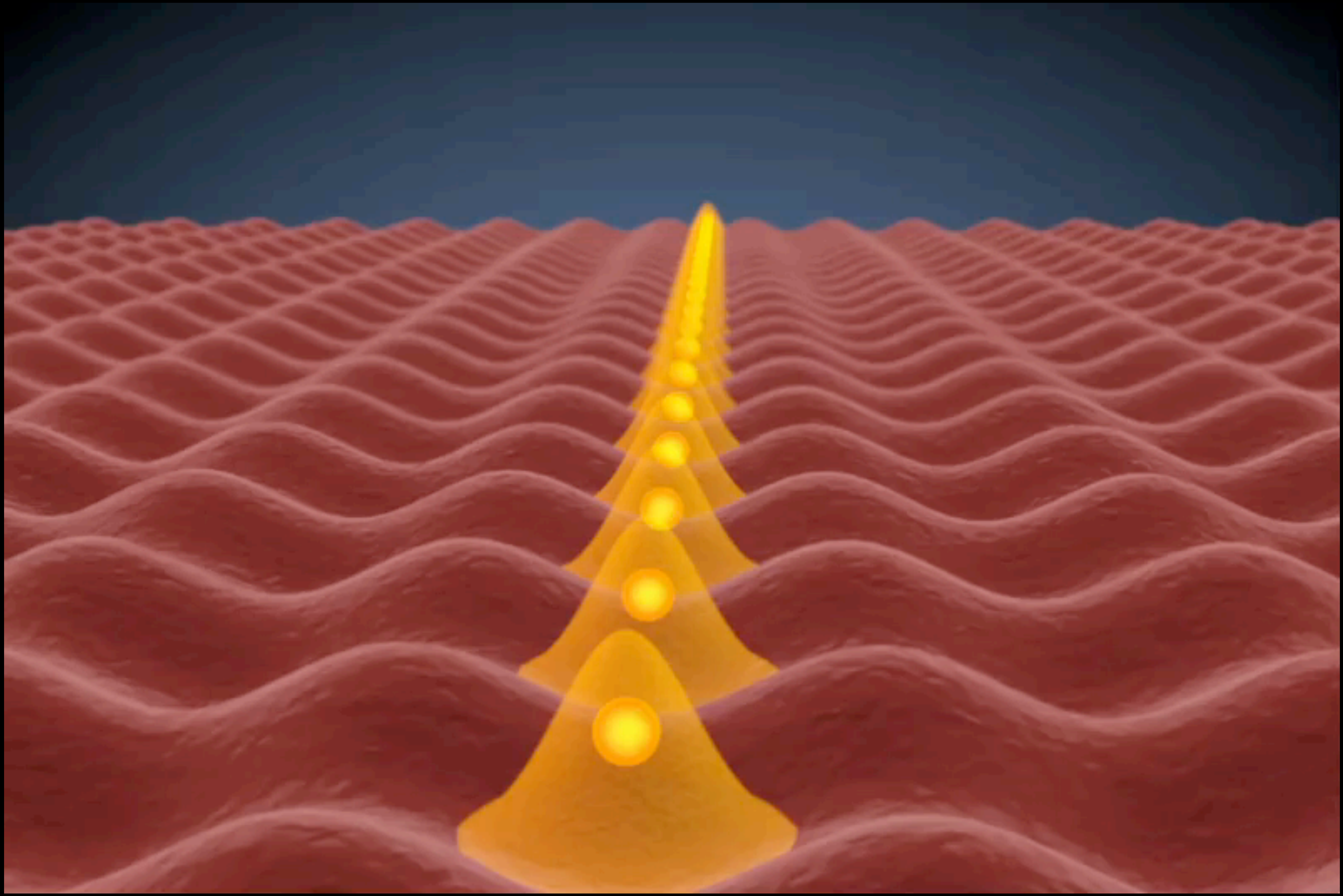
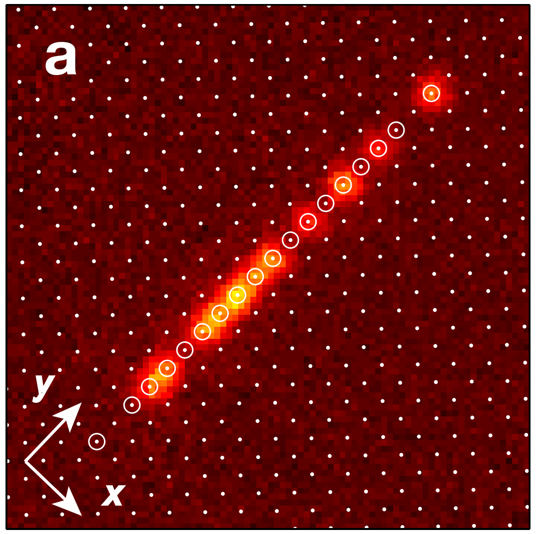
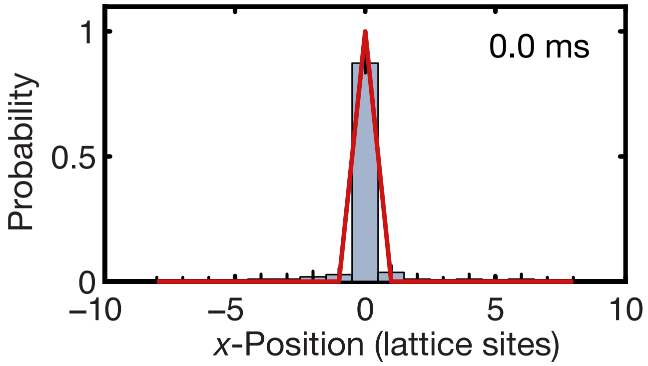
Single Atom Tunnelling

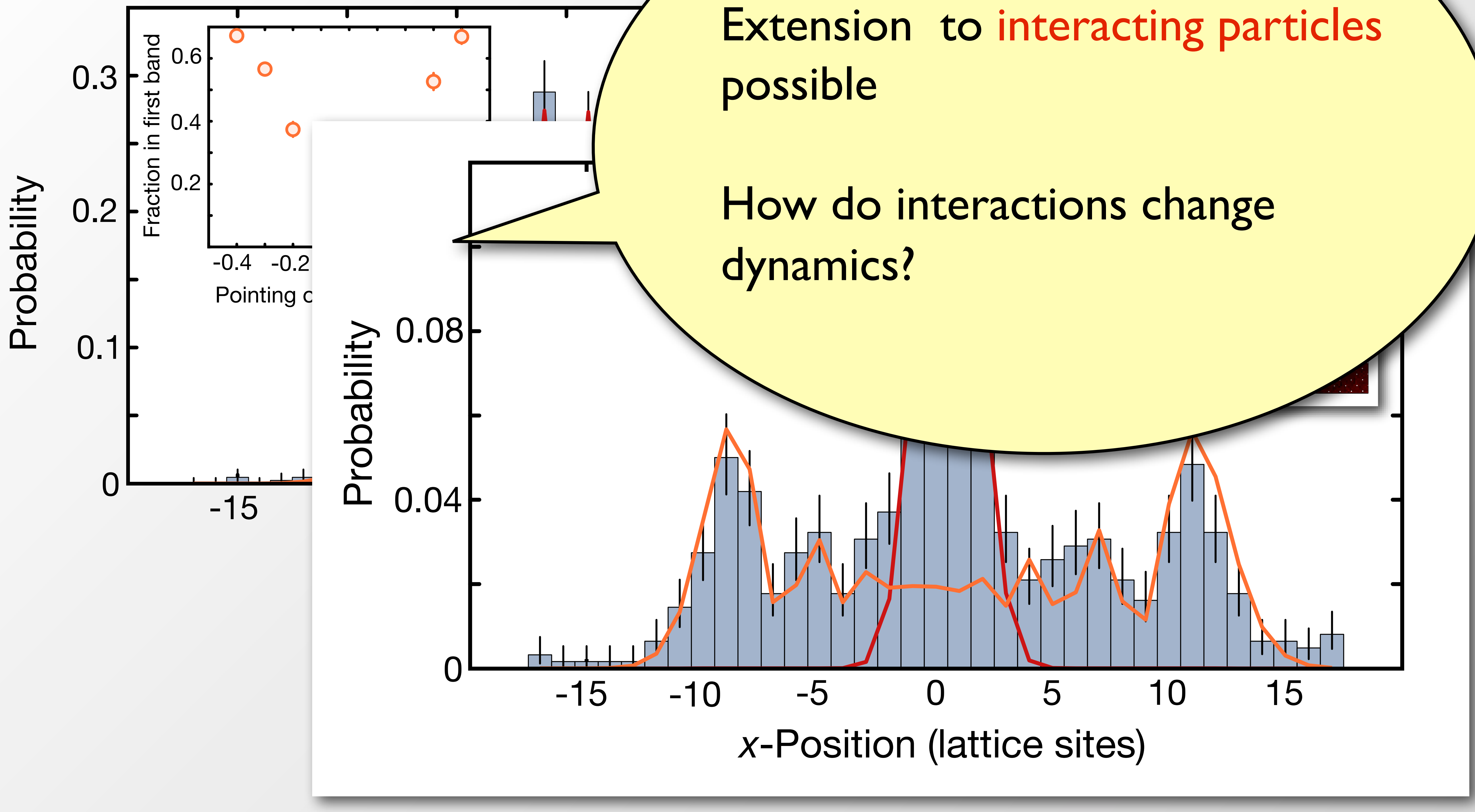


Motional State Affected?





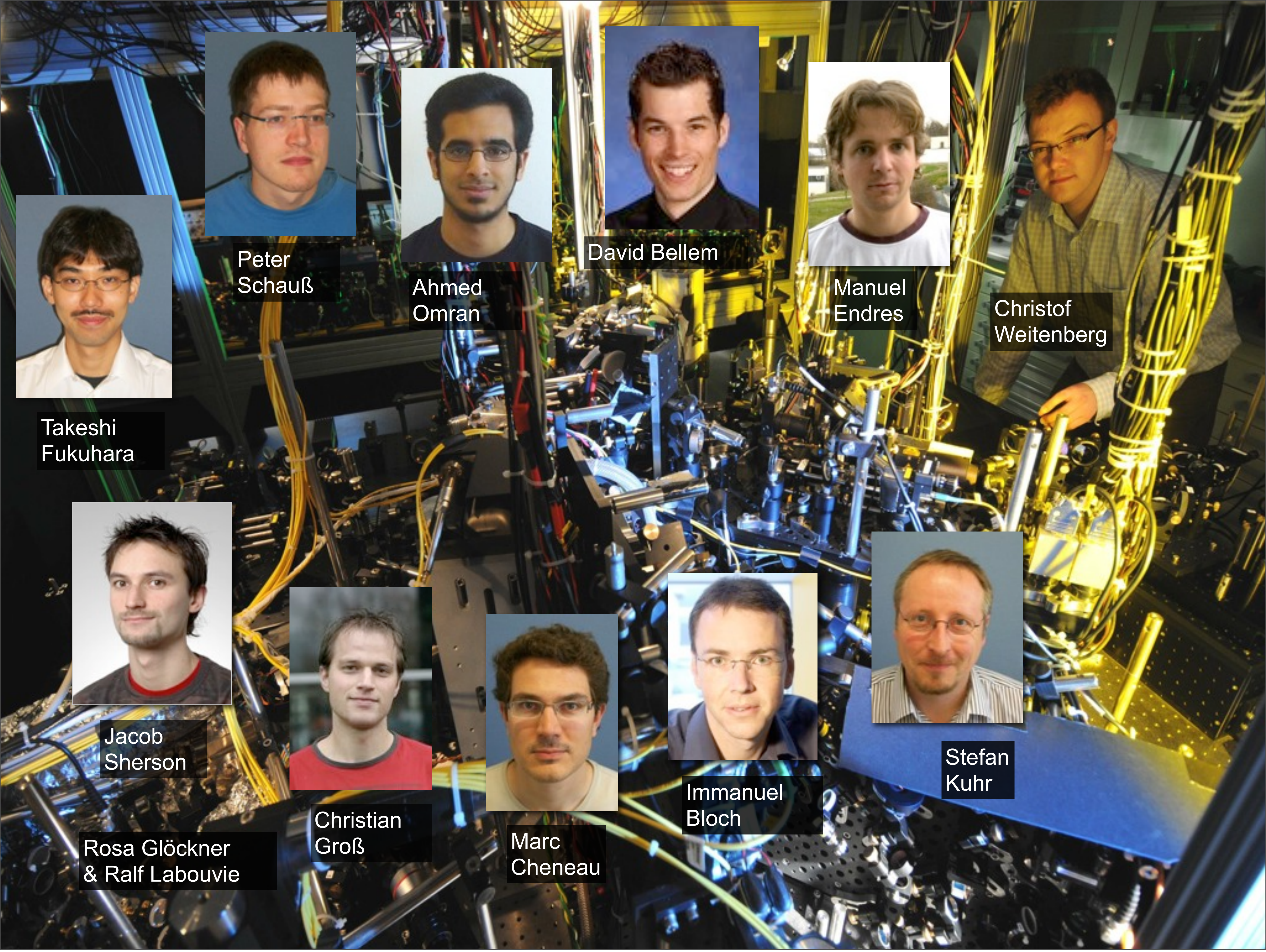
see exp: Y. Silberberg (photonic waveguides), D. Meschede & R. Blatt (quantum walks)...

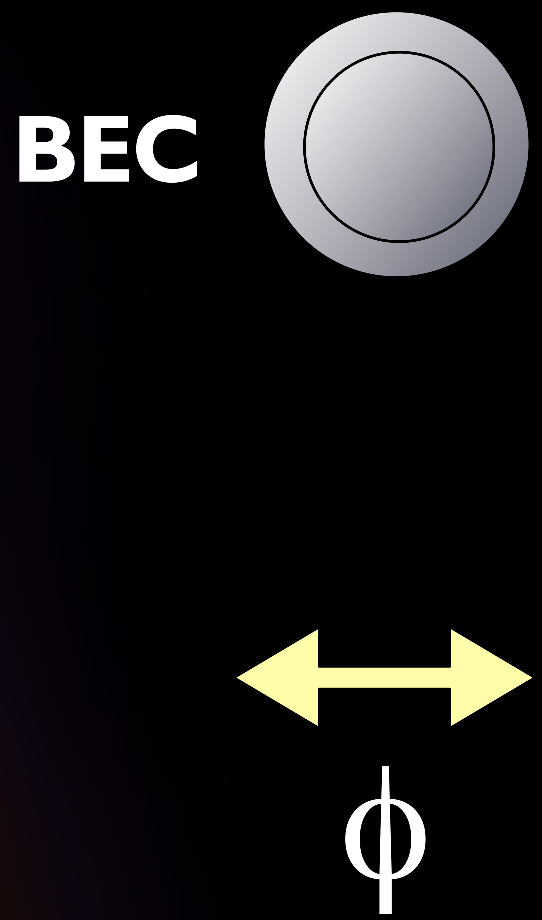


Extension to interacting particles possible

How do interactions change dynamics?

Excellent agreement with simulation.


Takeshi Fukuhara
Peter Schauß
Ahmed Omran
David Bellem
Manuel Endres
Christof Weitenberg
Jacob Sherson
Christian Groß
Marc Cheneau
Immanuel Bloch
Stefan Kuhr
Rosa Glöckner & Ralf Labouvie

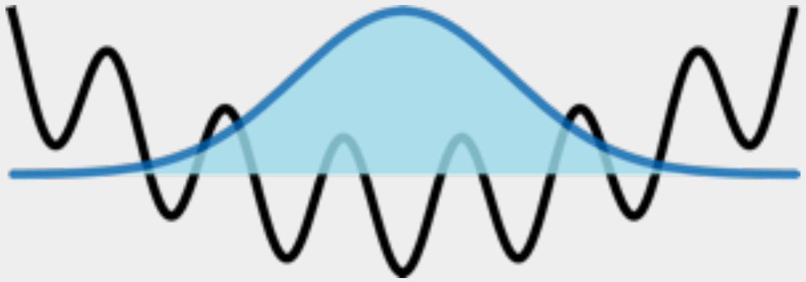


Probing Many-Body States via Quantum Phase Diffusion

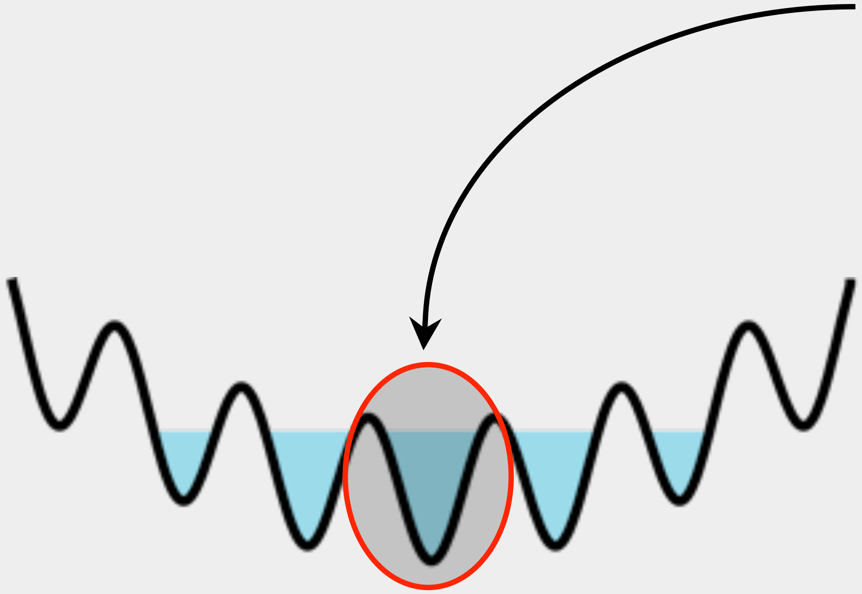
From BEC to a Superfluid in an Optical Lattice…

BEC in a harmonic trap…

…plus a weak lattice



Onsite picture:



$$c_0|\ \rangle + c_1|●\rangle + c_2|●●\rangle + c_3|●●●\rangle + \dots$$

Non-interacting, homogeneous case:

Coherent State

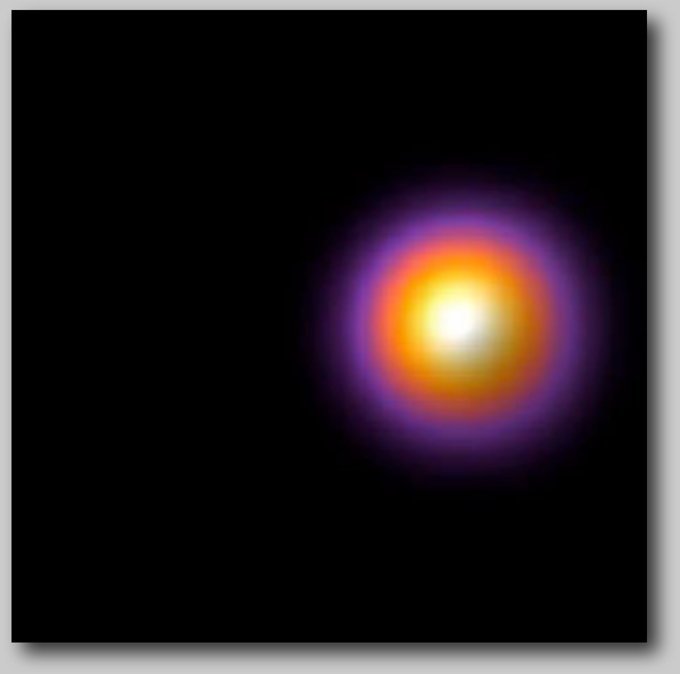
⬄

Poisson distribution

Phase Diffusion Dynamics

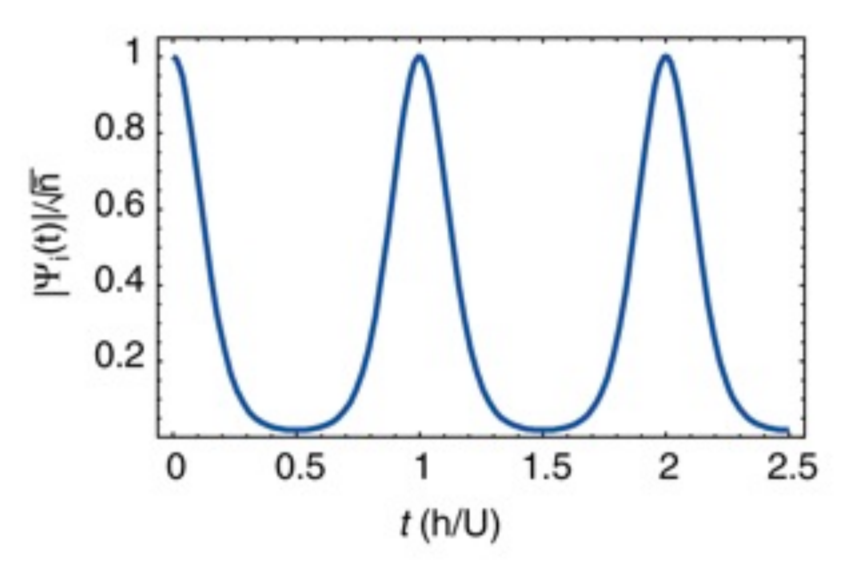
Quantum state in each lattice site (e.g. for a coherent state)

$$\left|\Psi(t)\right\rangle_i = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-i\frac{1}{2}U\,n(n-1)t/\hbar}\left|n\right\rangle$$



