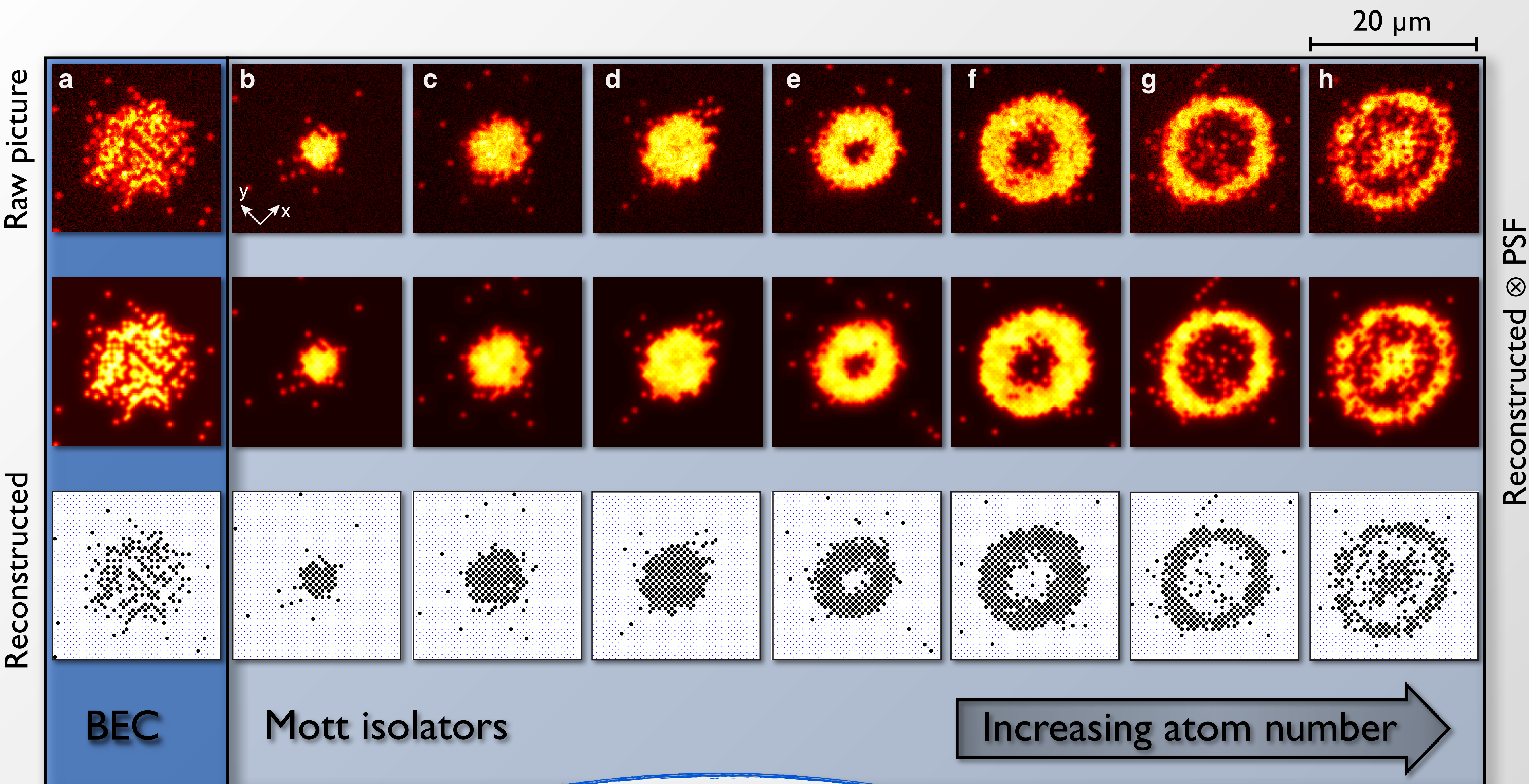
M.P.A. Fisher *et al.* PRB **40**, 546 (1989)
D. Jaksch *et al.* PRL **81**, 3108 (1998)



Inhomogeneous system:
effective local chemical potential

$$\mu_{loc} = \mu - \varepsilon_i$$

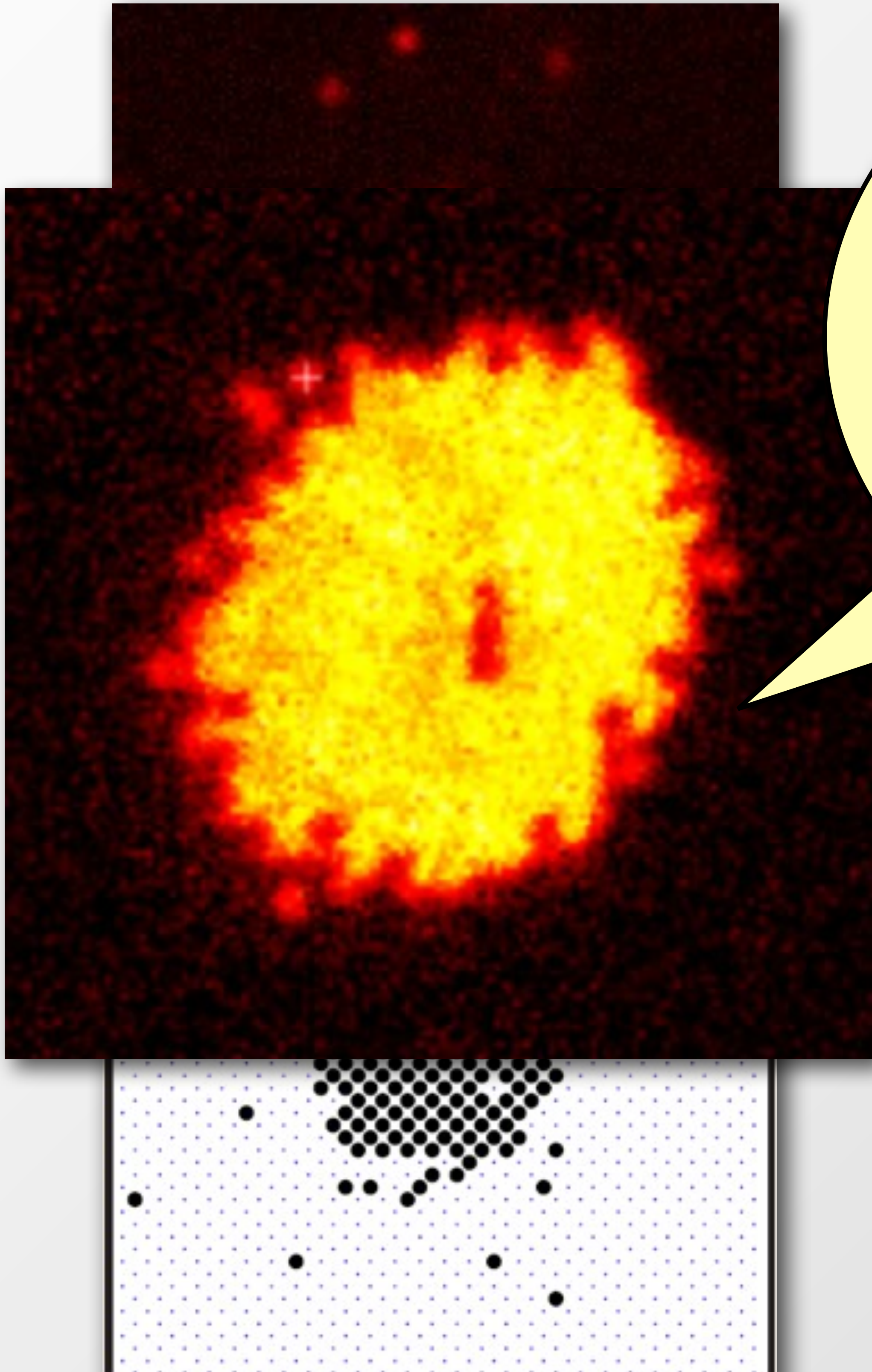
In-situ observation of a Mott insulator



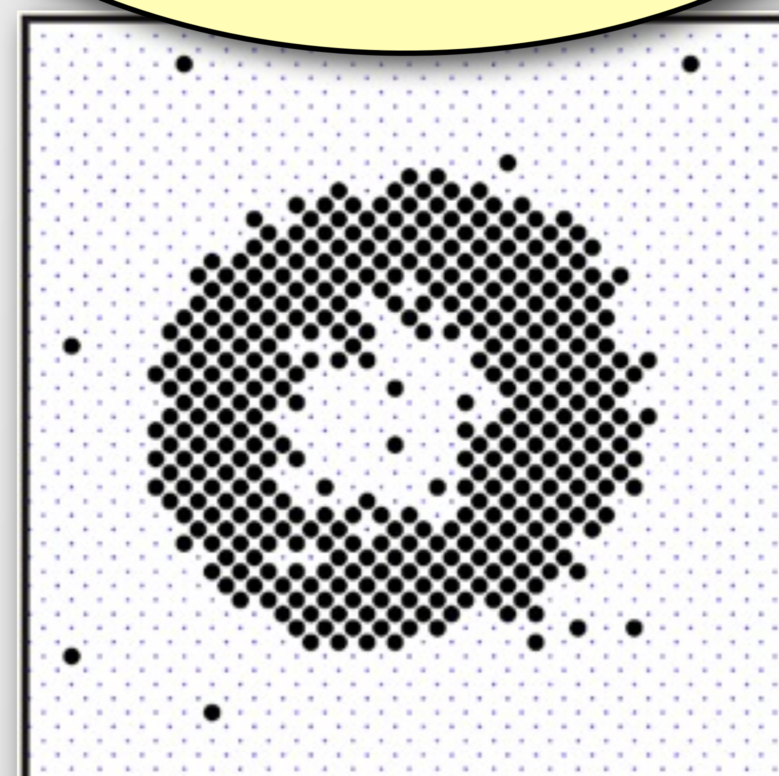
for the Mott insulators: U/J ~ 300 (critical U/J ~ 16)