Matter wave field on the i^{th} lattice site

$$\Psi_i(t) = {}_i\left\langle\Psi(t)\right|\hat{a}_i\left|\Psi(t)\right\rangle_i$$



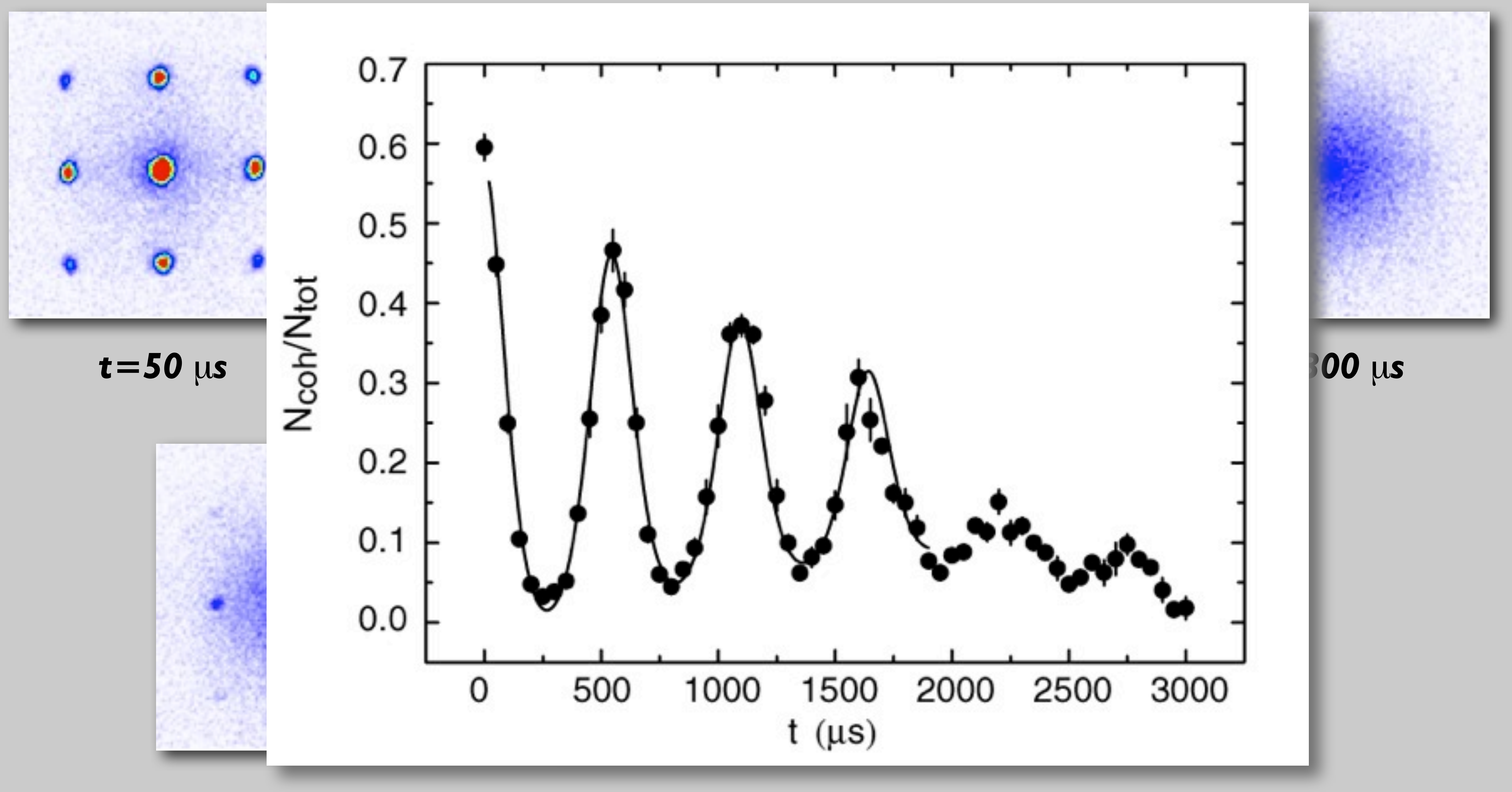
1. ***Matter wave field collapses but revives after times multiple times of h/U !***
2. ***Collapse time depends on the variance σ_N of the atom number distribution !***

Theory: Yurke & Stoler, 1986, F. Sols 1994; Wright et al. 1997; Imamoglu, Lewenstein & You et al. 1997, Castin & Dalibard 1997, E. Altman & A. Auerbach 2002,

Exp: M. Greiner et al 2002, G.-B. Jo et al 2006, J. Sebby-Strabley et al. 2007, see also M. Oberthaler

Similiar to Collapse and Revival of Rabi-Oscillations in Cavity QED !

Dynamical Evolution of the Interference Pattern



***t=50* μs**

***00* μs**

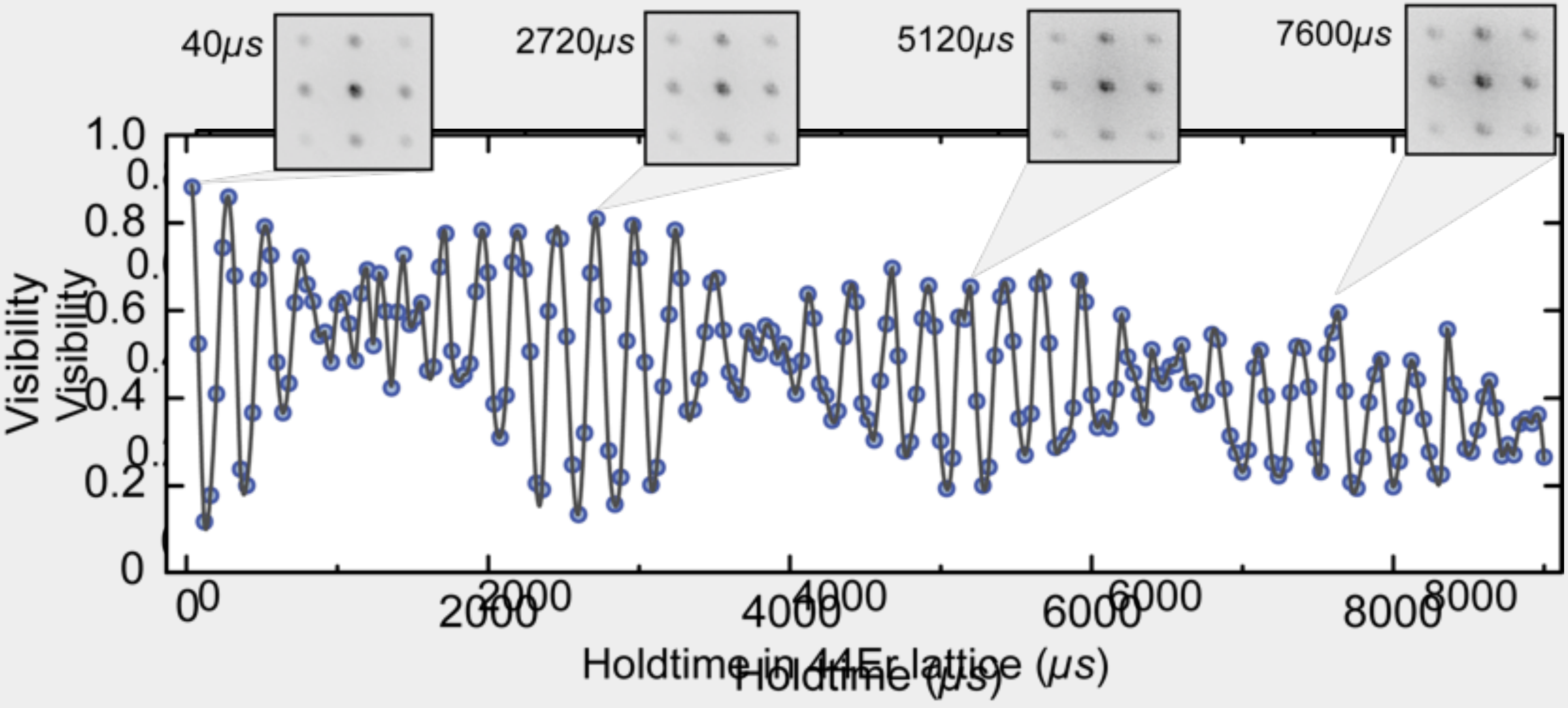
***t=400* μs**

***t=450* μs**

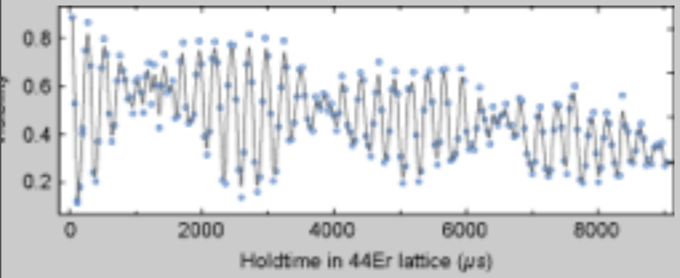
***t=600* μs**

After a potential jump from $V_A=8E_r$ to $V_B=22E_r$.

Collapse & Revival under Optimal Harmonic Confinement



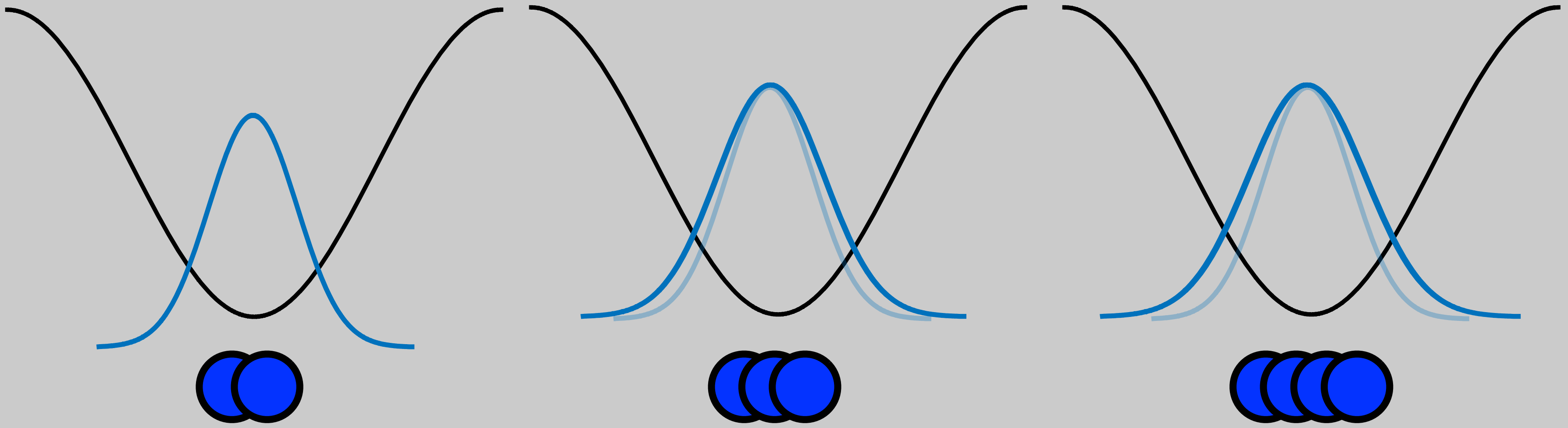
- Up to **70 revivals** can be detected!
- And: **Multiple frequency components!**



Why Multiple Frequencies?

$$\left|\Psi(t)\right\rangle_i = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}\, e^{-i\frac{1}{2}U\,n(n-1)t/\hbar}\left|n\right\rangle$$

We assume U to be constant, independent of filling....



n=2
U(2)

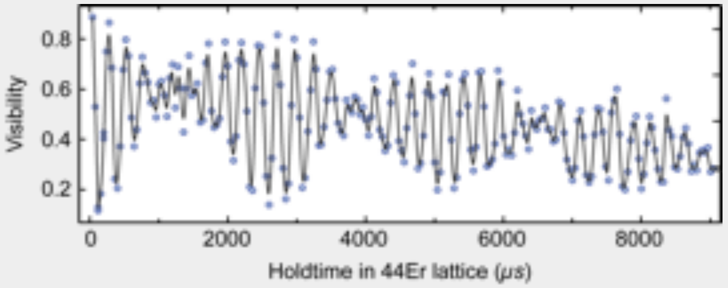
n=3
U(3)

n=4
U(4)

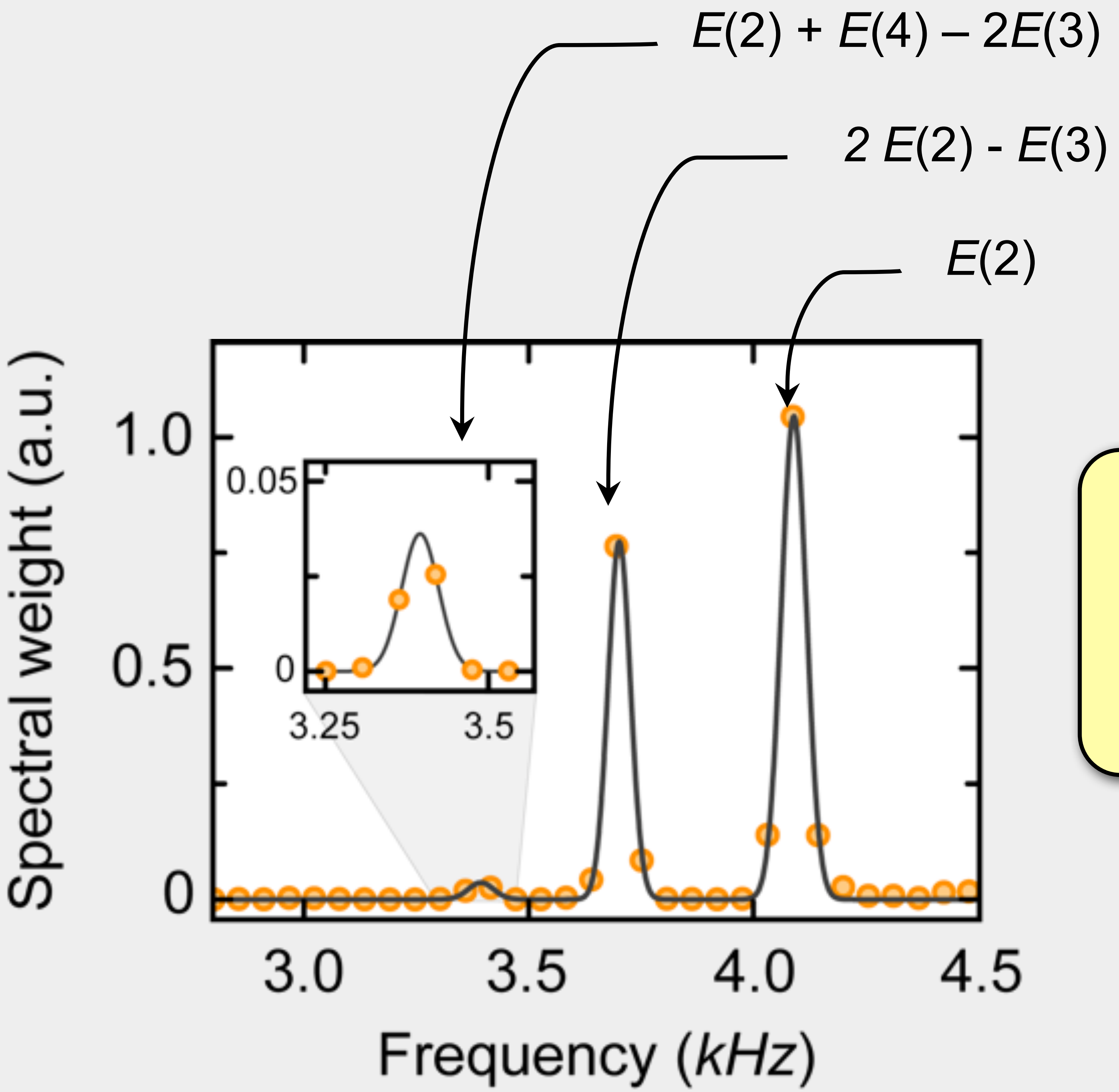
$$U = \frac{4\pi\hbar^2 a}{m}\int d^3x\,|w(\mathbf{x})|^4$$

Breakdown of single band approximation!

for differential measurement, see also: G. Campbell et al. Science (2006)



Fourier Spectrum



$E(2) + E(4) - 2E(3) \Rightarrow c_2 \cdot c_3^2 \cdot c_4$

$2\,E(2) - E(3) \Rightarrow c_1 \cdot c_2^2 \cdot c_3$

$E(2) \Rightarrow c_0 \cdot c_1^2 \cdot c_2$

Strong signal of

small contributions due to

heterodyning effect!

Coherent Three- and Four-Body Interactions in a Lattice

$$H_{int} = \frac{U(2)}{2}\hat{n}(\hat{n}-1) + \frac{\Delta U(3)}{6}\hat{n}(\hat{n}-1)(\hat{n}-2) + \frac{\Delta U(4)}{24}\hat{n}(\hat{n}-1)(\hat{n}-2)(\hat{n}-3)$$

Two-Body

Three-Body

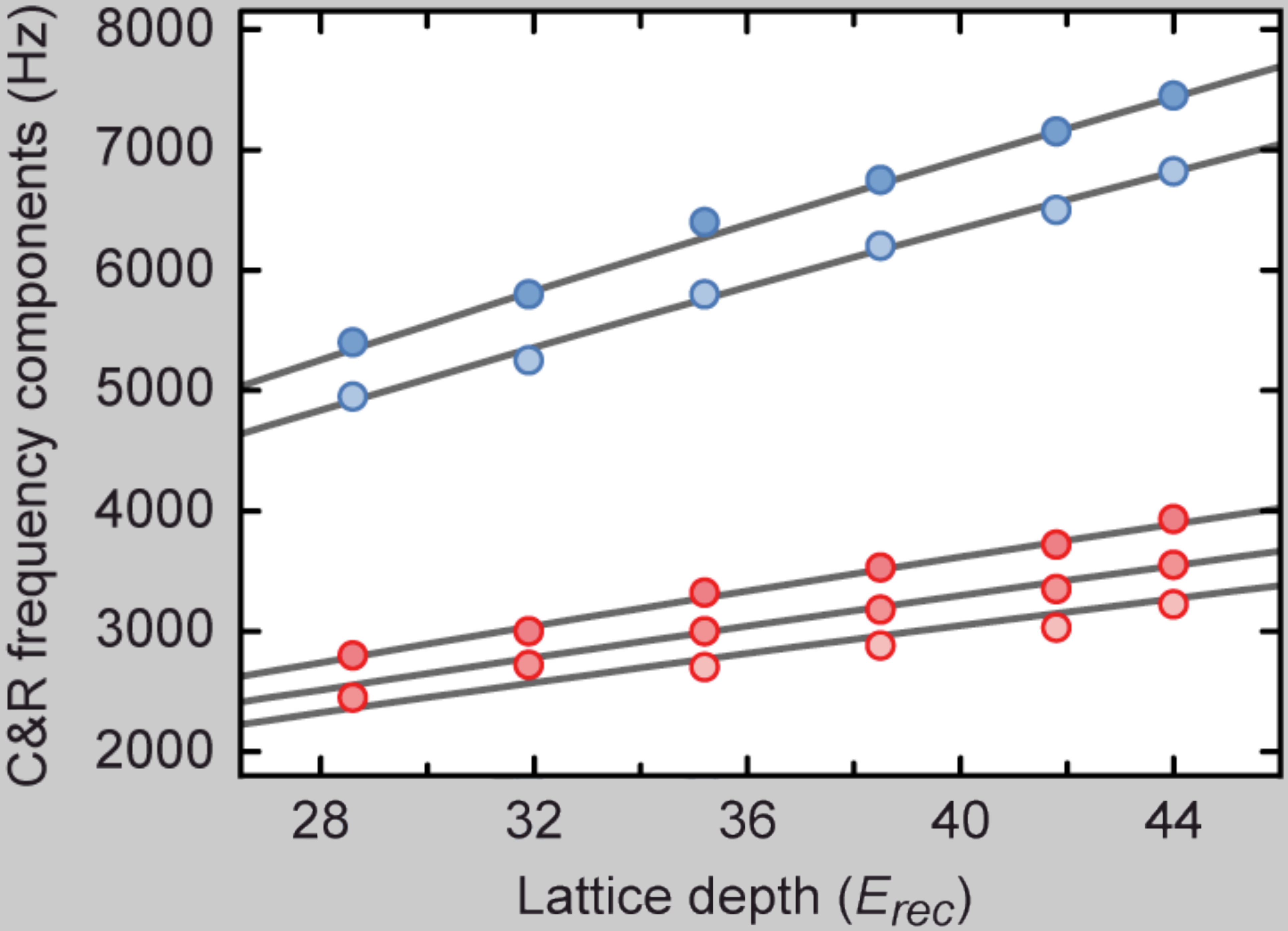
Four-Body

Virtual transitions to higher orbitals
induce effective three- and four-body interactions!

P.R. Johnson, E. Tiesinga, J.V. Porto, C.J. Williams
arXiv:0812.1387

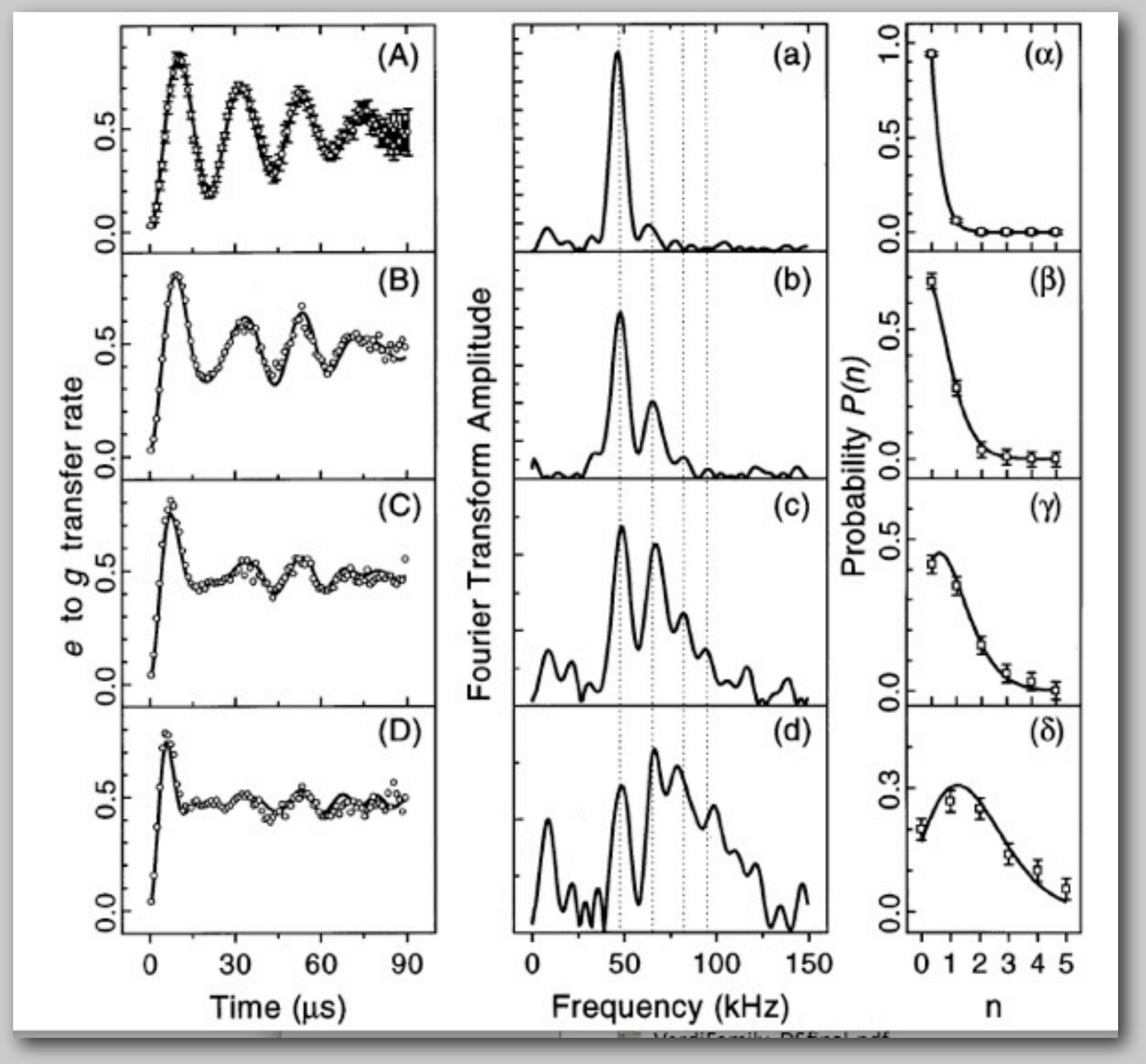
detection of multi-particle interactions **via enhanced losses** for three-body systems (Efimov states), Atom-Molecule collisions
see exps. Innsbruck, JILA, Heidelberg

Comparison with Exact Diagonalization



Theory: exact diagonalization D. Lühmann (Univ. Hamburg)

Quantum Rabi Oscillations in CQED

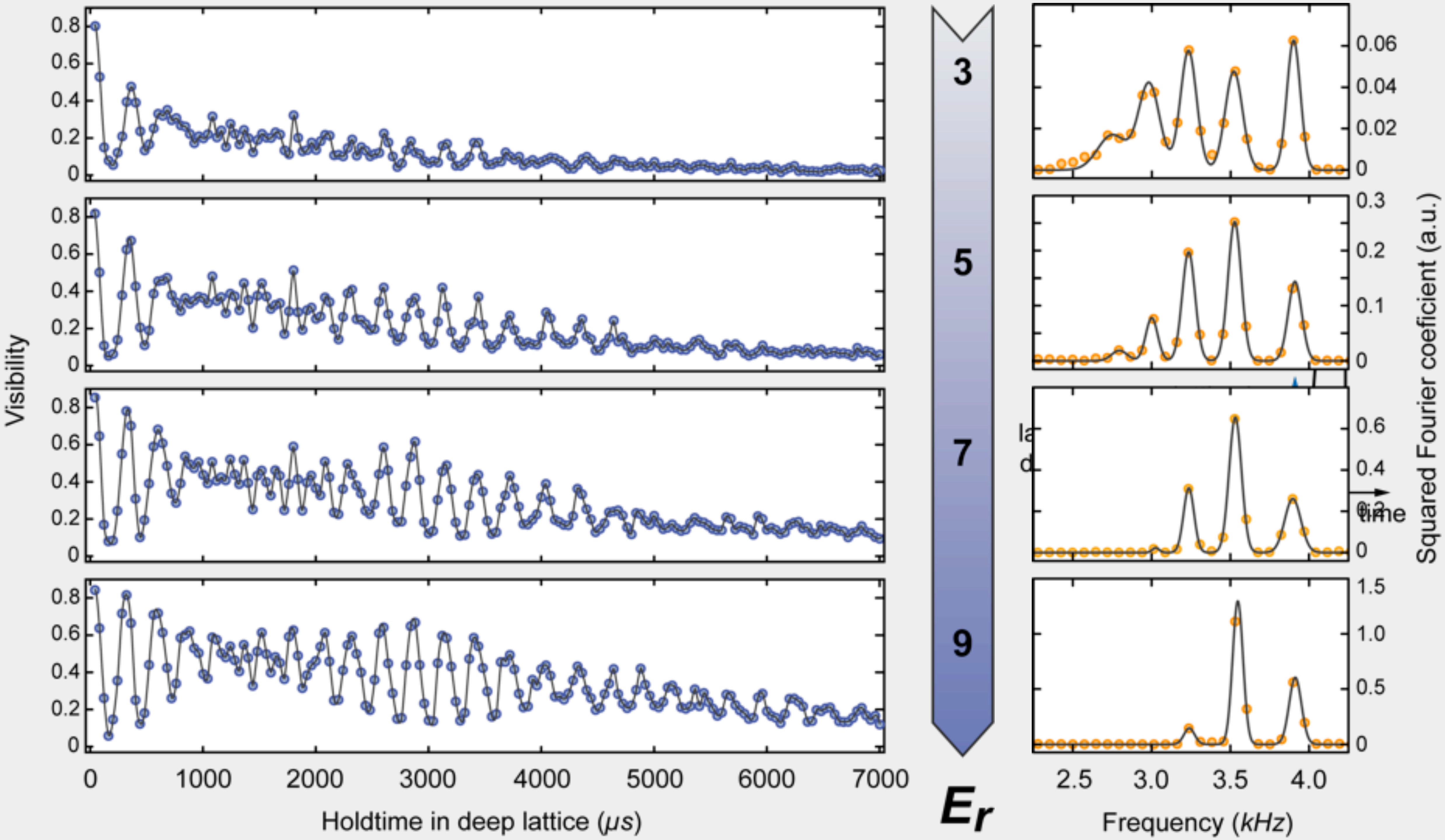


$$\sqrt{n+1}\,\Omega_0$$

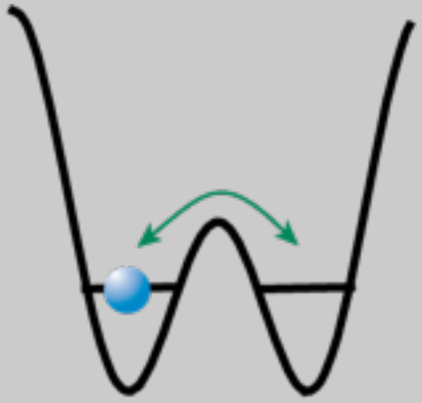
Rabi-Oscillations quantized!

M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond, and S. Haroche
PRL, **76** 1800 (1996)

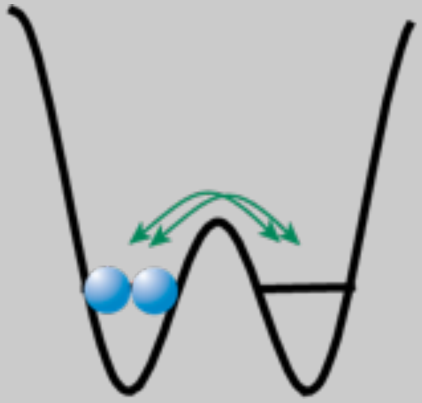
Atom distribution along the SF to MI transition:



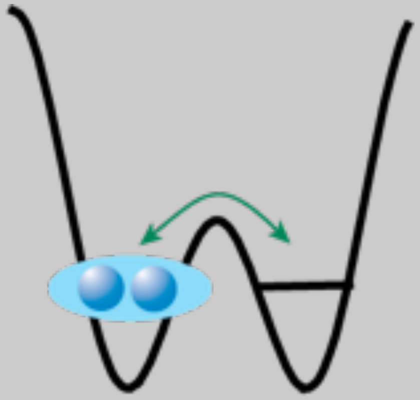
Tunneling of one or two atoms



1) **Resonant tunneling between the two wells with frequency $2J$**

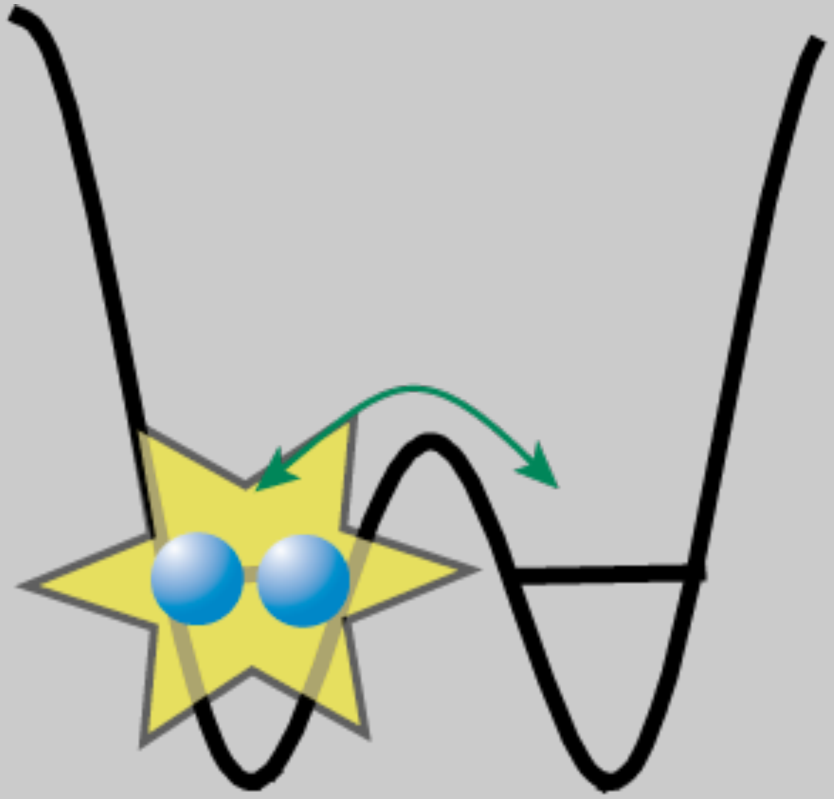


2) **Two atoms, no interaction: tunneling is independent**



3) **Cooperative tunneling of attractively bound objects (Cooper pairs, molecules)**

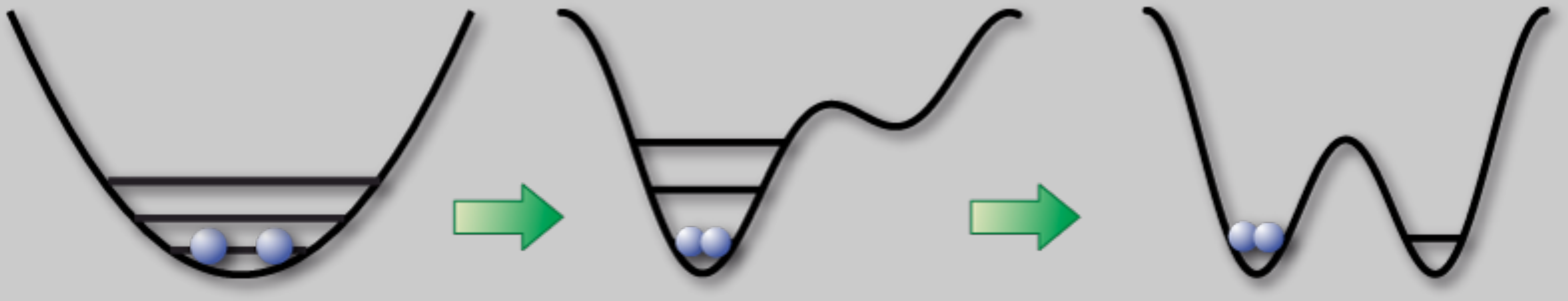
What about interacting atoms?



???