⇒ only thermal fluctuations

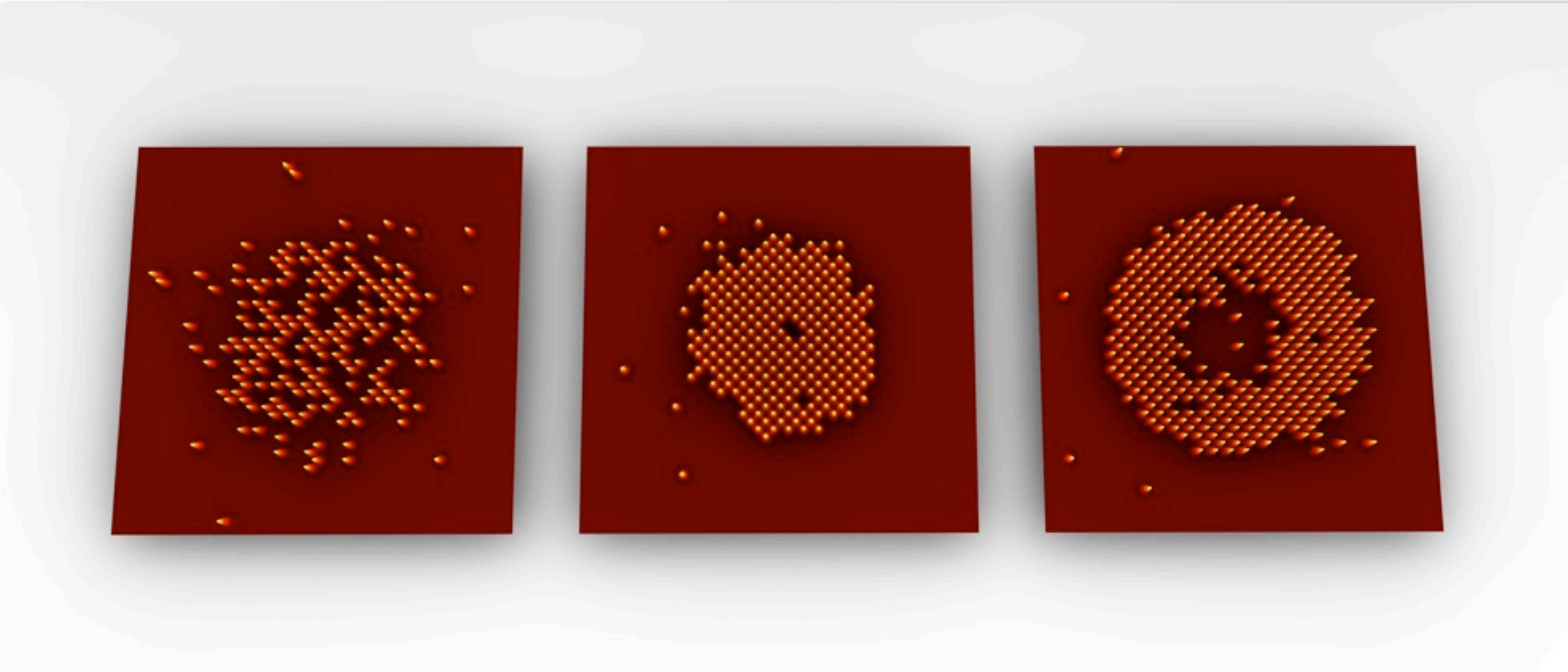
Images of Mott Insulators



Towards larger n=1 MI
300-400 Atoms
>97-98% preparation fidelity



Reconstructed Atom Distribution



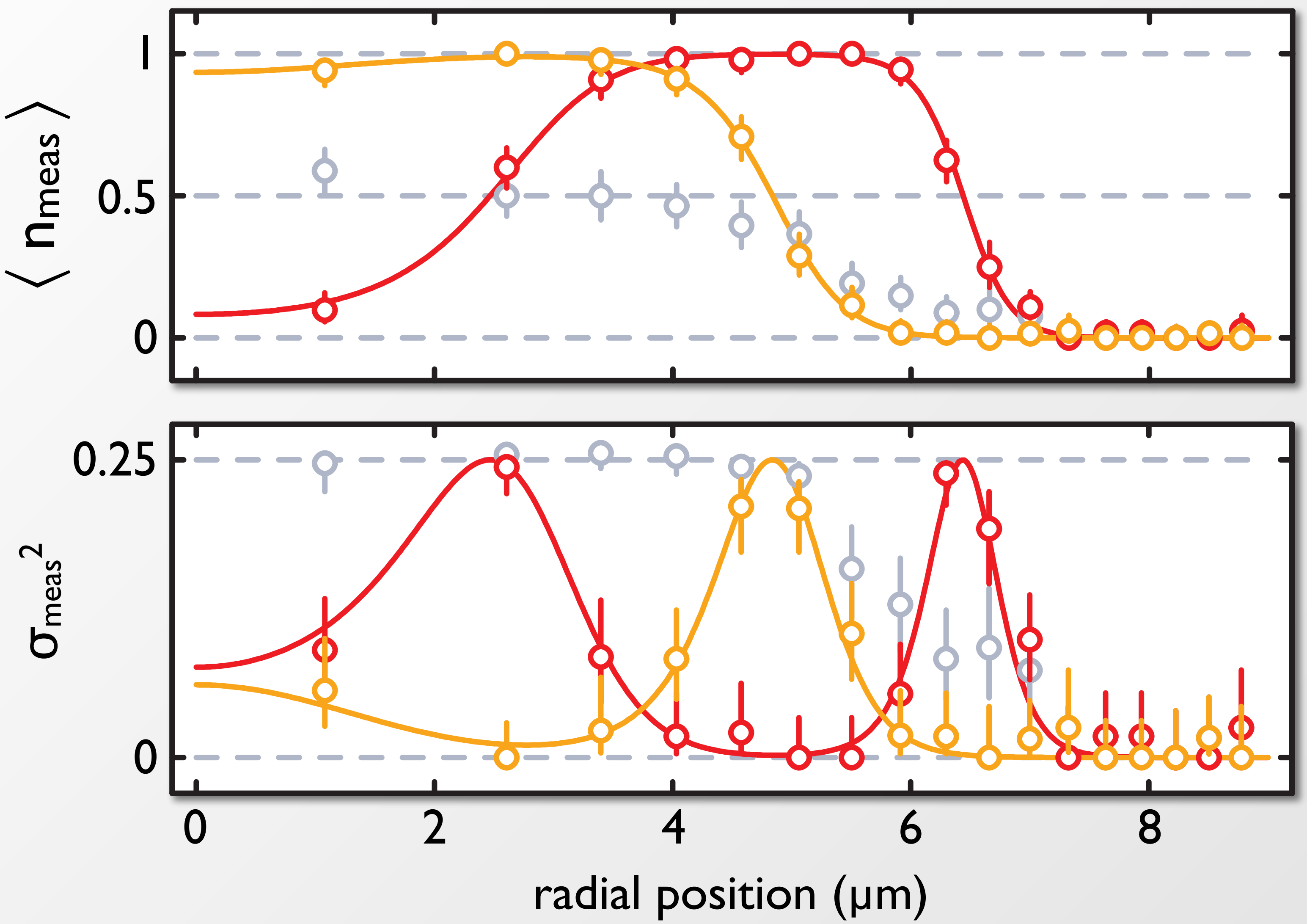
BEC

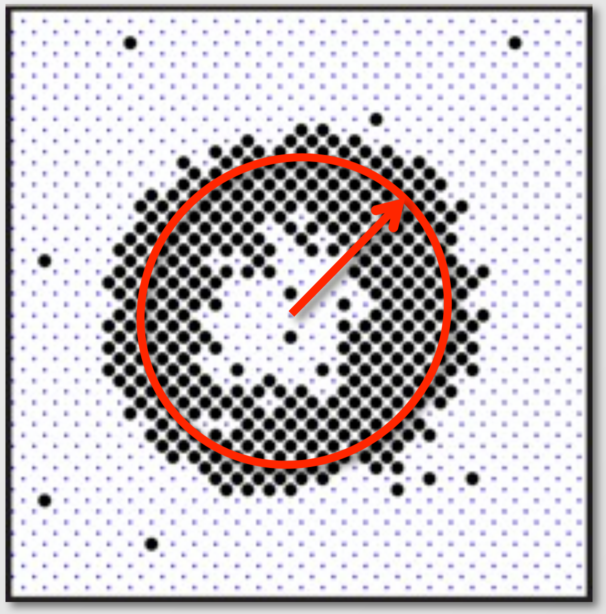
n=1
Mott Insulator

n=1 & n=2
Mott Insulator

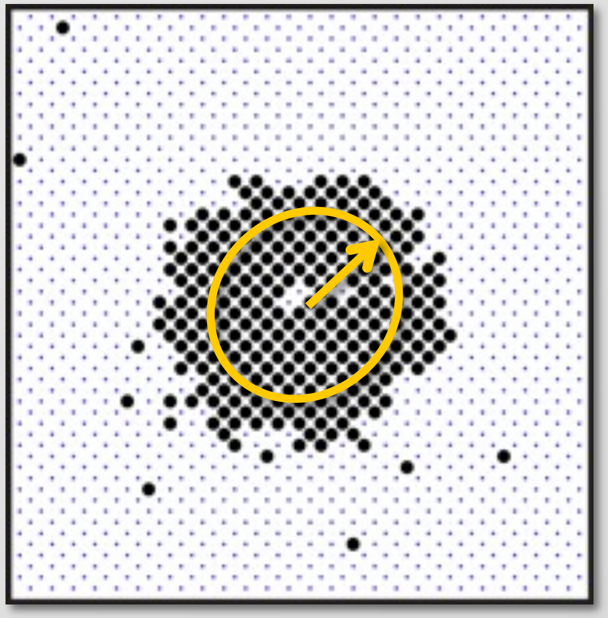
J. Sherson *et al.* Nature **467**, 68 (2010)

Analysis of the site occupancy





T = 0.074(5) U/k_B, μ = 1.17(1) U
N = 610(20)



T = 0.090(5) U/k_B, µ = 0.73(3) U
N = 300(20)

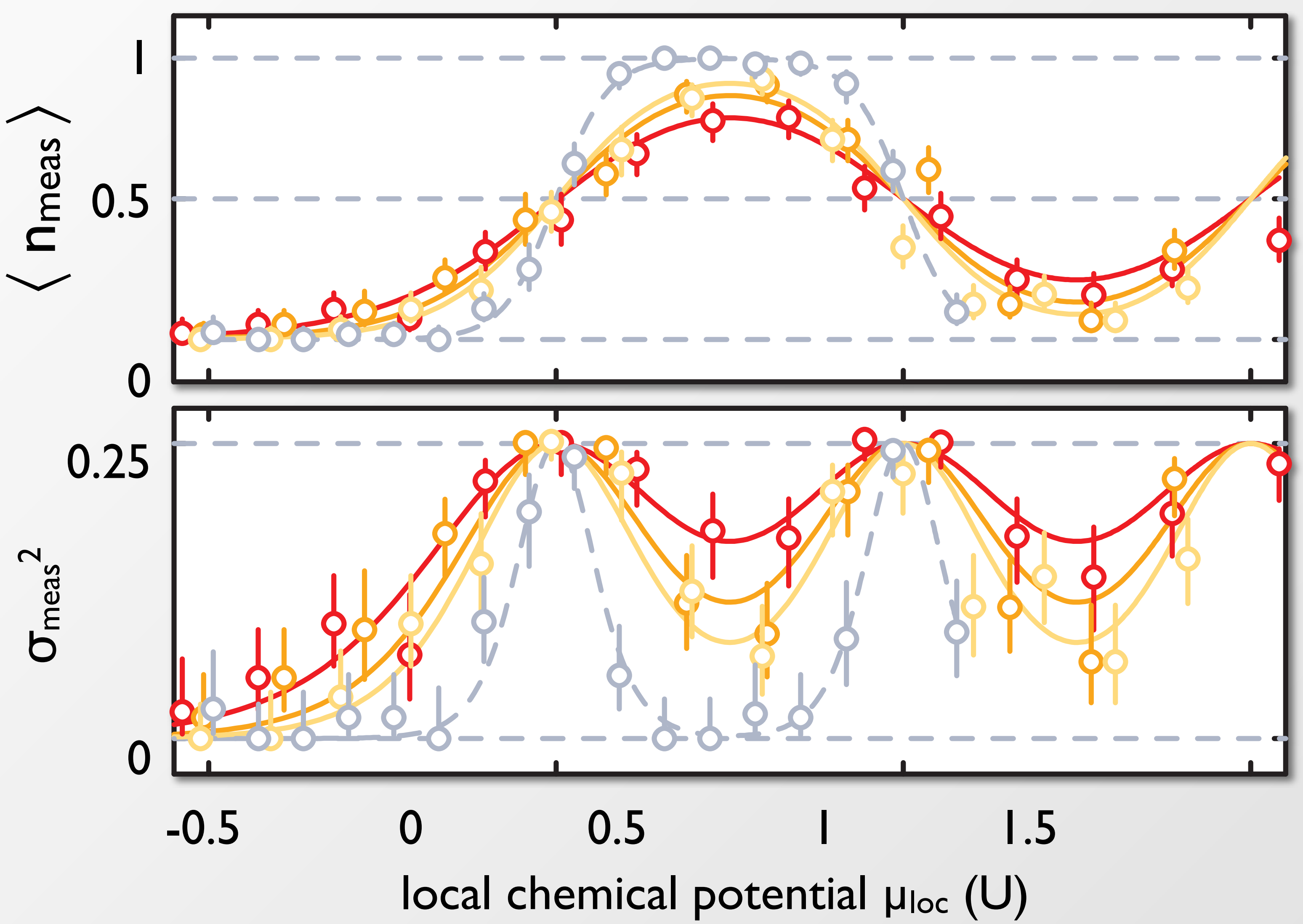
Simple Theory - Atomic Limit Mott Insulator

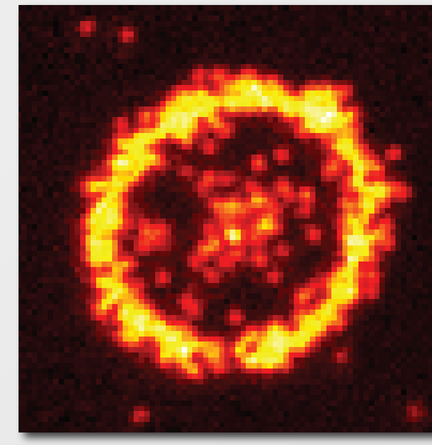
occupation probability: $p_n(r) = \dfrac{e^{-\beta(E_n - \mu(r)n)}}{Z(r)}$

interaction energy: $E_n = \dfrac{1}{2} U n(n-1)$

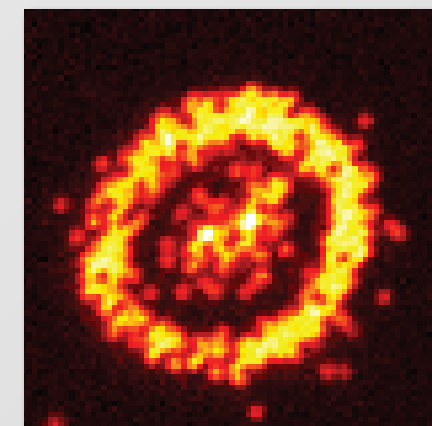
fit parameters: $T/U, \mu/U, U/\omega^2$