S. Fölling et al., Nature **448**, (2007)
stability of pairs, see: K. Winkler et al, Nature **441**, (2006)

State Preparation

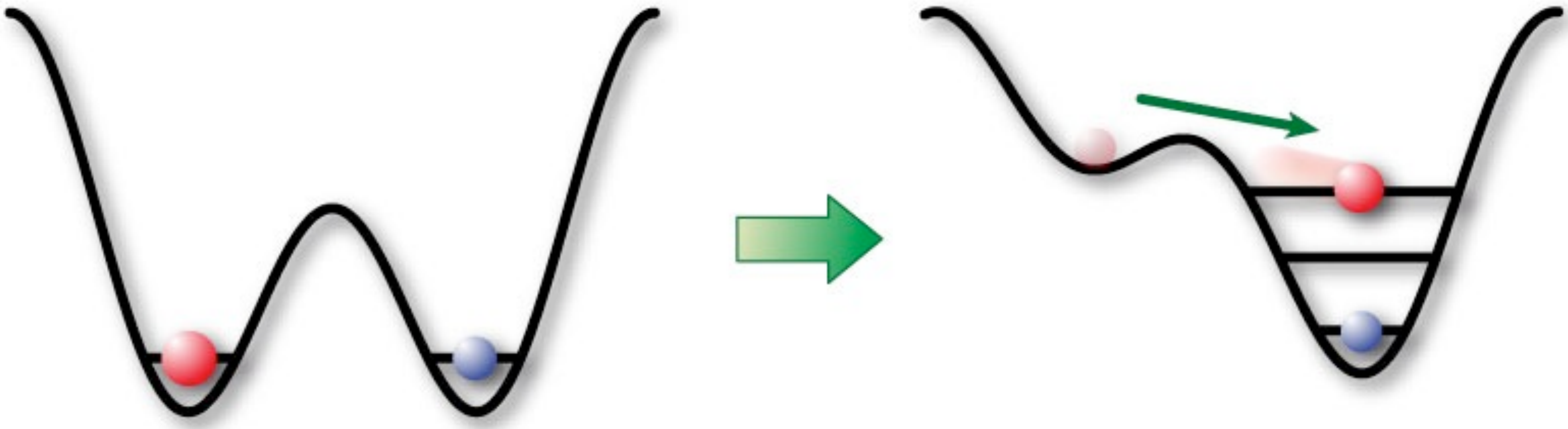


**Two Atoms in
„Big Well“**

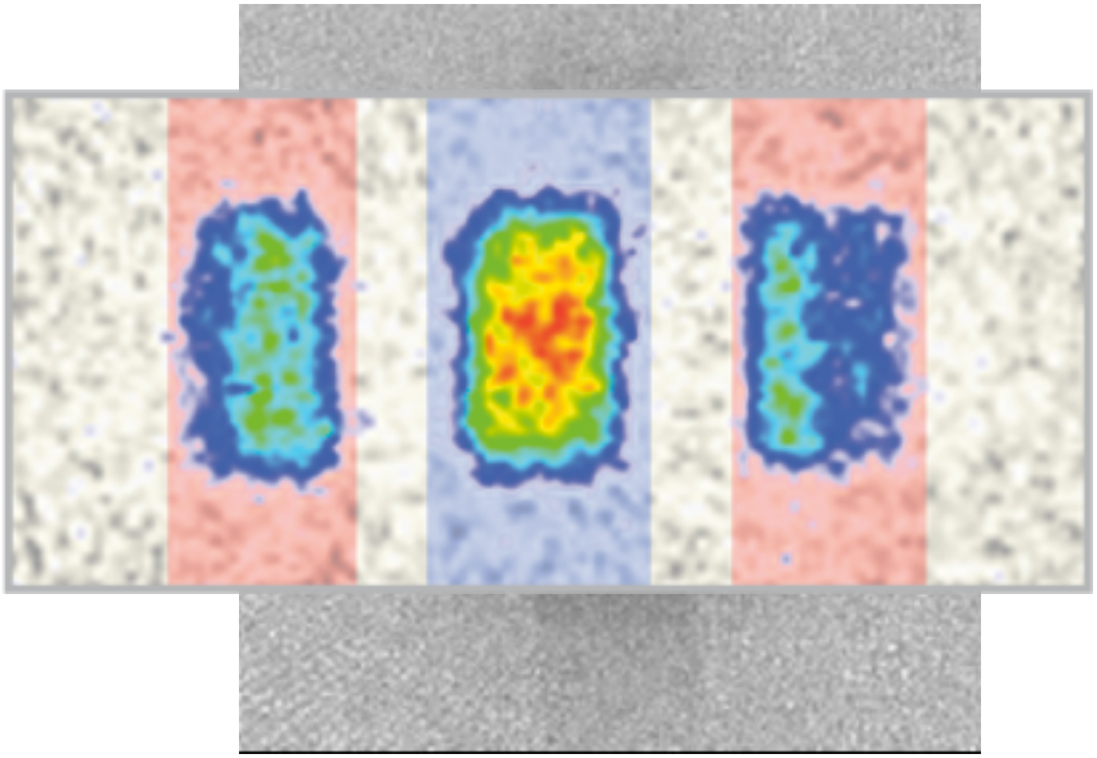
Tilted Double Well

Balanced Double Well

Population Imbalance Measurement



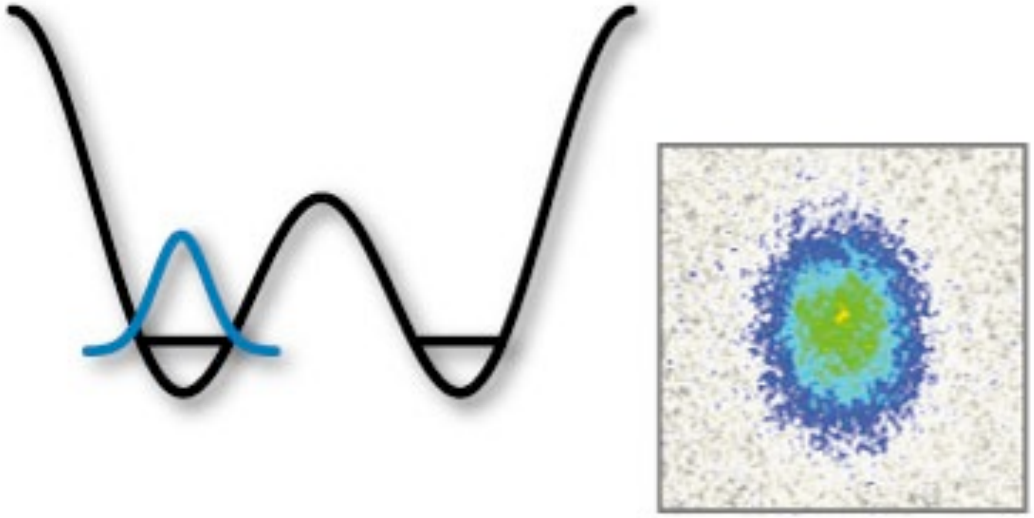
Map Left-Right Populations onto Band Populations



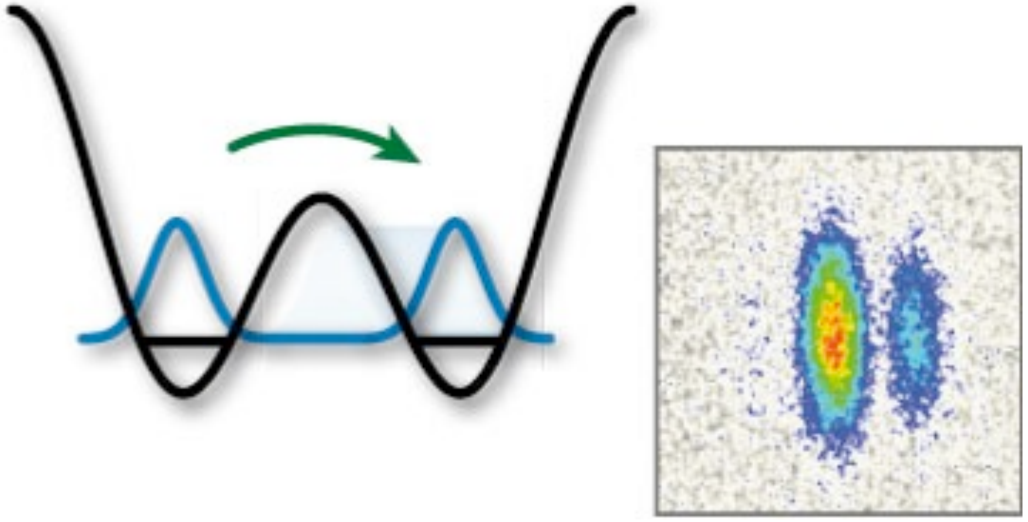
see also: Sebby-Strabley et al., PRL **98**, 200405 (2007)

Phase Measurement

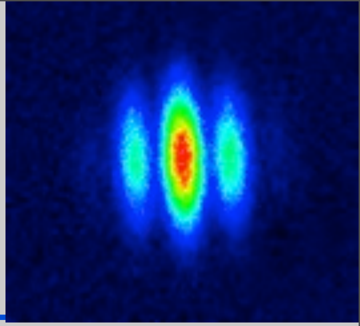
$$c_1|\psi_L\rangle + c_2 e^{i\varphi}|\psi_R\rangle$$



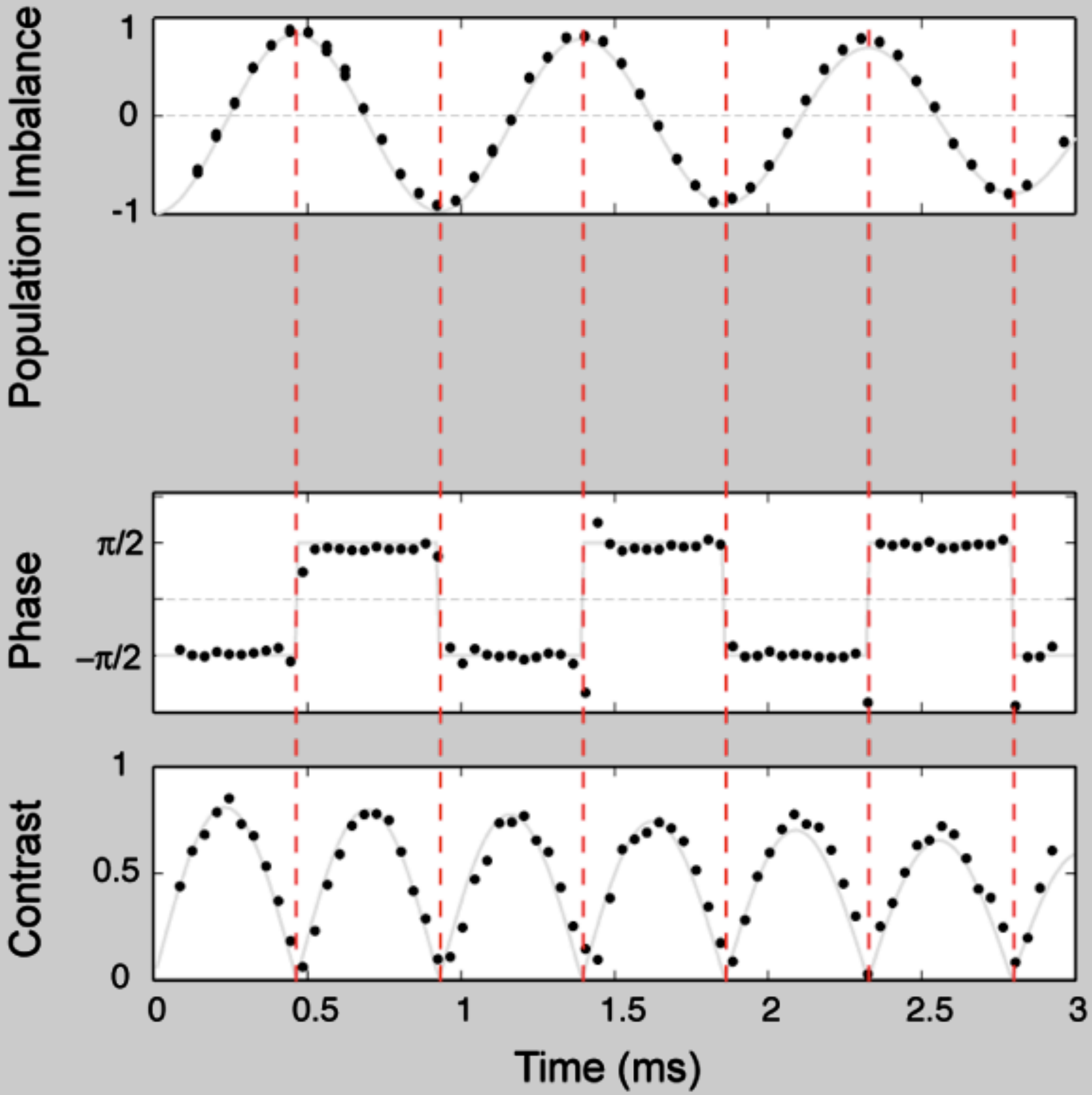
Localized Particle yields no interference pattern



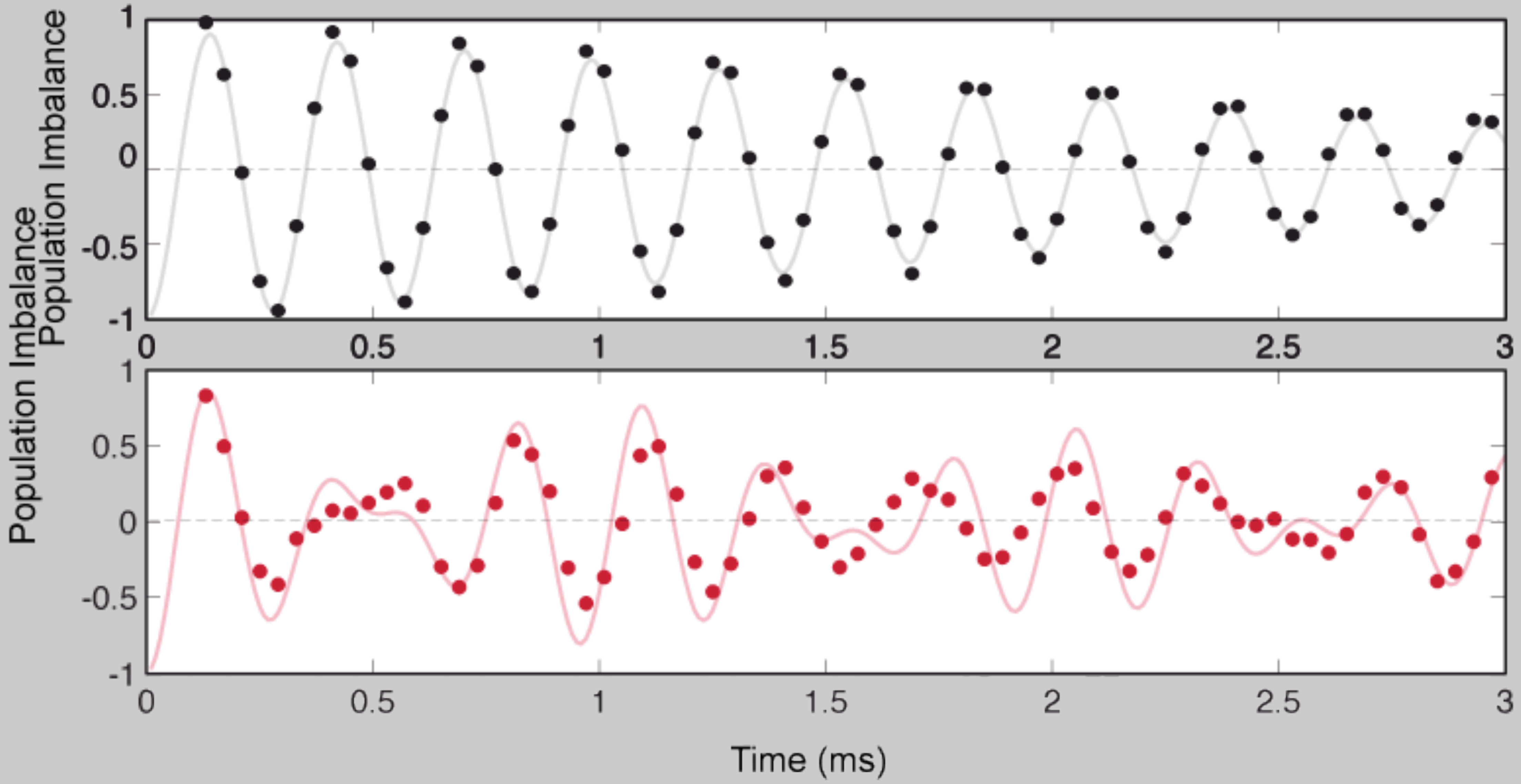
Phase of superposition state can be read out through phase of interference pattern.



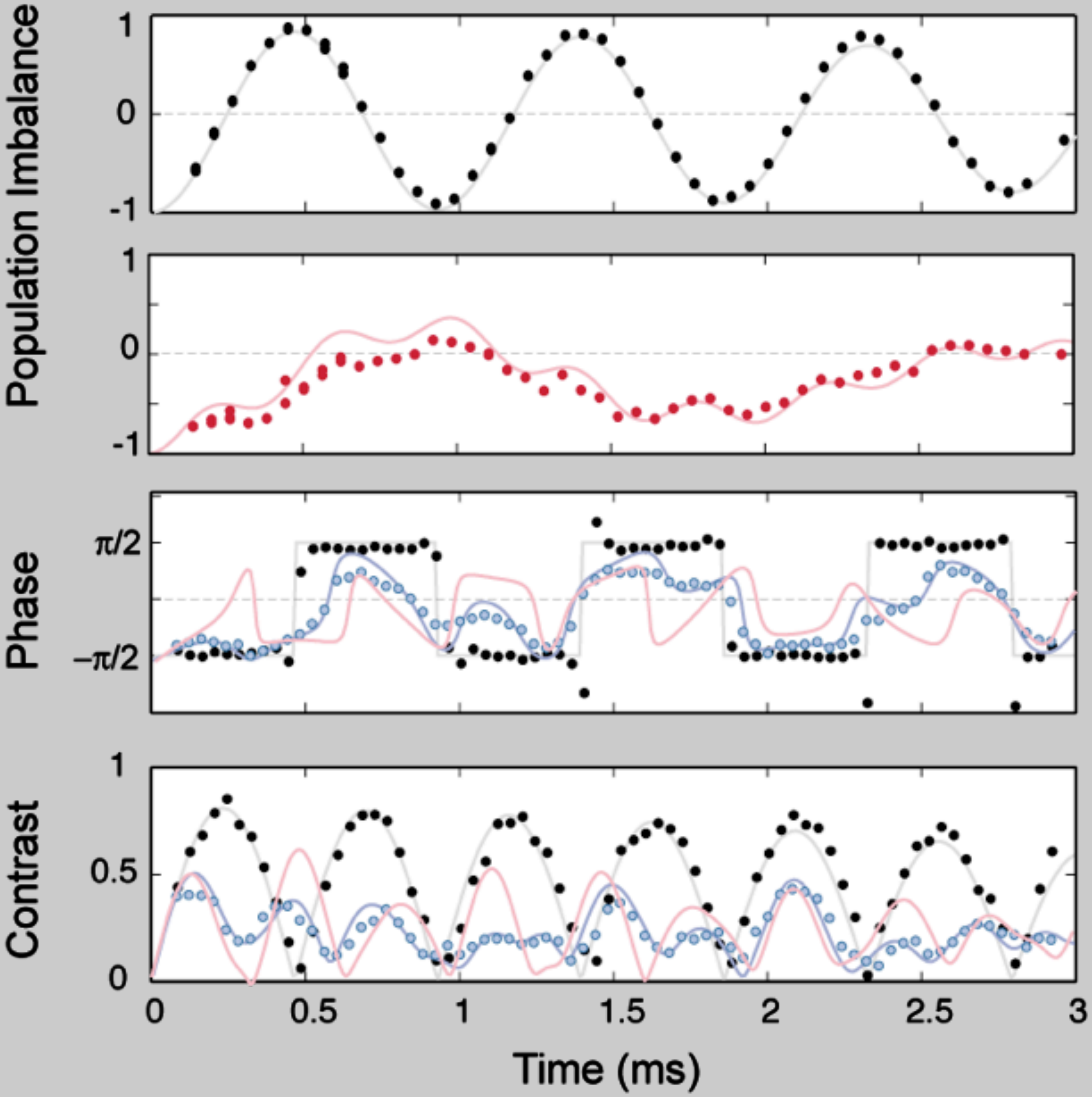
Single particle tunneling



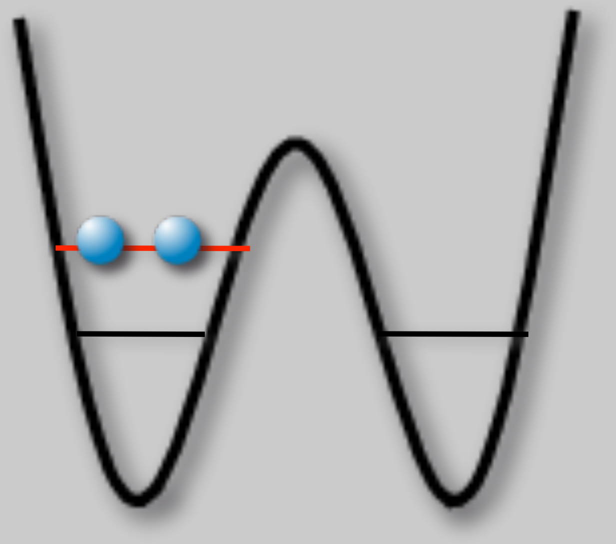
Correlated Pair Tunneling J/U=1.5



Correlated Particle Tunneling J/U=0.2

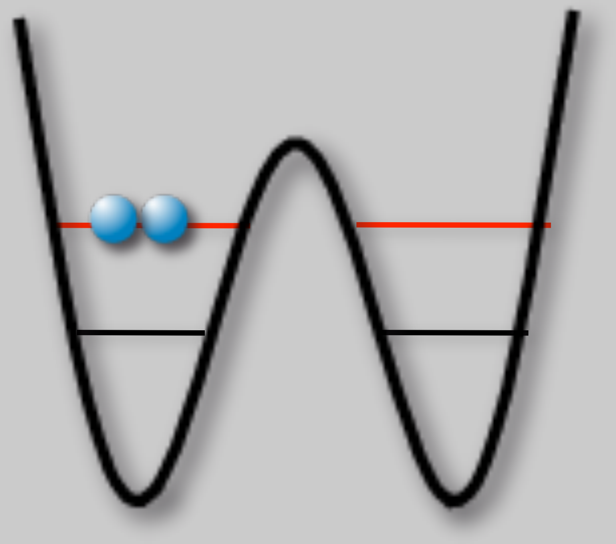


Tunneling under Repulsive Interactions



Single atom tunneling
Transition is detuned by *U*
Off-resonant tunneling between the two wells with frequency