"Melting" of a Mott insulator

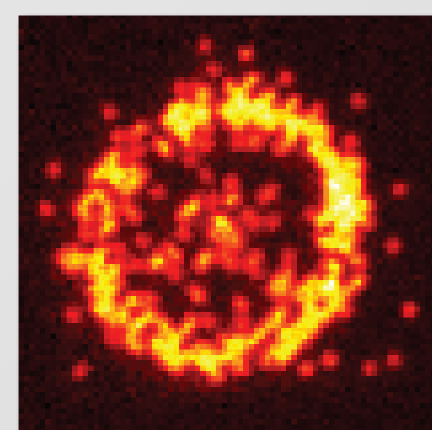




T = 0.17(1) U/kB
μ = 2.08(4) U



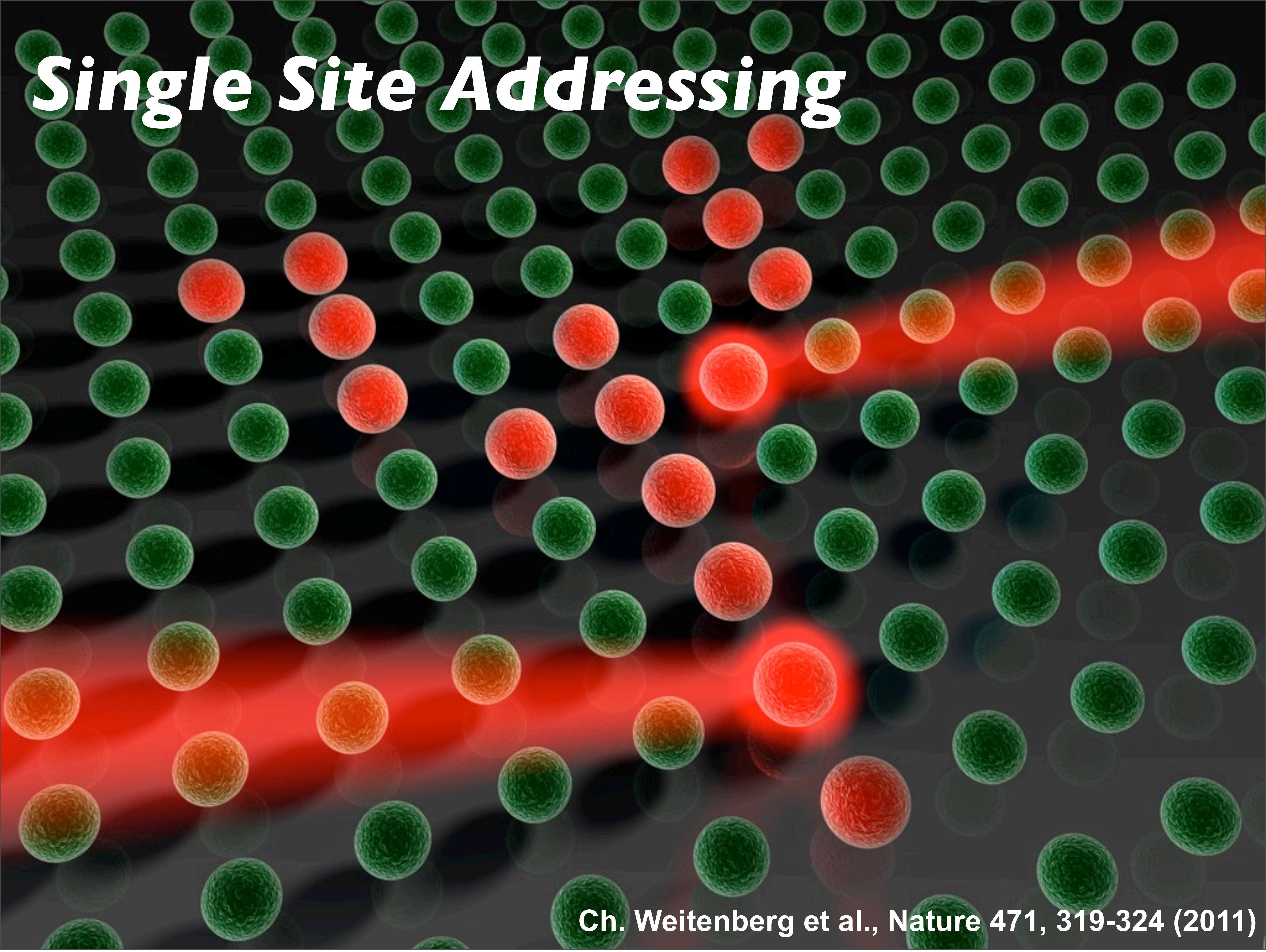
T = 0.20(2) U/kB
μ = 2.10(5) U



T = 0.25(2) U/kB
μ = 2.06(7) U

F. Gerbier, *PRL* **99**, 120405 (2007)

Single Site Addressing



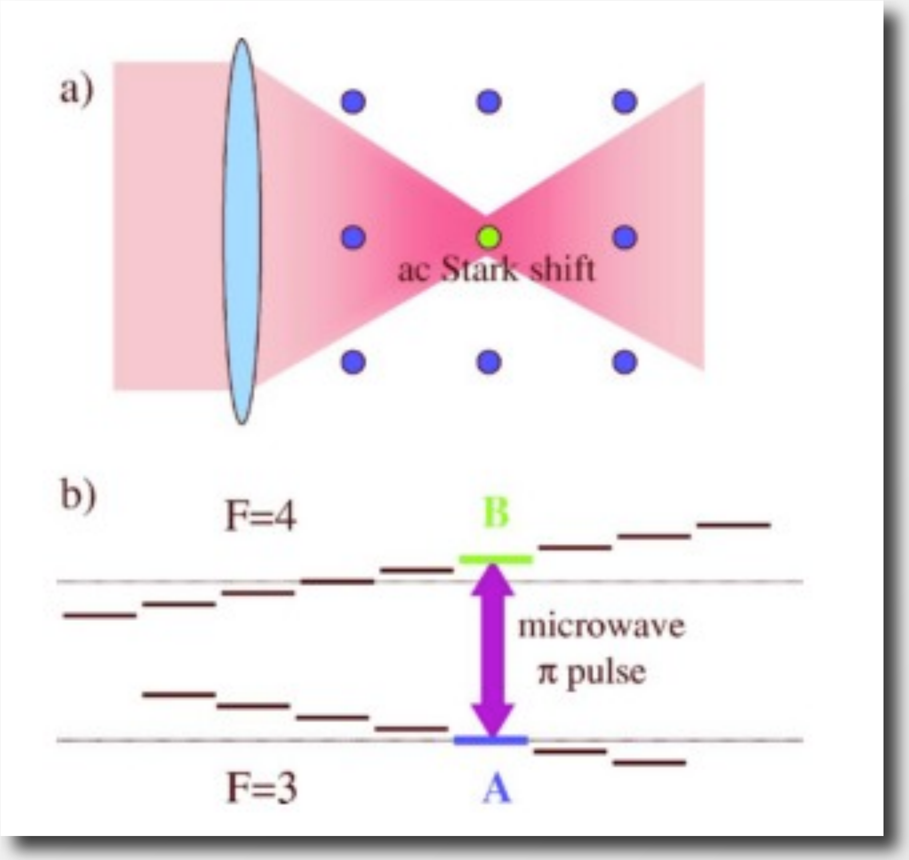
Ch. Weitenberg et al., Nature 471, 319-324 (2011)

Overview/Requirements

We want:

- Single Site resolution in 2D (sub-lattice resolution)
- Single Atom sensitivity
- Coherent control single atom

- In Degenerate MI samples (short lattice spacing)

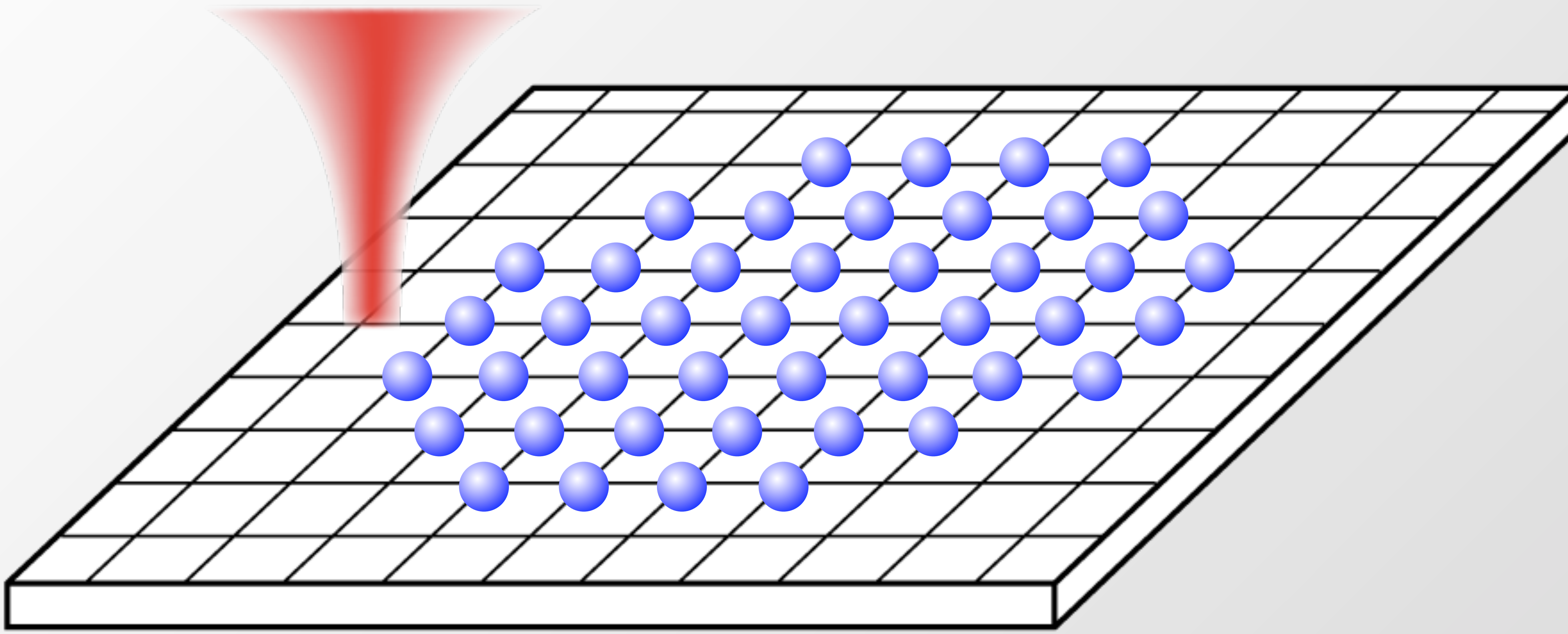


idea: localized differential Stark shift+Microwave

D.S. Weiss et al., PRA **70**, 040302 (2004),
Zhang *et al.*, PRA **74**, 042316 (2006)

see exp: ion traps (Blatt, Wineland, ...), atomic arrays (Meschede, Grangier, Weitz, Ott,...)

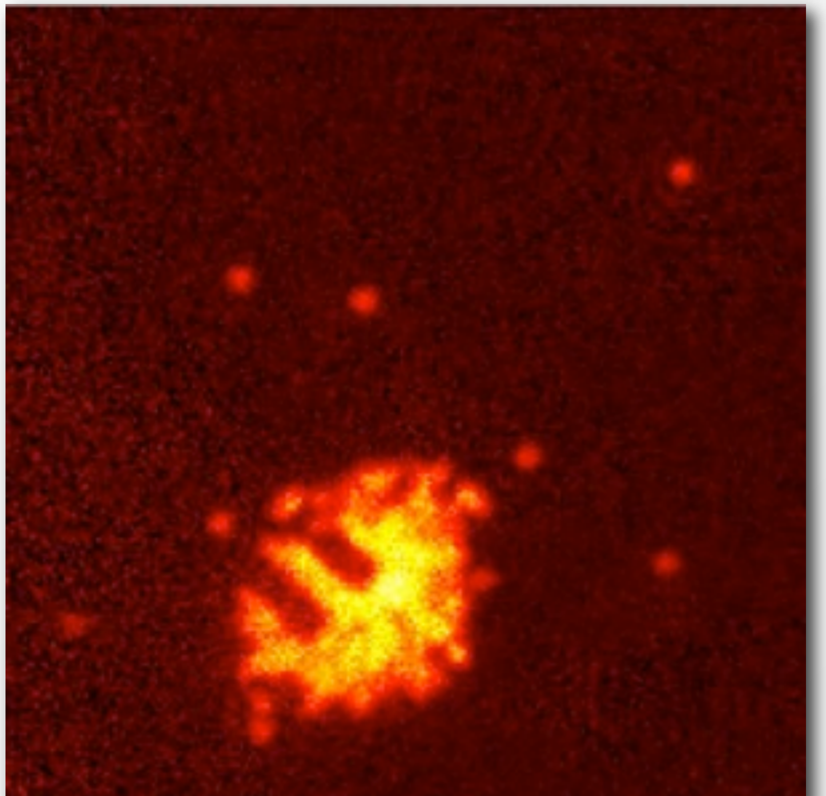
Addressing Atoms

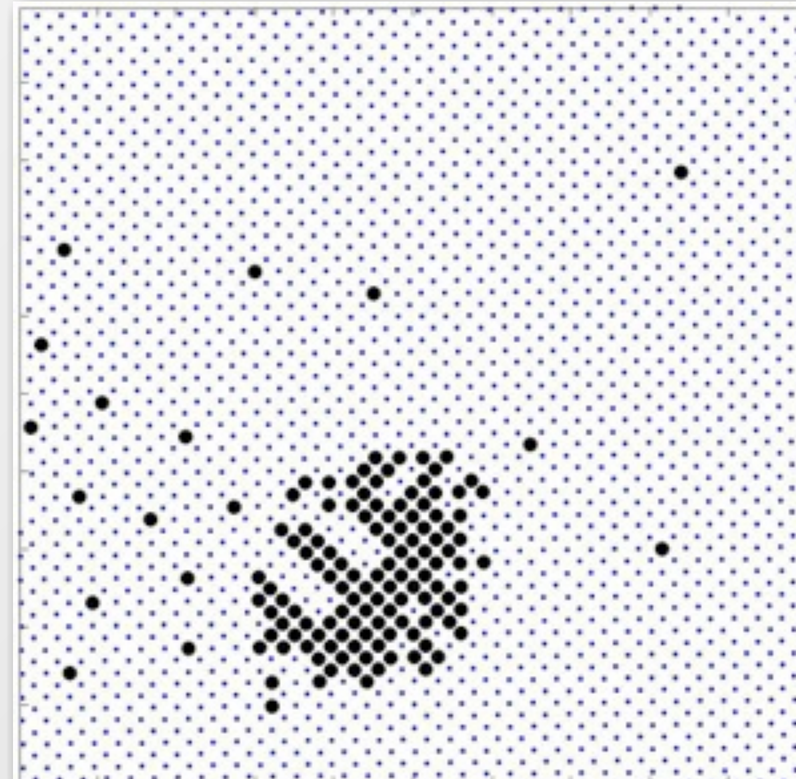


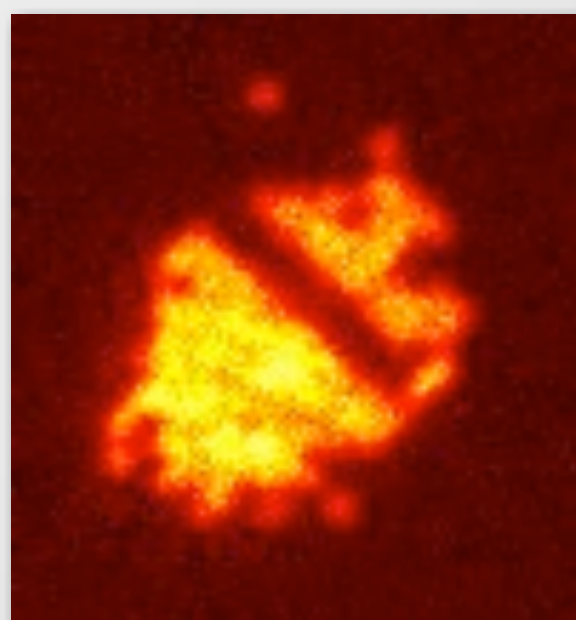
Atoms are pulled out of sites by focussed dipole trap beam!

‘Vacuum Cleaner Mode’