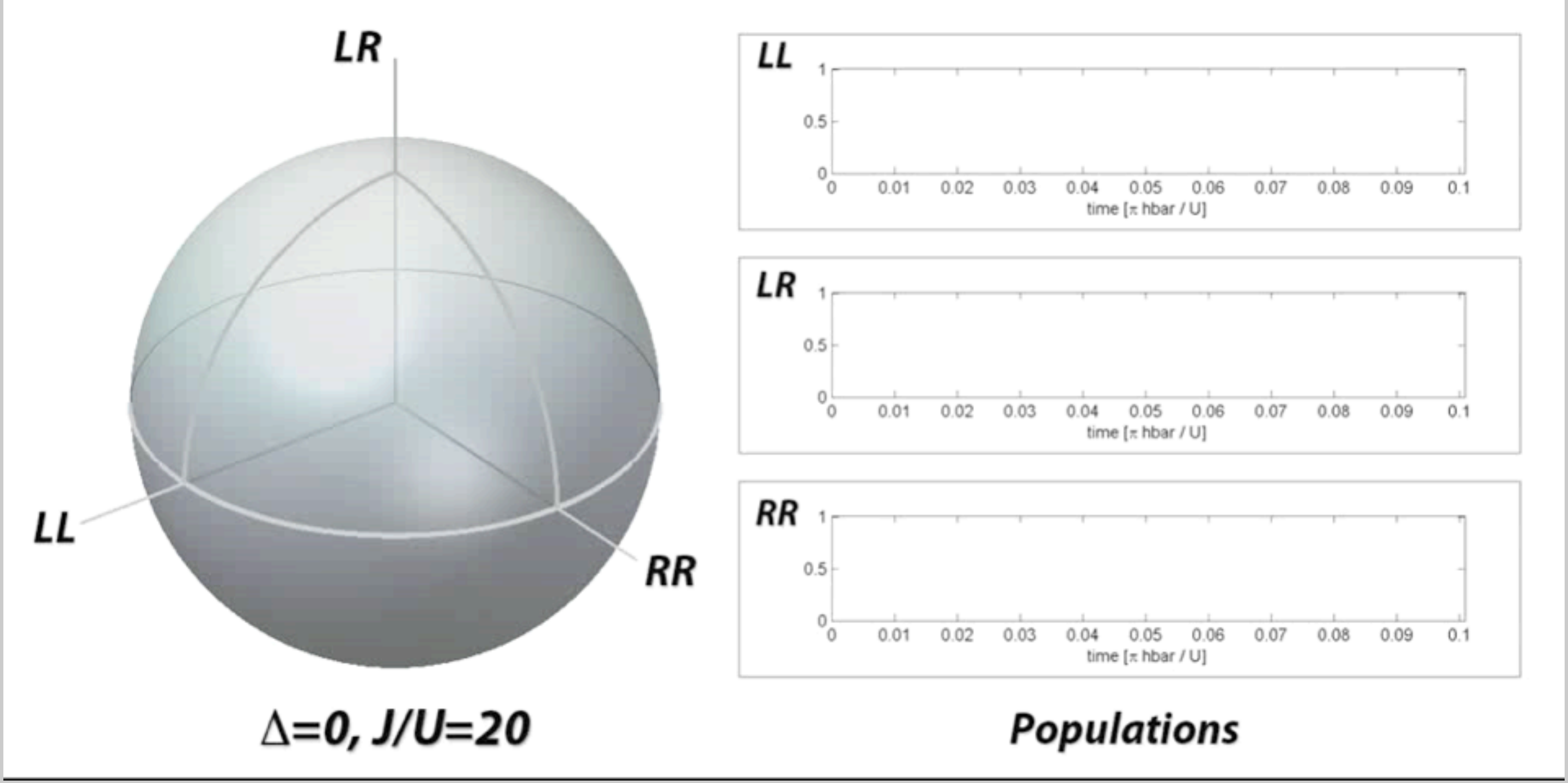
$$2\sqrt{4J^2+U^2}$$



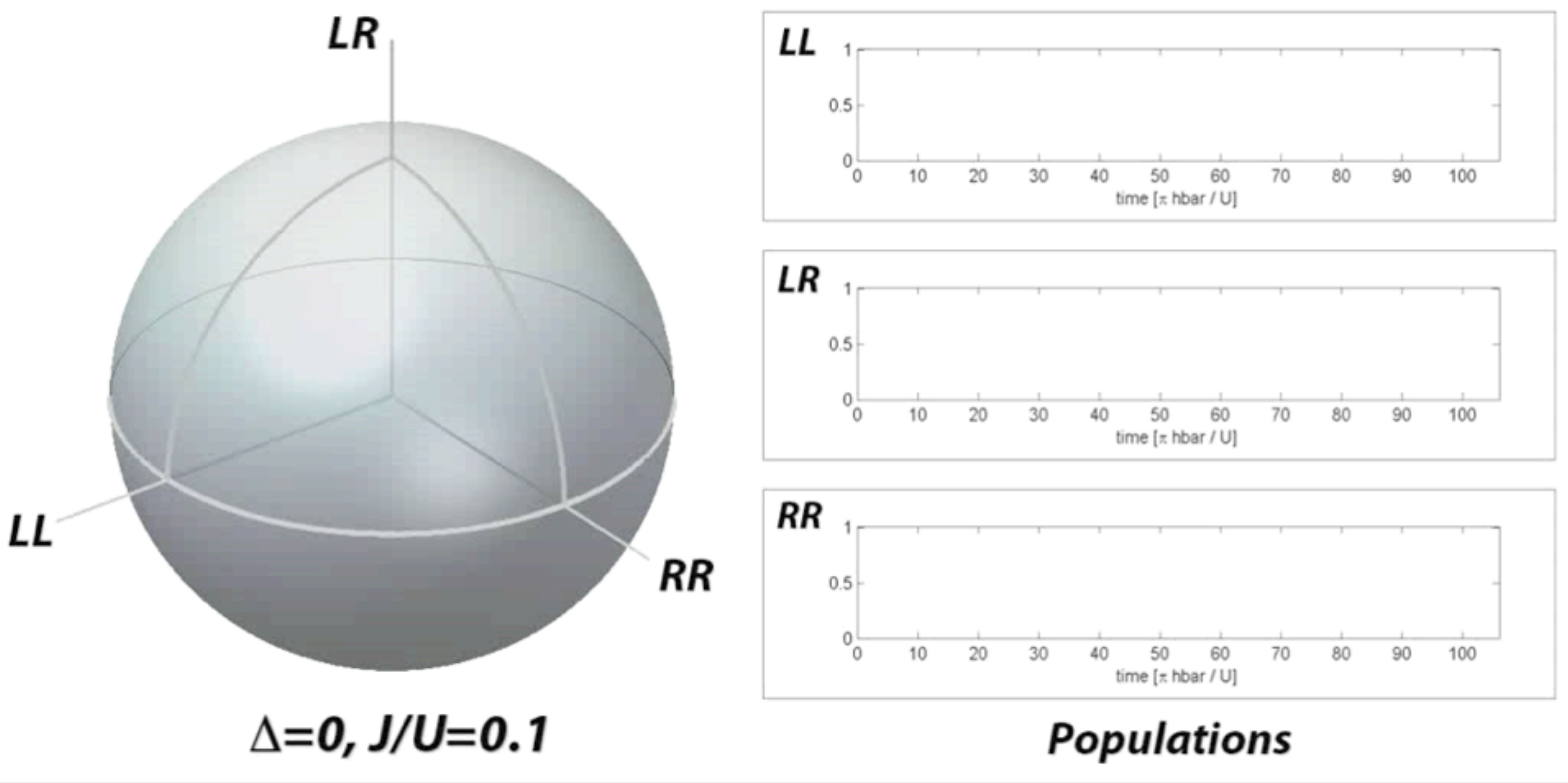
Simultaneous tunneling is resonant – with tunneling rate – co-tunnelling

$$J^2/U$$

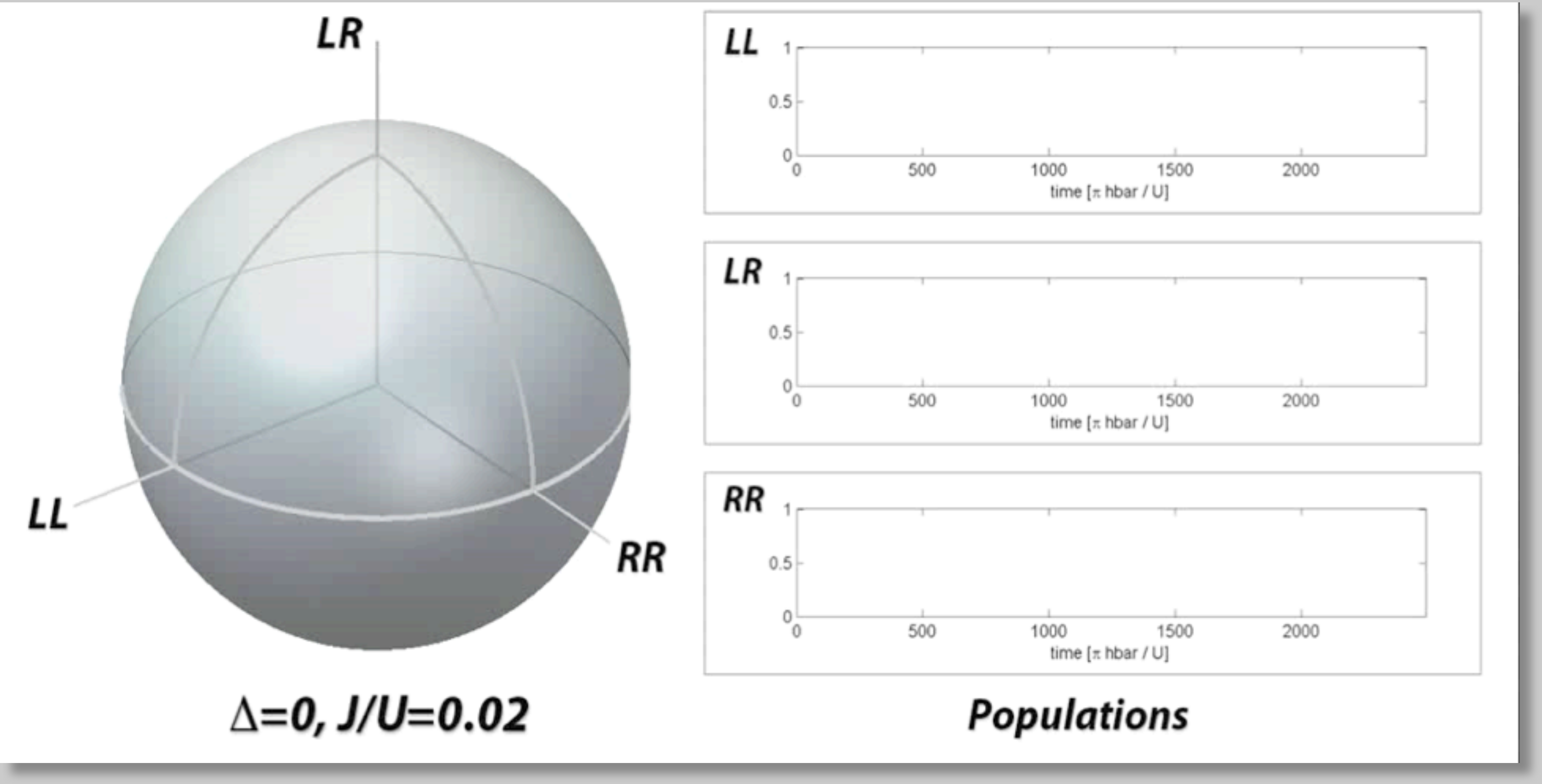
Atom Pair Tunneling J/U=20



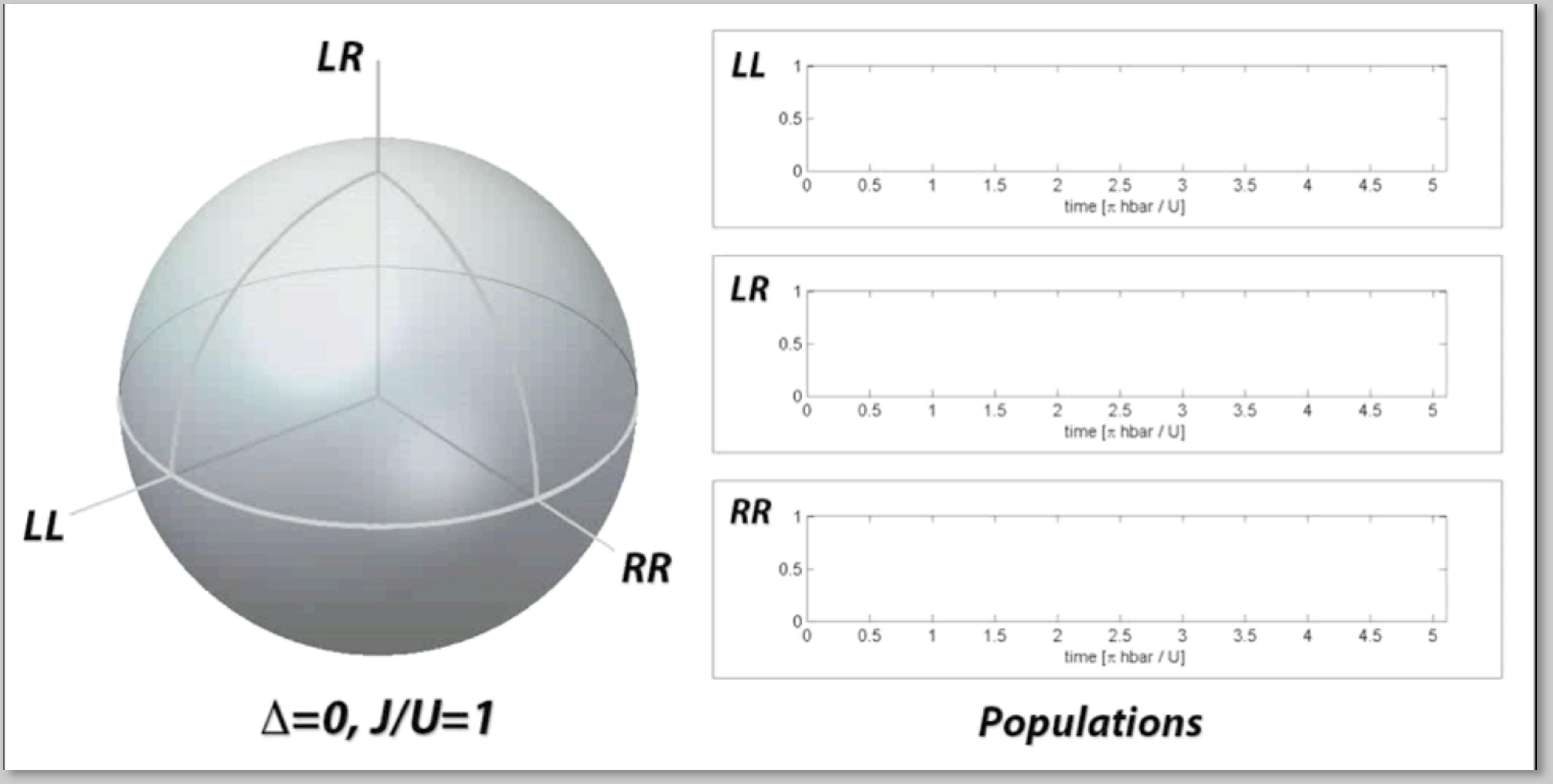
Atom Pair Tunneling J/U=0.1



Atom Pair Tunneling J/U=0.02



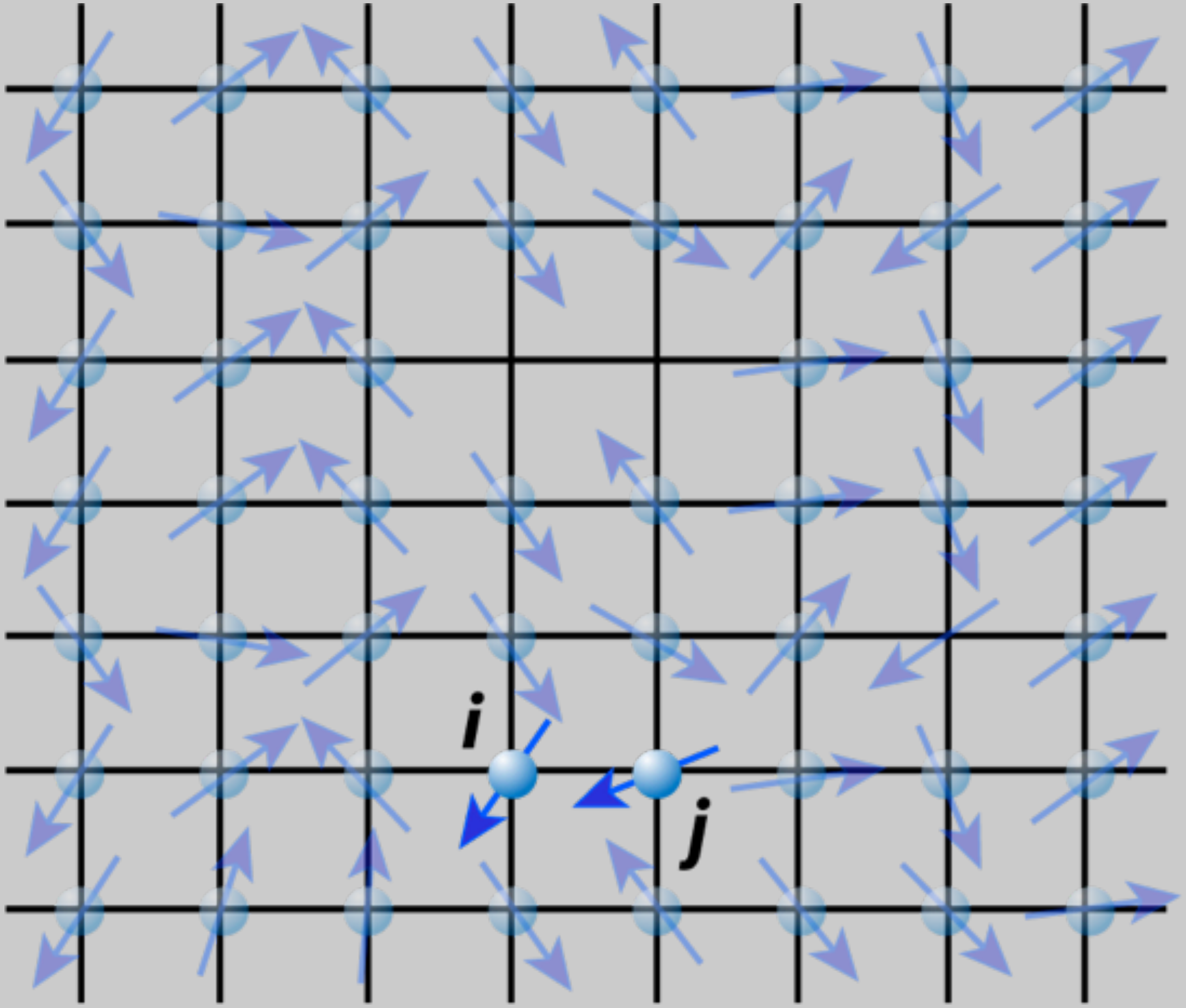
Atom Pair Tunneling J/U=1



Controlling Superexchange Interactions

S. Trotzky et al. Science (2008)

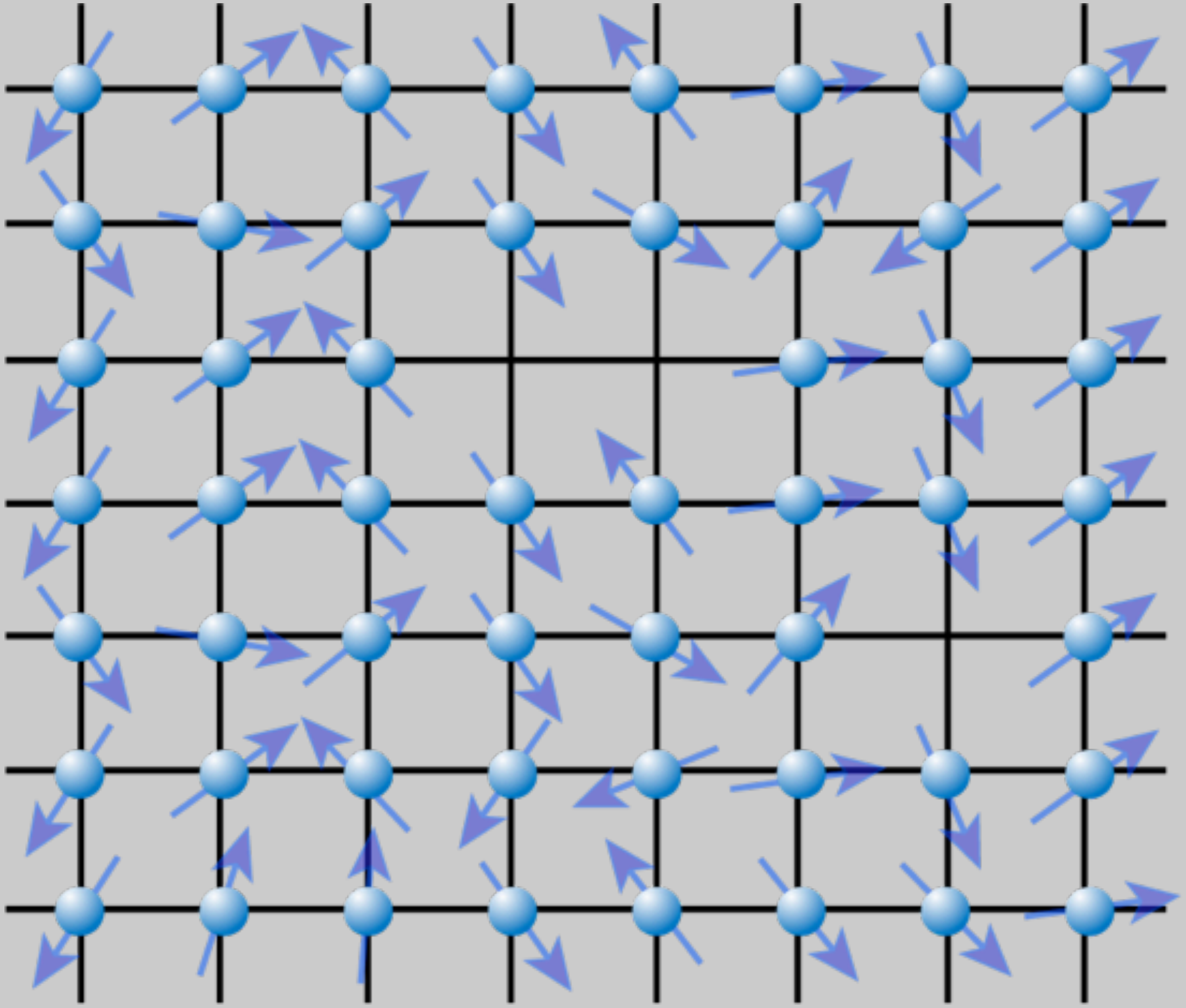
Quantum Spin Systems in Optical Lattices