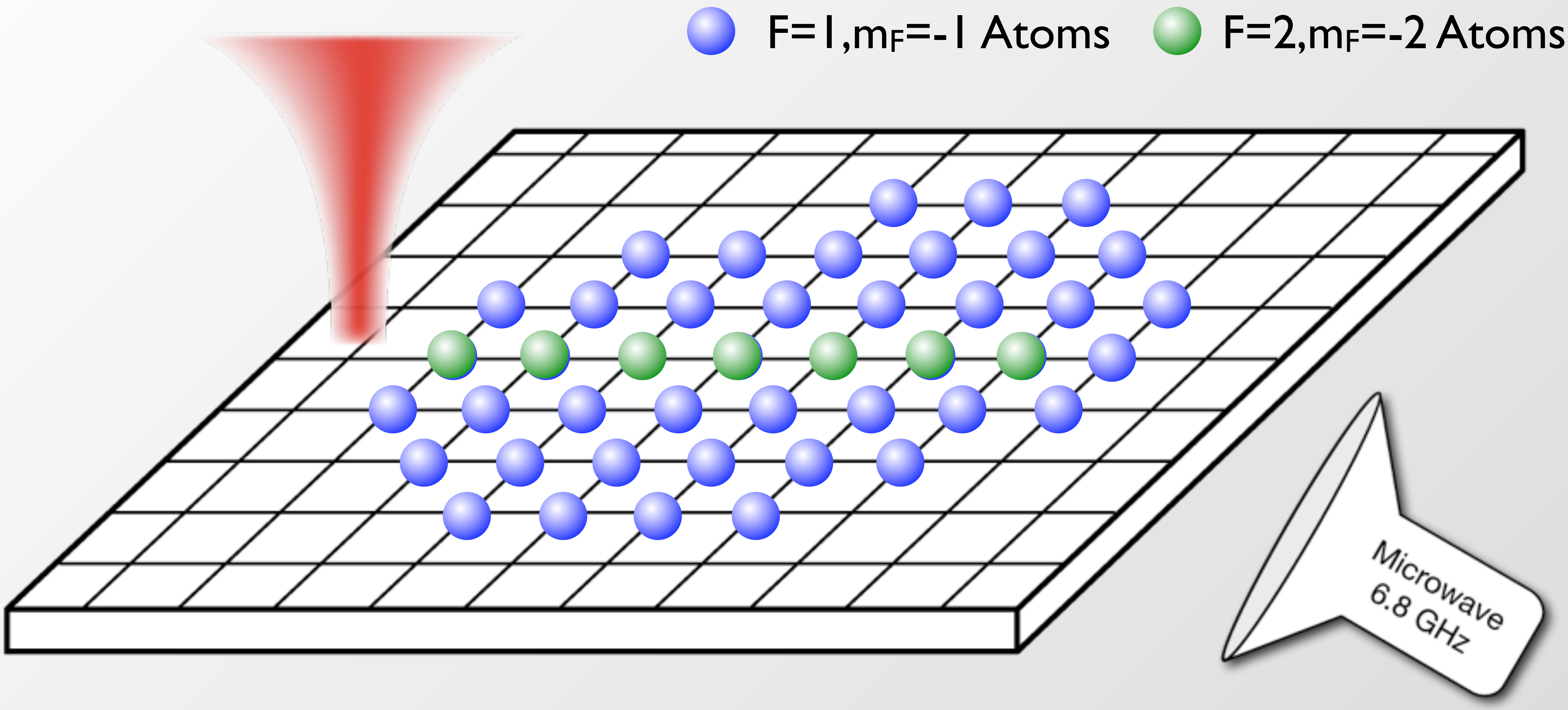
Sucking Out Atoms







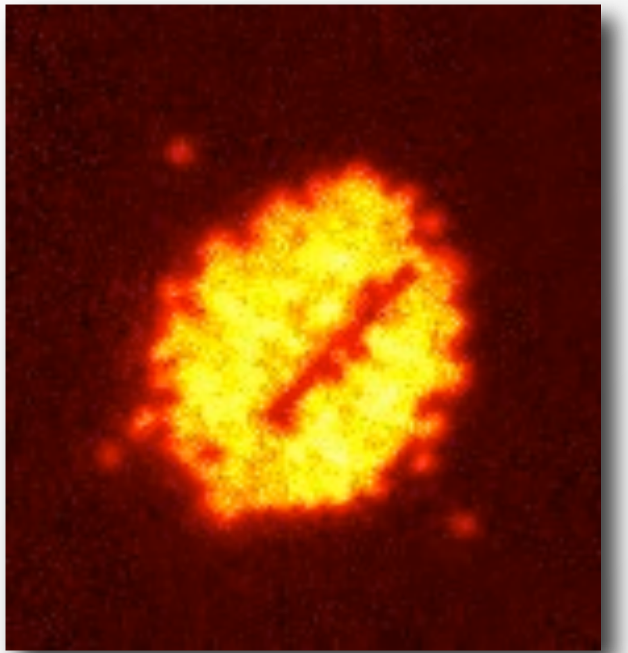
Coherent Addressing of Atoms



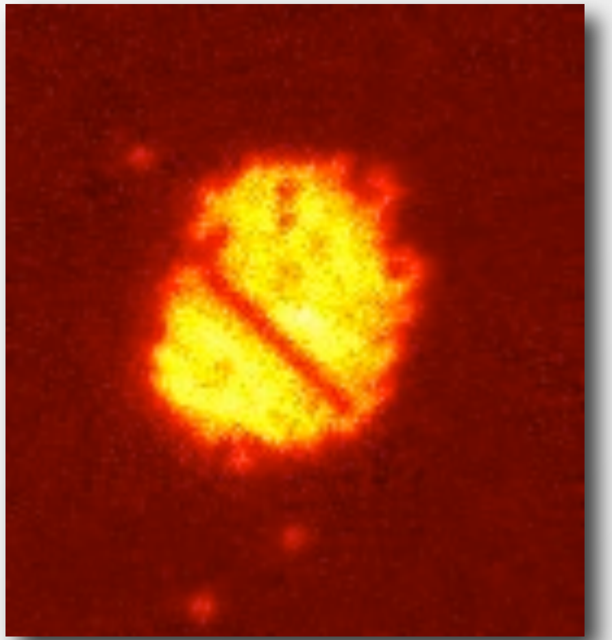
Differential light shift allows to coherently address single atoms!

Landau-Zener Microwave sweep to coherently convert atoms between spin-states.