In strongly correlated electron system spin-spin interactions exist.

$$-J_{ex}\vec{S}_i \cdot \vec{S}_j$$

Quantum Spin Systems in Optical Lattices

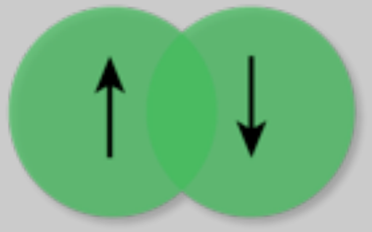


***Double occupancy suppressed* in strongly interacting regime of Mott insulator.**

Origin of Spin-Spin Interactions – Exchange Interactions

$$-J_{ex}\vec{S}_1\cdot\vec{S}_2$$
$$J_{ex}>0$$

In Atoms
(e.g. excited state Helium)

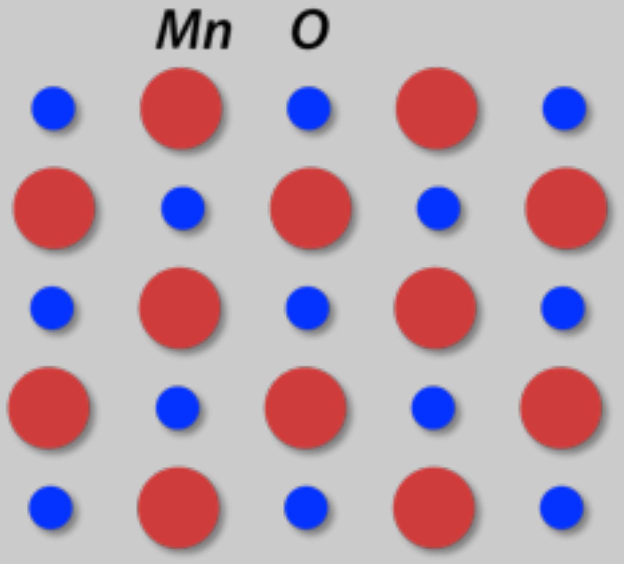


$$-J_{ex}\vec{S}_i\cdot\vec{S}_j$$
$$J_{ex}<0$$

In Molecules
(e.g. In molecule)

Direct overlap of electronic wave functions determines strength of exchange interactions
(typically very short ranged)

Origin of Spin-Spin Interactions – Exchange Interactions



?

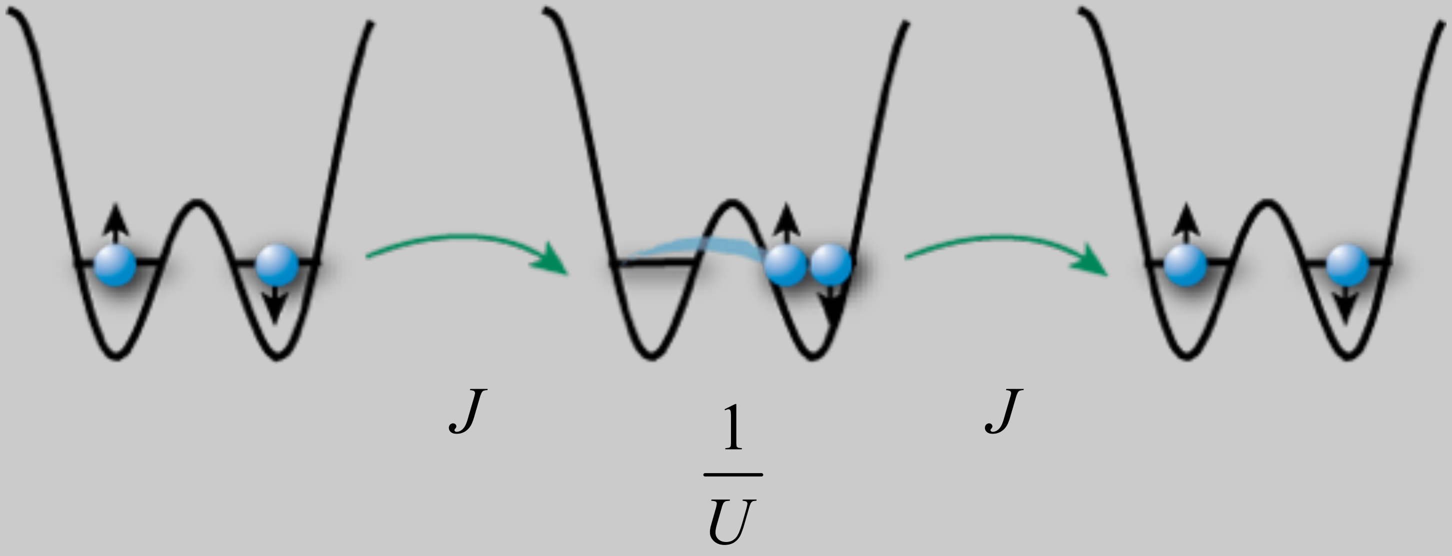
Important ionic solids with no direct exchange between magnetic ions show magnetic ordering (MnO, CuO)!

„Super“-exchange interactions must be at work!

P.W. Anderson, Phys. Rev. **79**, 350 (1950)

Quantum Magnetism

Second order hopping processes form the basis of superexchange interactions! (see e.g. A. Auerbach, Interacting Electrons and Quantum Magnetism)



$$-J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j+\frac{1}{2}U\sum_i\hat{n}_i(\hat{n}_i-1)$$

$$\boxed{H=-J_{ex}\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j}$$

$$\boxed{J_{ex}\propto\frac{J^2}{U}}$$

Ultracold atoms allow tuning of Spin-Hamiltonians

$$H=\sum_{\langle i,j\rangle}\left[\lambda_{\mu z}\hat{\sigma}_i^z\hat{\sigma}_j^z\pm\lambda_{\mu\perp}\left(\hat{\sigma}_i^x\hat{\sigma}_j^x+\hat{\sigma}_i^y\hat{\sigma}_j^y\right)\right]$$

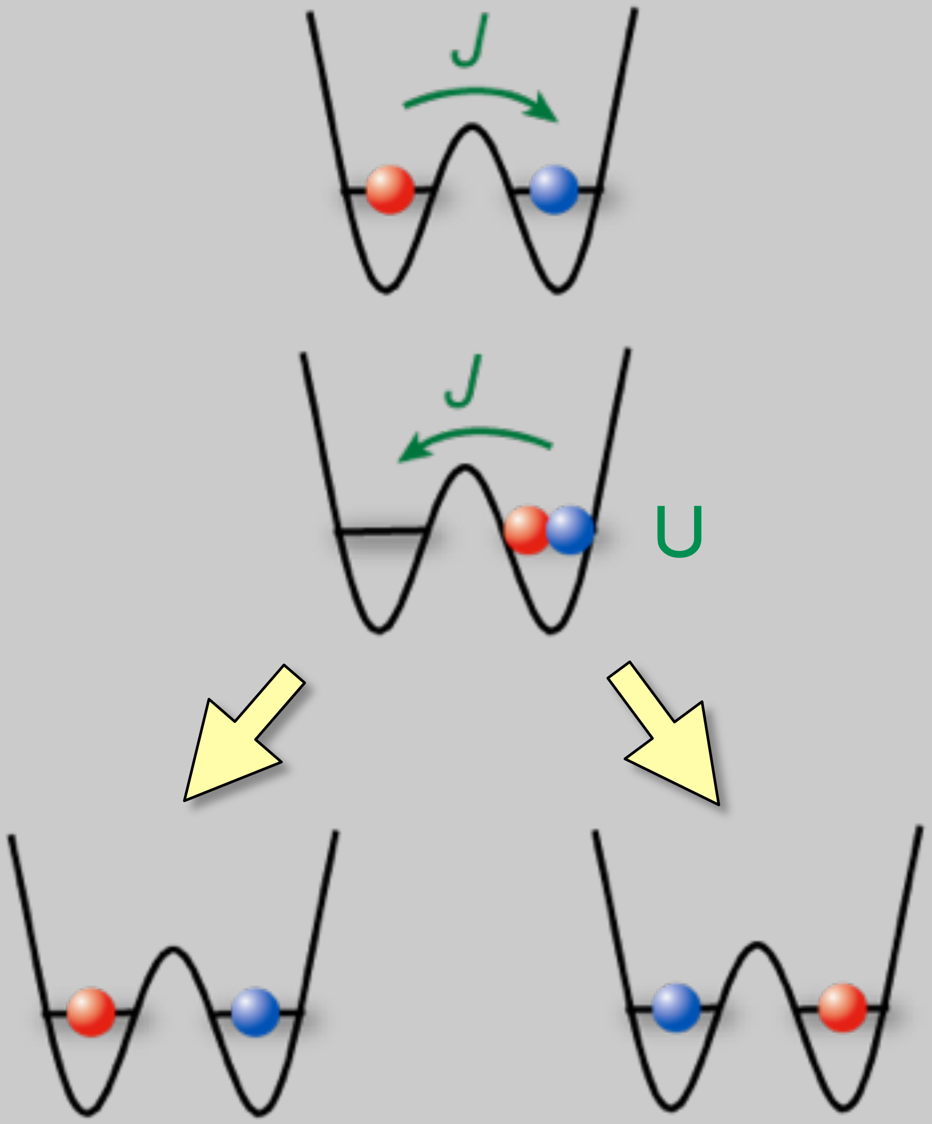
$$\lambda_{\mu z}=\frac{t_{\mu\uparrow}^2+t_{\mu\downarrow}^2}{2U_{\uparrow\downarrow}}-\frac{t_{\mu\uparrow}^2}{U_\uparrow}-\frac{t_{\mu\downarrow}^2}{U_\downarrow}$$

$$\lambda_{\mu\perp}=\frac{t_{\mu\uparrow}t_{\mu\downarrow}}{U_{\uparrow\downarrow}}$$

L.M. Duan et al., PRL **91**, 090402 (2003),
E. Altman et al., NJP **5**, 113 (2003), A.B. Kuklov et al. PRL **90**, 100401 (2003)

Deriving the Effective Spin Hamiltonian (1)

How do we get from $-J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j+\frac{1}{2}U\sum_i\hat{n}_i(\hat{n}_i-1)$ to $H=-J_{ex}\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j$?



$$\hat{1} \quad + \quad \hat{X}_{LR}$$

Deriving the Effective Spin Hamiltonian (2)

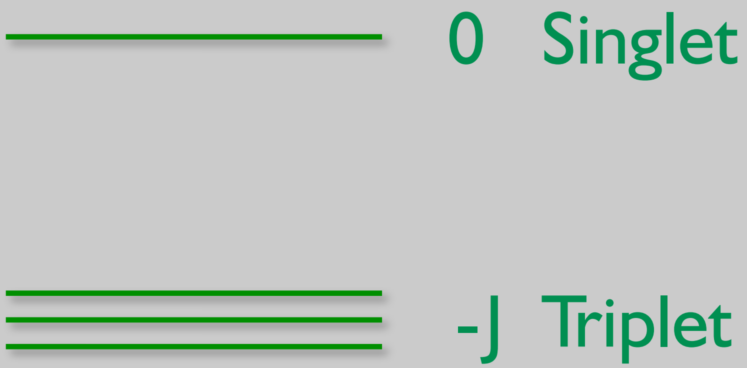
Second order hopping can be written as

$$H = -2\frac{J^2}{U}\left(1+\hat{X}_{LR}\right)$$

$$\hat{X}_{LR}\left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right] = -\left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right]$$

$$\hat{X}_{LR}\left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}\right] = +\left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}\right]$$

$$H = -J_{ex}\hat{P}_{\text{triplet}}$$



Deriving the Effective Spin Hamiltonian (3)

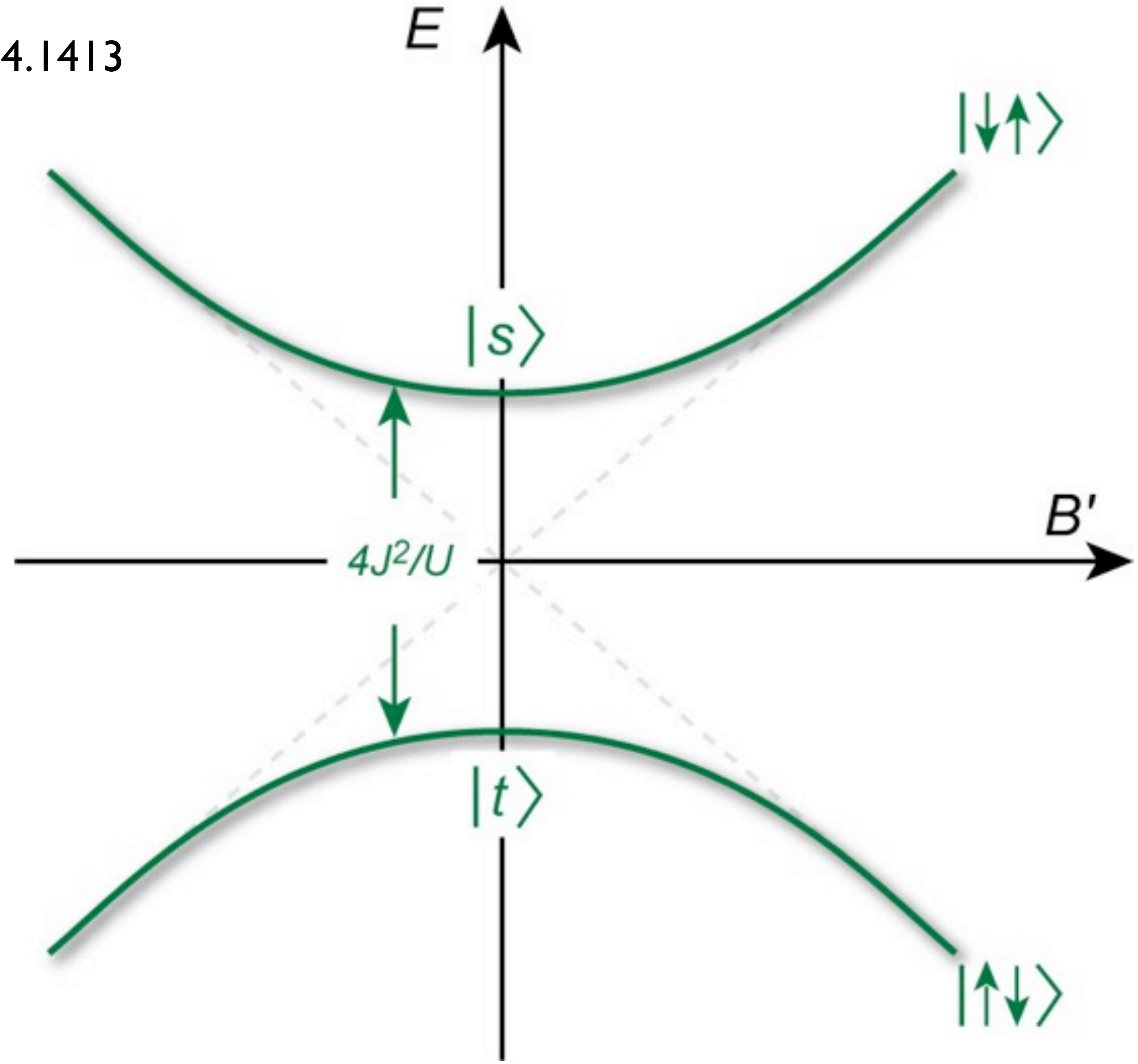
$$\hat{P}_{\text{triplet}} = \hat{P}_{S=1}$$

$$\begin{aligned}\mathbf{S}_L \cdot \mathbf{S}_R &= \frac{(\mathbf{S}_L + \mathbf{S}_R)^2}{2} - \frac{3}{4} \\ &= \frac{S(S+1)}{2} - \frac{3}{4} \\ &= \hat{P}_{S=1} - \frac{3}{4}\end{aligned}$$

$$\boxed{H = -J_{ex}\left(\mathbf{S}_L \cdot \mathbf{S}_R + \frac{3}{4}\right)}$$

Direct Detection of Superexchange Interactions

A. M. Rey et al., cond-mat/0704.1413

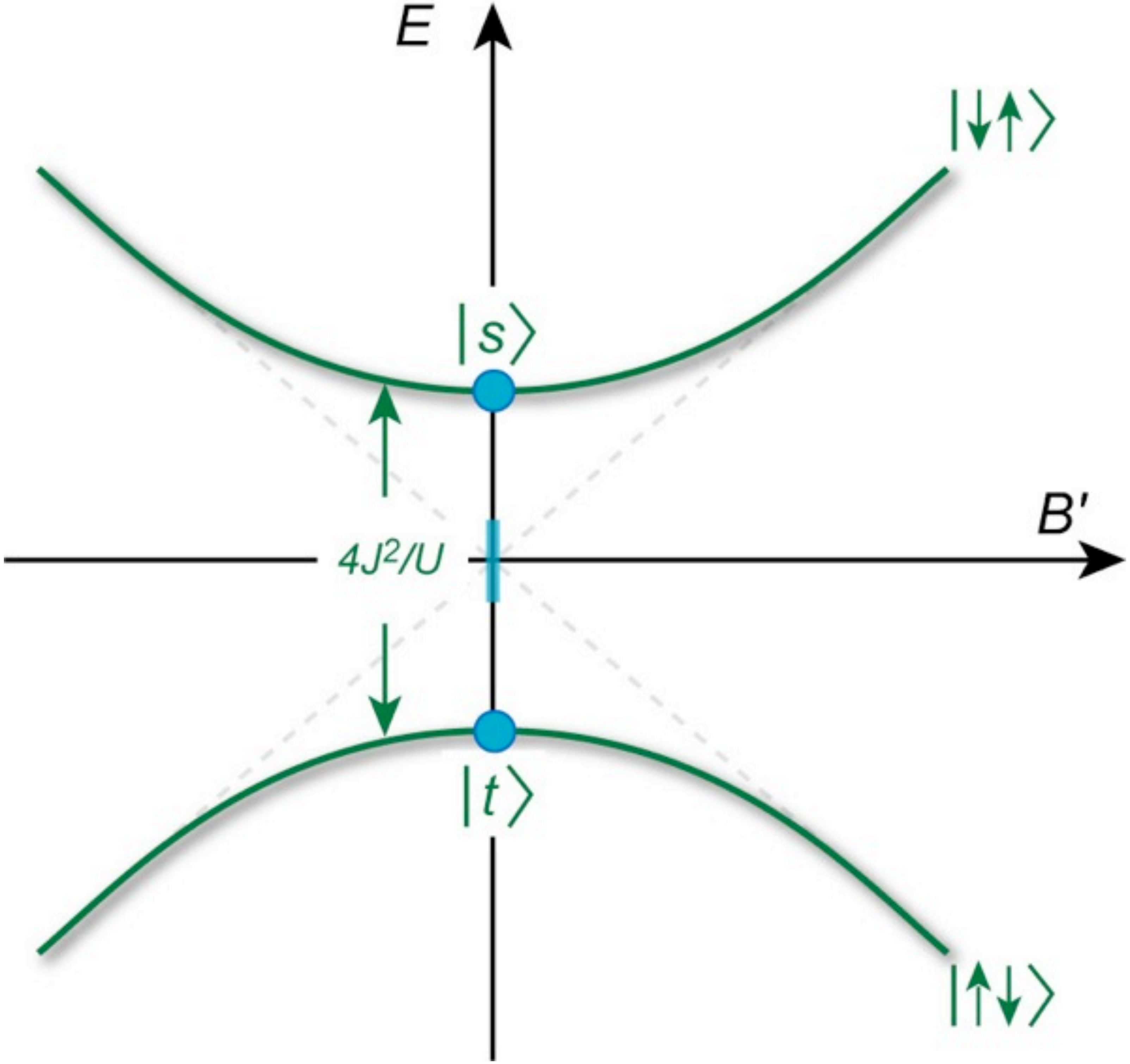


$$|s\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

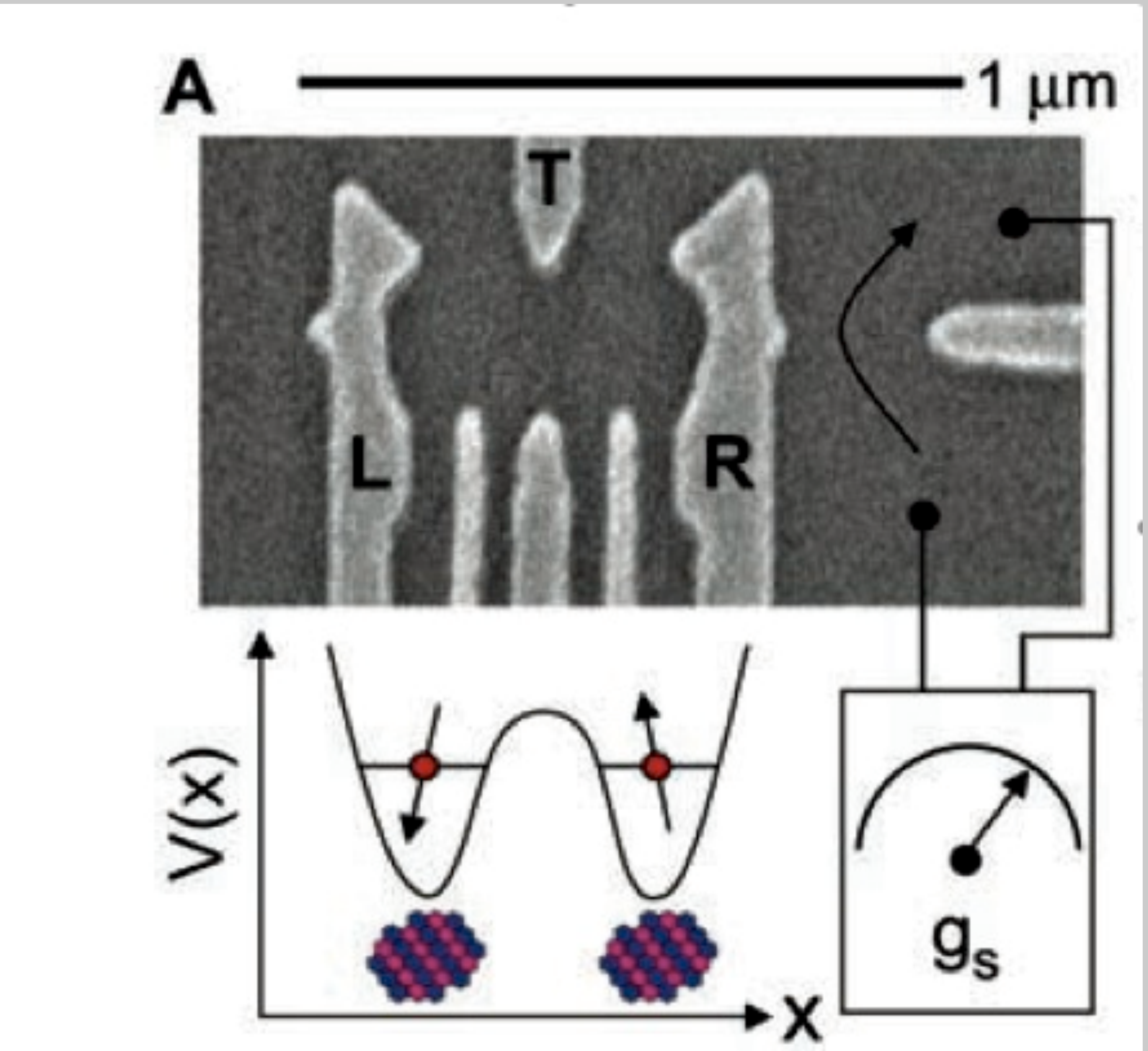
$$|t\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

$$H_{eff} = -J_{ex}\,\vec{S}_L \cdot \vec{S}_R - \mu_B B'\left(S_{z,L} - S_{z,R}\right)$$

Direct Detection of Superexchange Interactions (2)

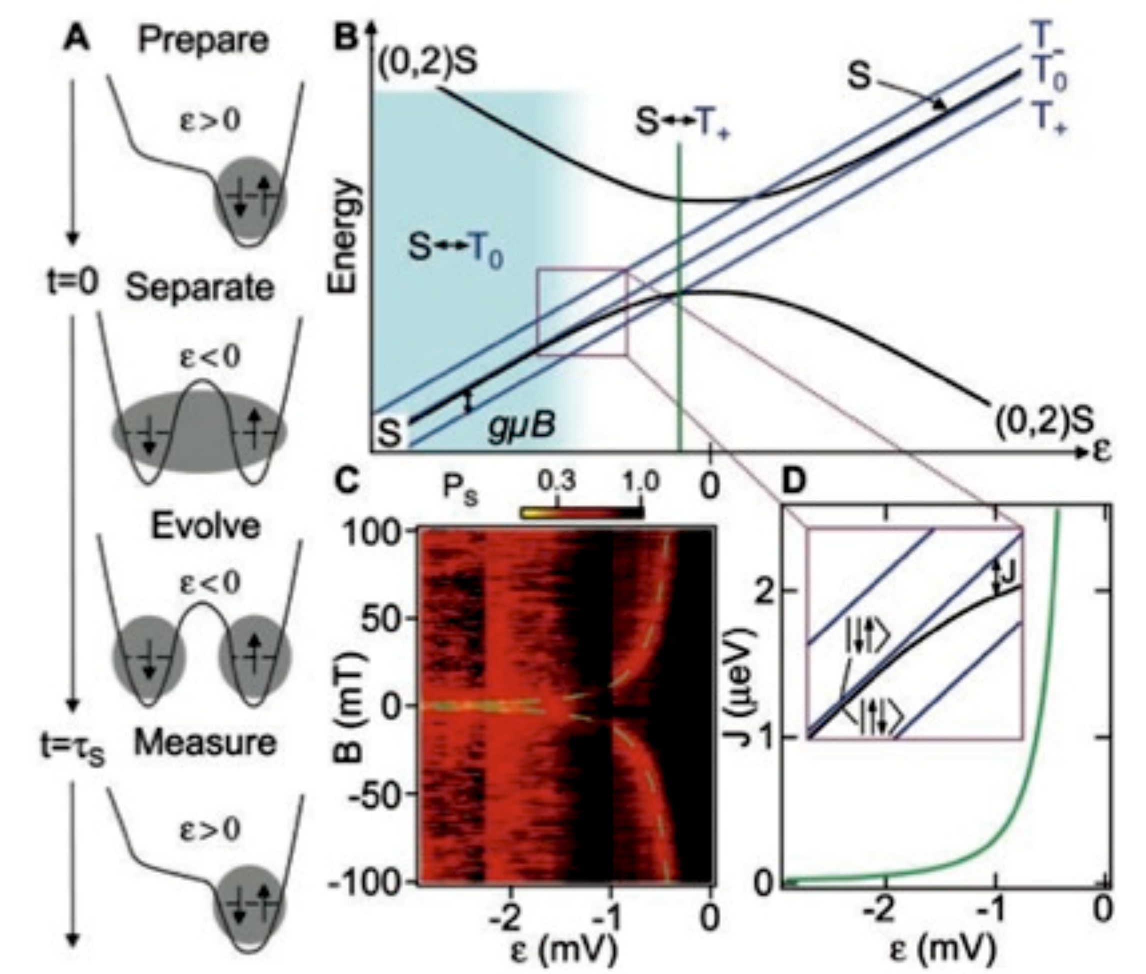


Superexchange Coupling in Quantum Dots



J.R. Petta et al., Science **309**, 2180 (2005)

Coherent Manipulation of Coupled Electron Spins in Semiconductor Quantum Dots

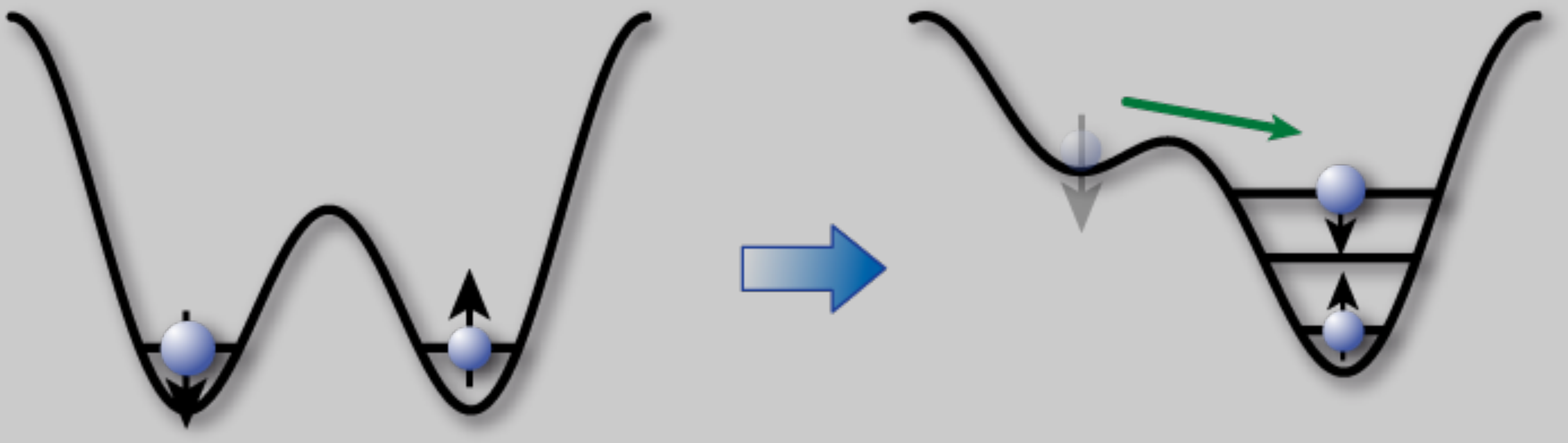


Local control of spin states & interactions between spin states.

Superexchange induced flopping

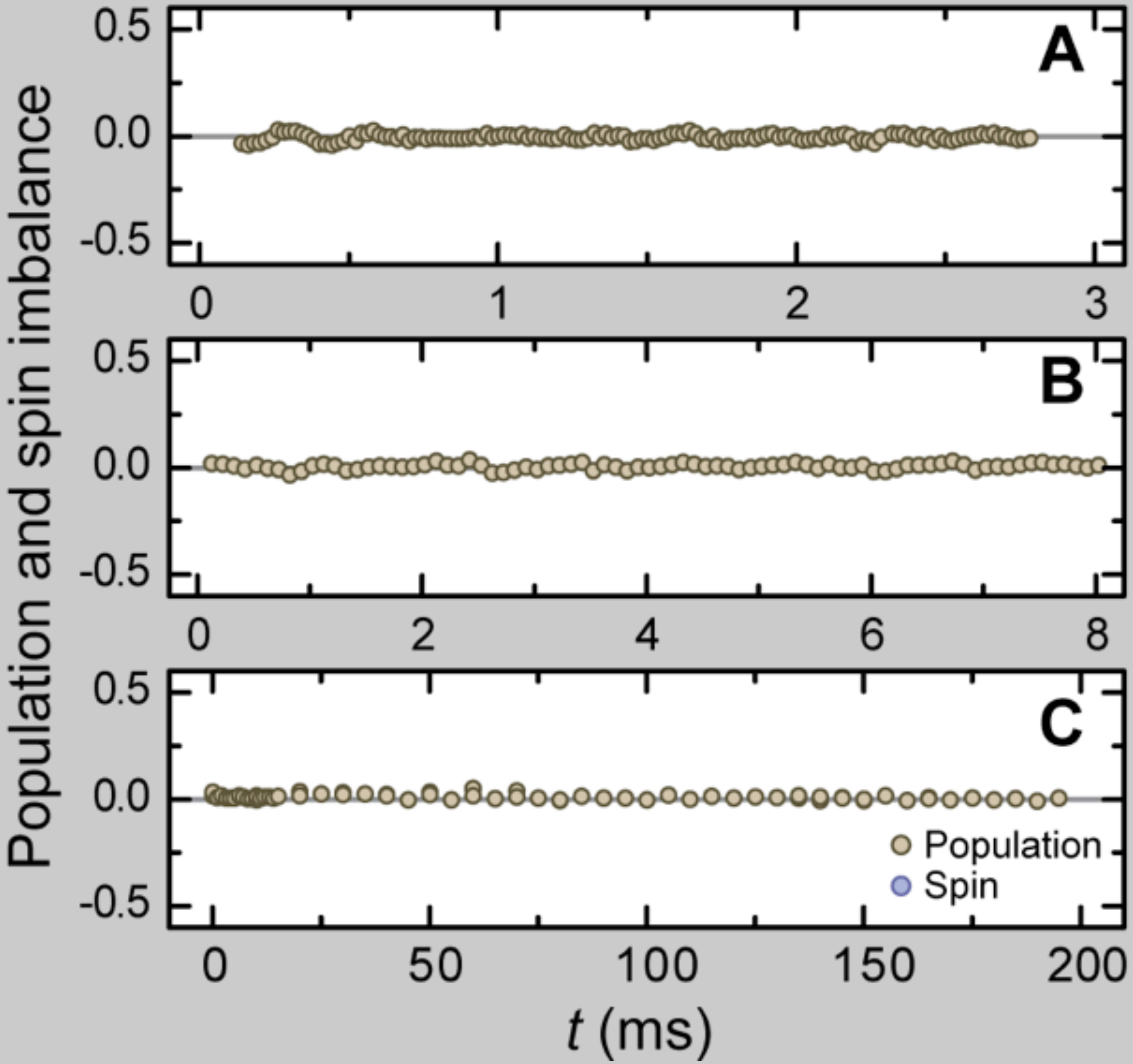
$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j$$
$$= -\frac{J_{ex}}{2}\left(\hat{S}_i^+\hat{S}_j^- + \hat{S}_i^-\hat{S}_j^+\right) - J_{ex}\hat{S}_i^z\hat{S}_j^z$$

Mapping the Spins



Initial AF order verified in the experiment!