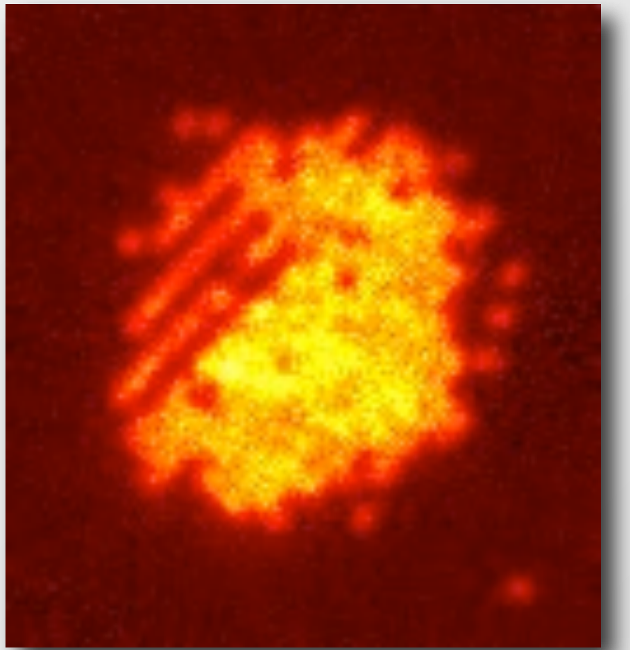
Coherent Spin Flips - Negative Imaging



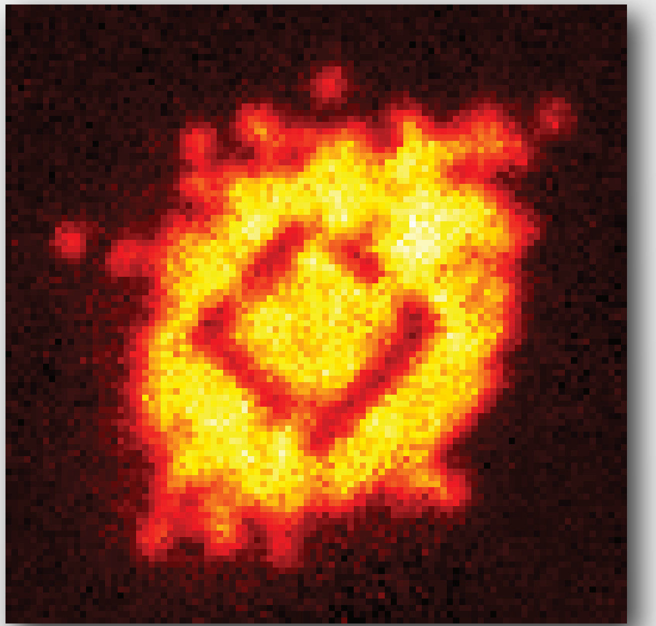
Single Line

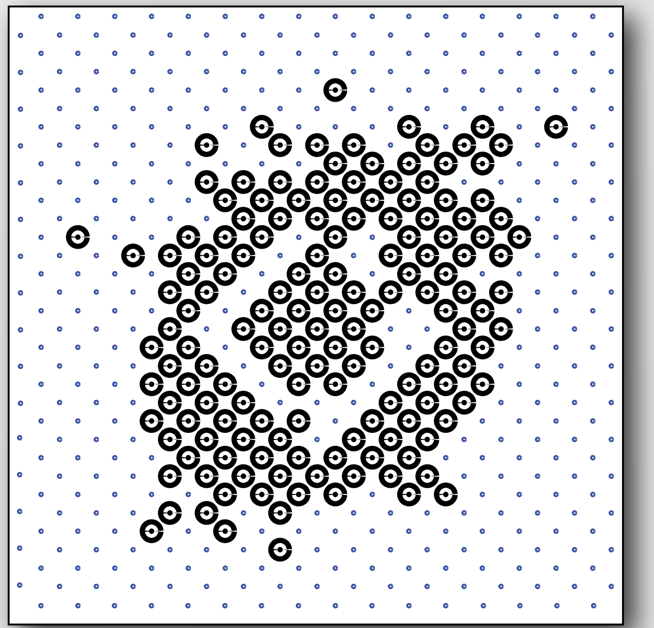


Single Line



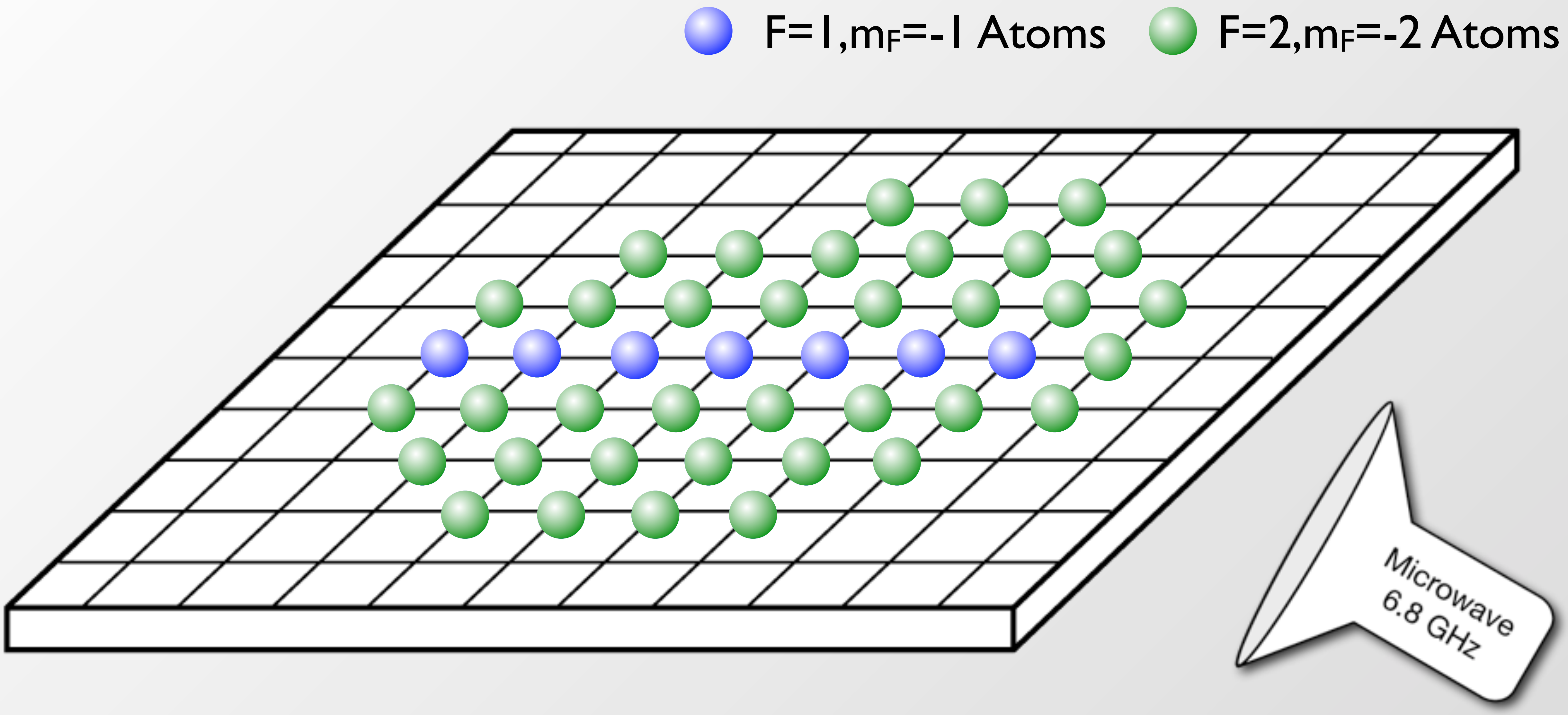
Three Line





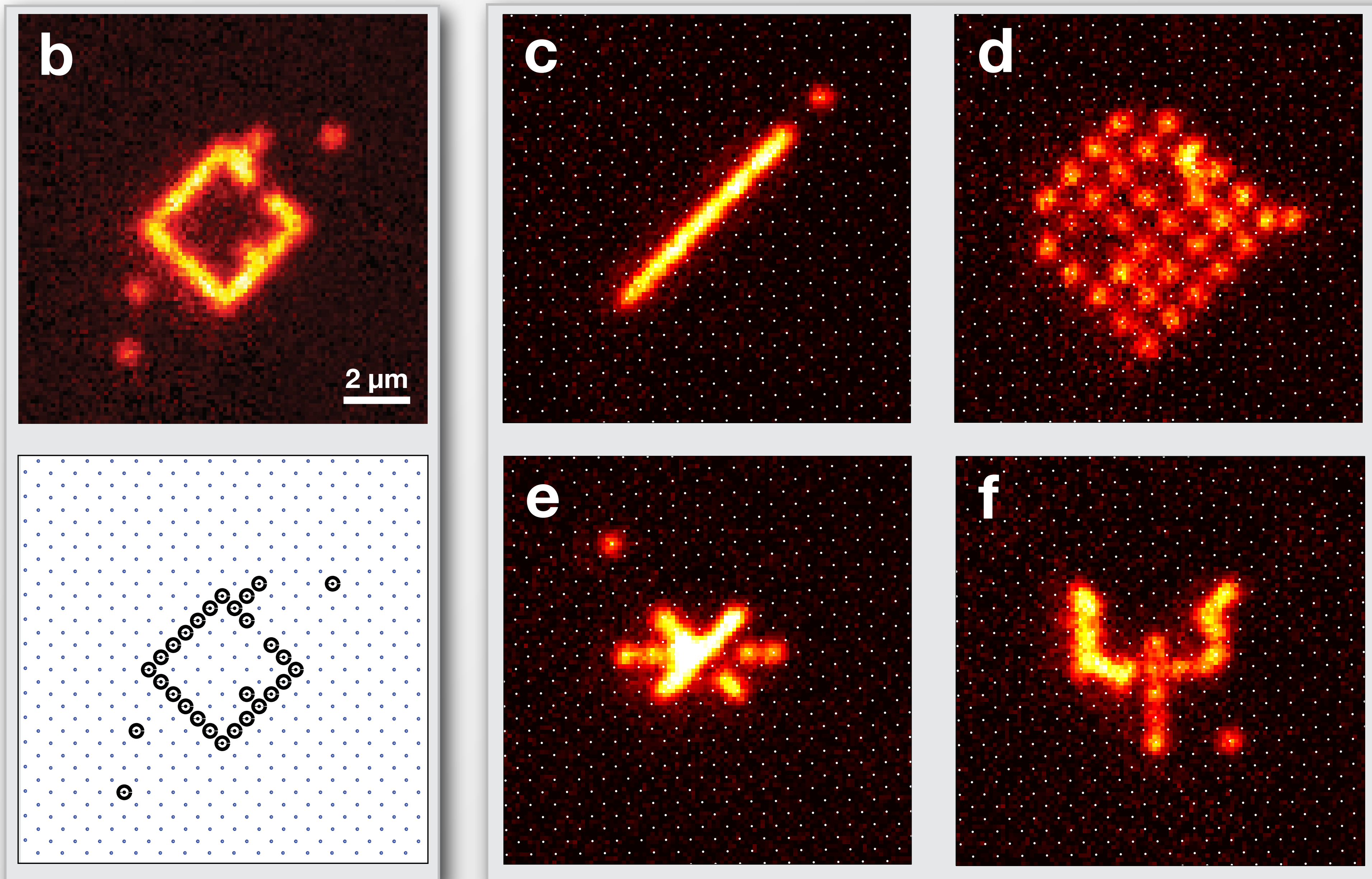
7x7 Square

Coherent Addressing of Atoms Imaging



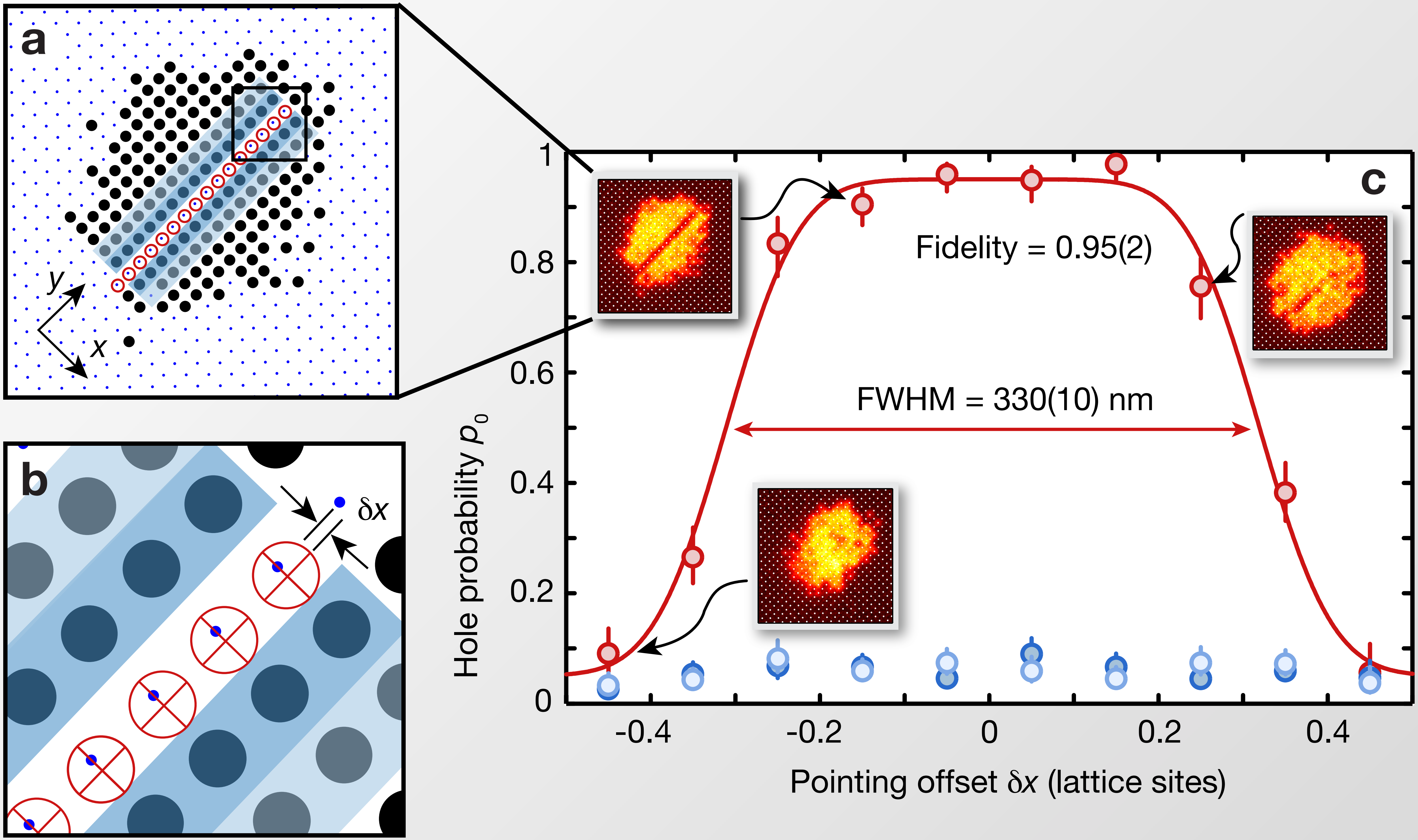
Inverting population allows to image addressed atoms directly

Coherent Spin Flips - Positive Imaging



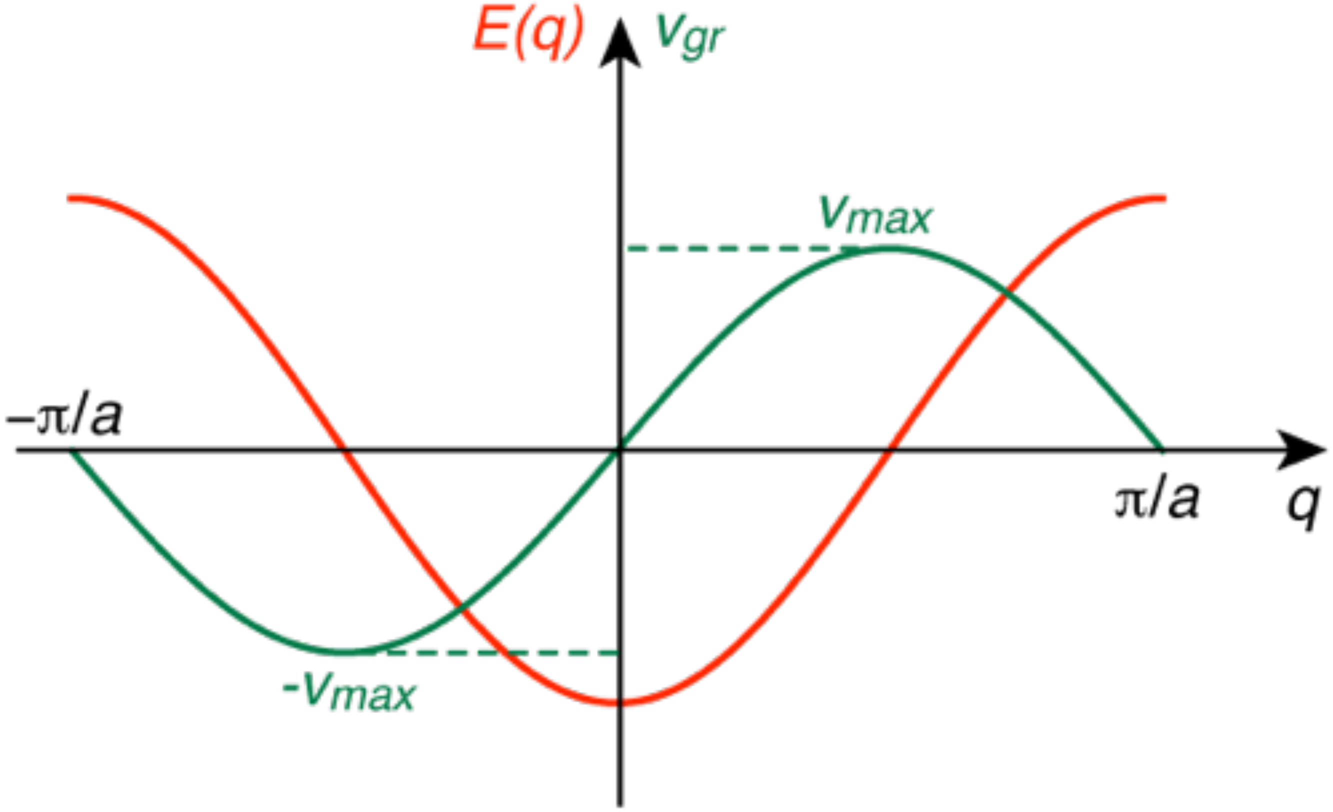
Ch. Weitenberg et al., Nature **471**, 319-324 (2011)

Addressing Fidelity

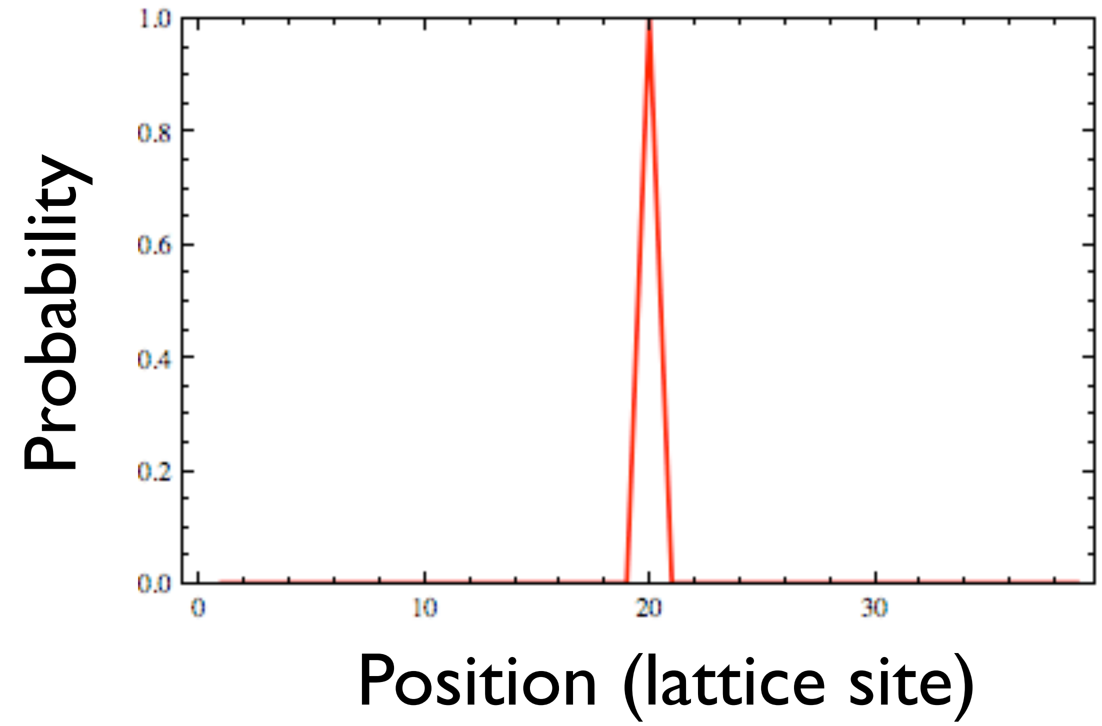


LMU

Tunneling of a Single Atom



$$v_{max} = \frac{2Ja}{\hbar}$$



$$H = -J^{(0)} \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \frac{1}{2} m\omega^2 a_{\text{lat}}^2 i^2 \hat{n}_i$$