Superexchange induced flopping

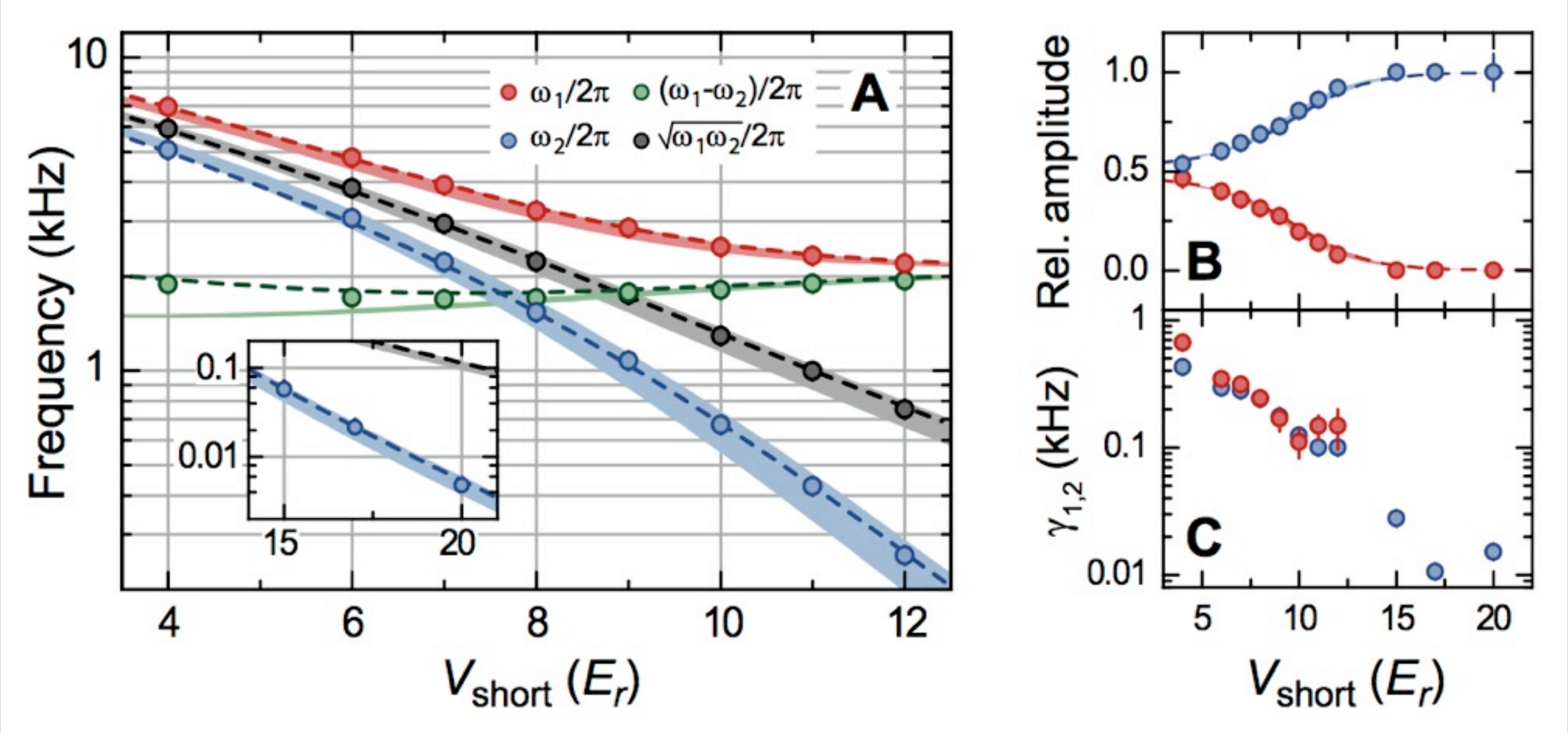


$J/U=1.25$
$V_{short}=6\ E_r$

$J/U=0.26$
$V_{short}=11\ E_r$

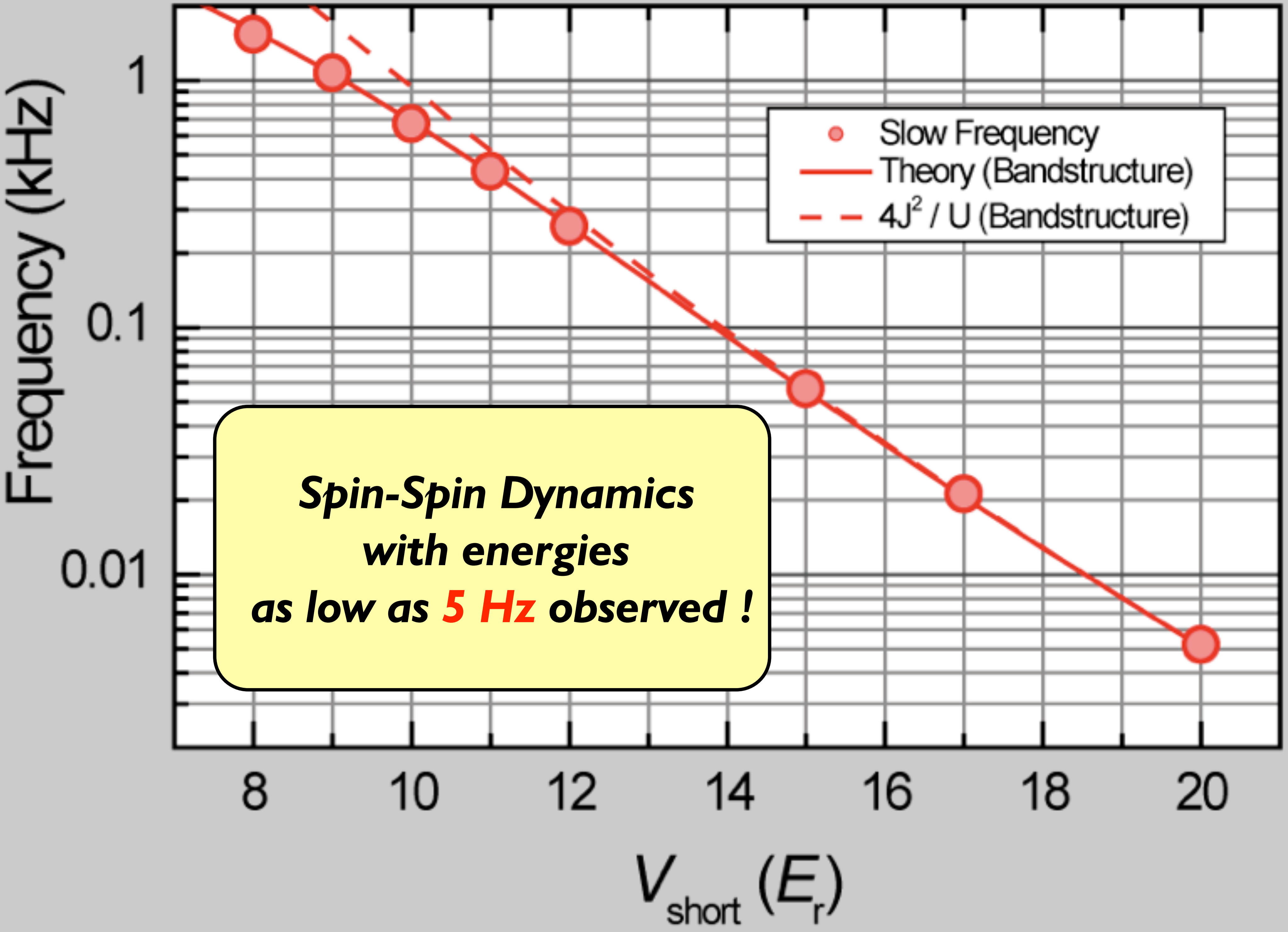
$J/U=0.05$
$V_{short}=17\ E_r$

Measured Frequencies



$$U' = U + 3U_{LR} \qquad J'_{\text{ex}} = 2J'^2/U - U_{\text{LR}}$$

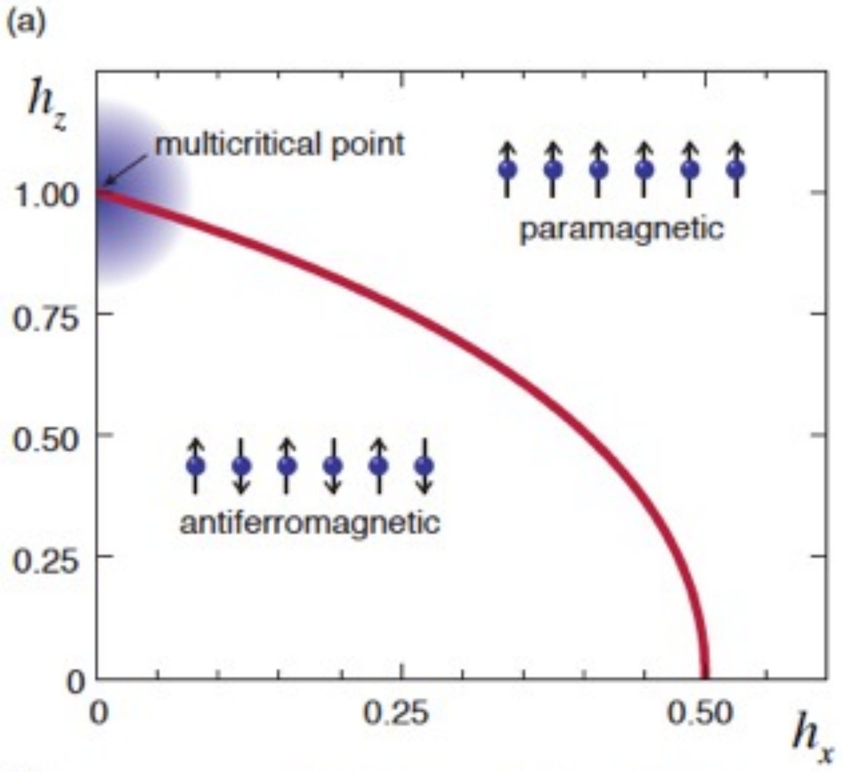
Oscillation Frequencies (2)



Spin-Spin Dynamics with energies as low as 5 Hz observed !

QMag

Quantum Magnetism in Tilted Lattices

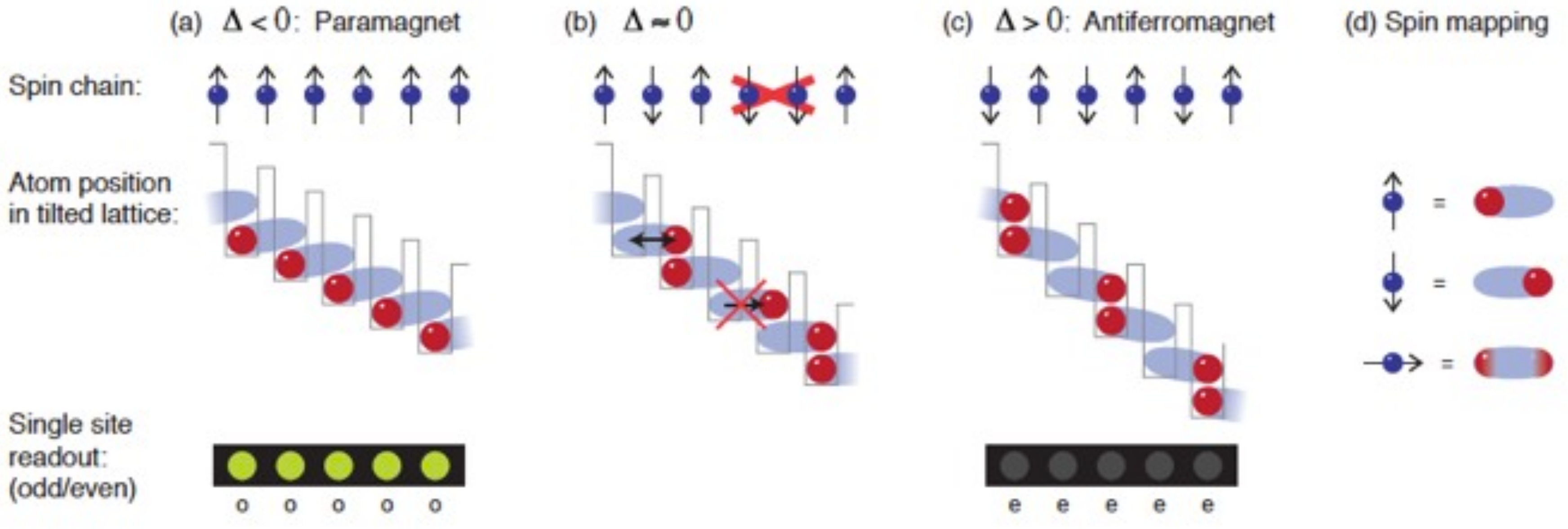


(b)

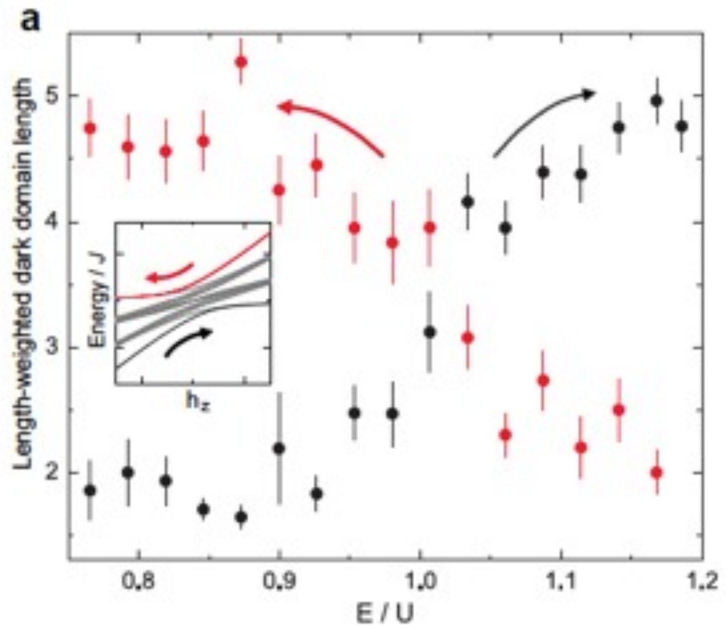
$$H = J\sum_i \overbrace{S_z^i S_z^{i+1}}^{\text{realizes constraint}} \overbrace{-\underbrace{(1-\tilde{\Delta})}_{h_z} S_z^i - \underbrace{2^{3/2}\tilde{t}}_{h_x} S_x^i}^{\text{drives quantum phase transition}}$$

magnetic fields: longitudinal transverse

Mapping onto
Quantum Ising Model



Pseudospin in density sector of MI



Domain size
approx 5 sites

J. Simon et al. Nature (2011)
S. Sachdev et al., PRB (2002) & S. Pielawa et al. arXiv:1101.2897v2

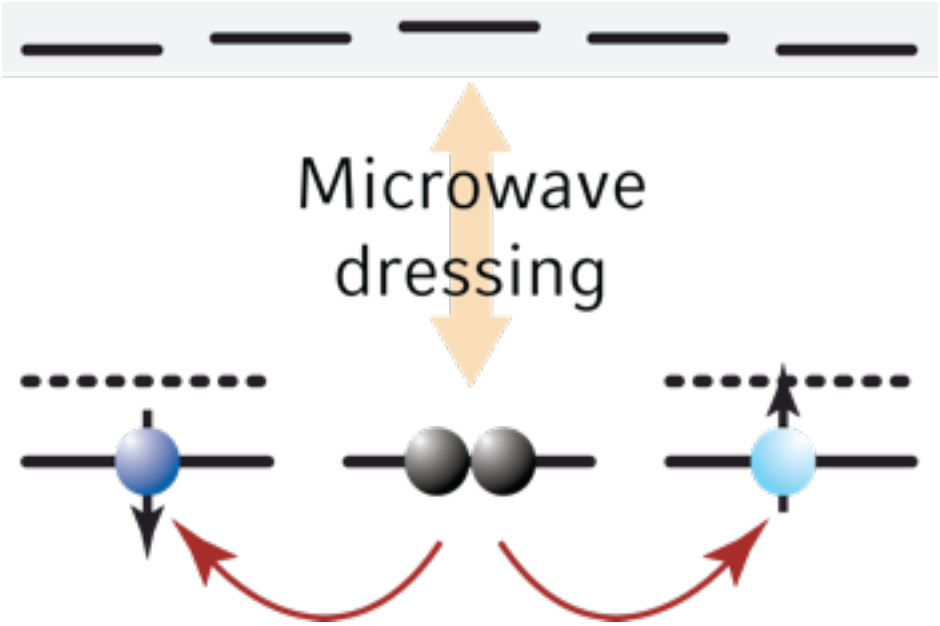
Controlling and Detecting Spin Correlations

S. Trotzky et al., Phys. Rev. Lett **105**, 265303 (2010)

Loading Spin Pairs

- Atom pairs in long-lattice wells
- Initialize in $|F=1, m_F=0\rangle$
- Microwave-dressed spin-changing collisions $\rightarrow$ **Spin-pairs** in $|F=1, m_F=\pm1\rangle$

*A. Widera et al., PRL **95** (2005)*
*F. Gerbier et al., PRA **73** (2006)*



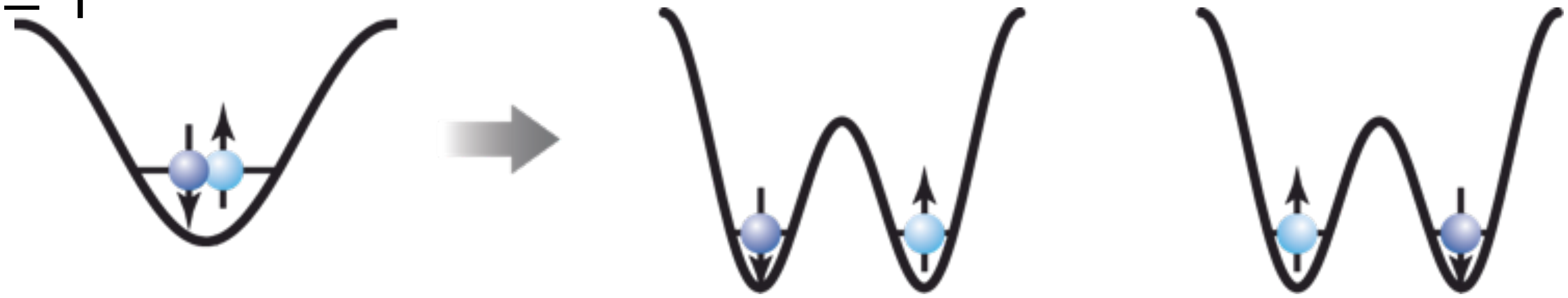
Collisionally driven Rabi-oscillations

Splitting a spin pair

- **Spin pairs** in $|F=1, m_F=\pm 1\rangle \equiv |\uparrow\rangle, |\downarrow\rangle$ (repulsive)
- Barrier raised *slowly* to split

→ Crossing a miniature Mott-transition: $n_{\text{Left}} = n_{\text{Right}} = 1$

J. Sebby-Strabley et al., PRL ***98*** *(2007)*



Details on the loading of the Spin-pairs:
S.T., P. Cheinet et al., Science ***319*** *(2008)*

Splitting a spin pair

- **Spin pairs** in $|F=1, m_F=\pm 1\rangle \equiv |\uparrow\rangle, |\downarrow\rangle$
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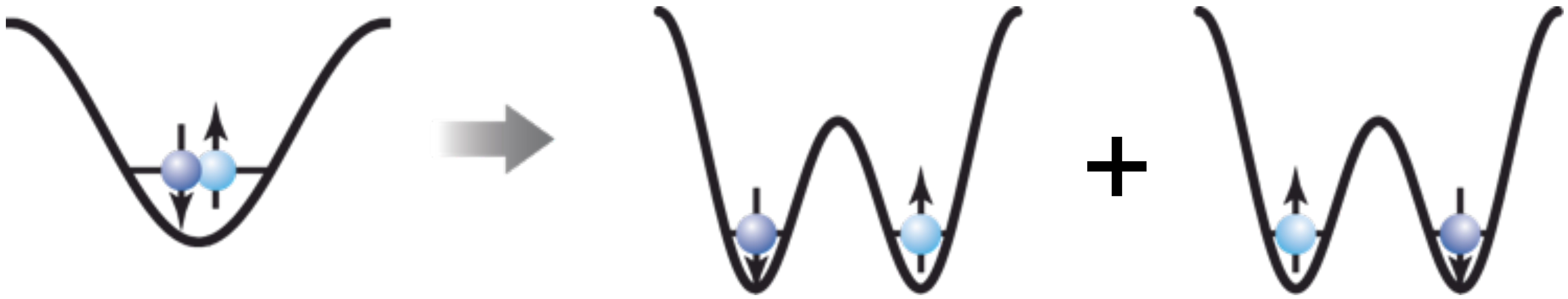


- **Bosons**: Symmetric wavefunction → Triplet $|t_0\rangle$

(Fermions: Antisymmetric wavefunction → Singlet $|s\rangle$)

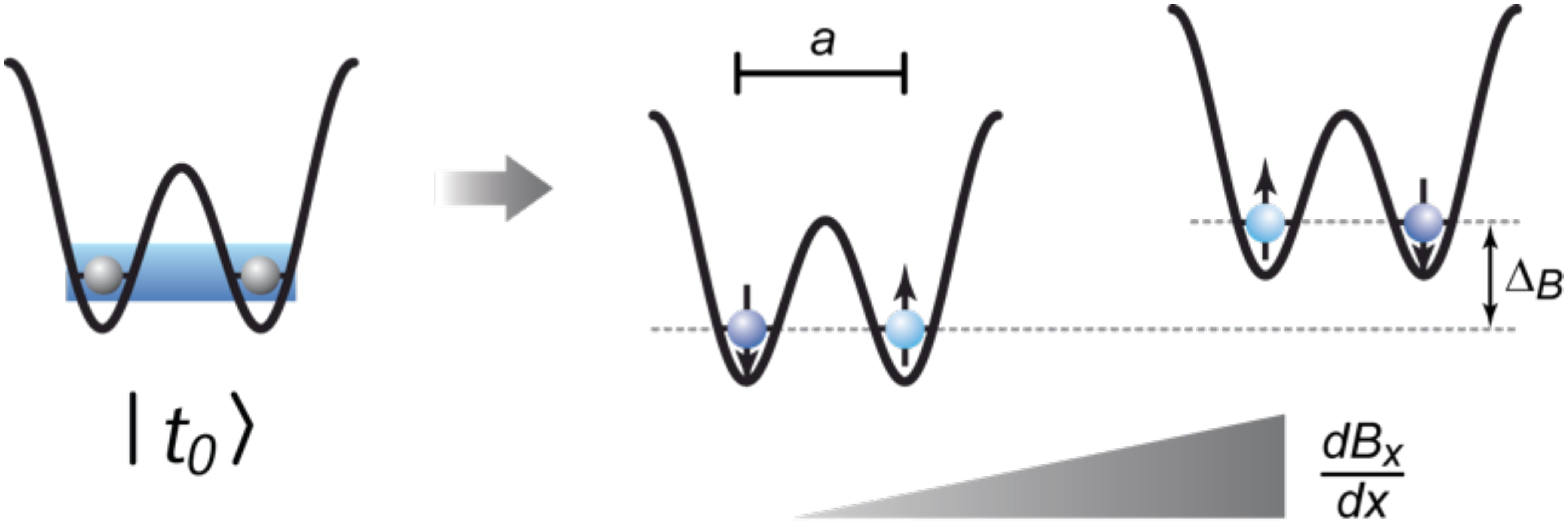
*A.-M. Rey at al., PRL **99** (2007)*

Details on the loading of the Spin-pairs:
*S.T., P. Cheinet et al., Science **319** (2008)*

Driving Triplet-Singlet oscillations

- **Magnetic field gradient** lifts degeneracy:

$$\Delta_B \propto a \cdot \partial_x B_x$$



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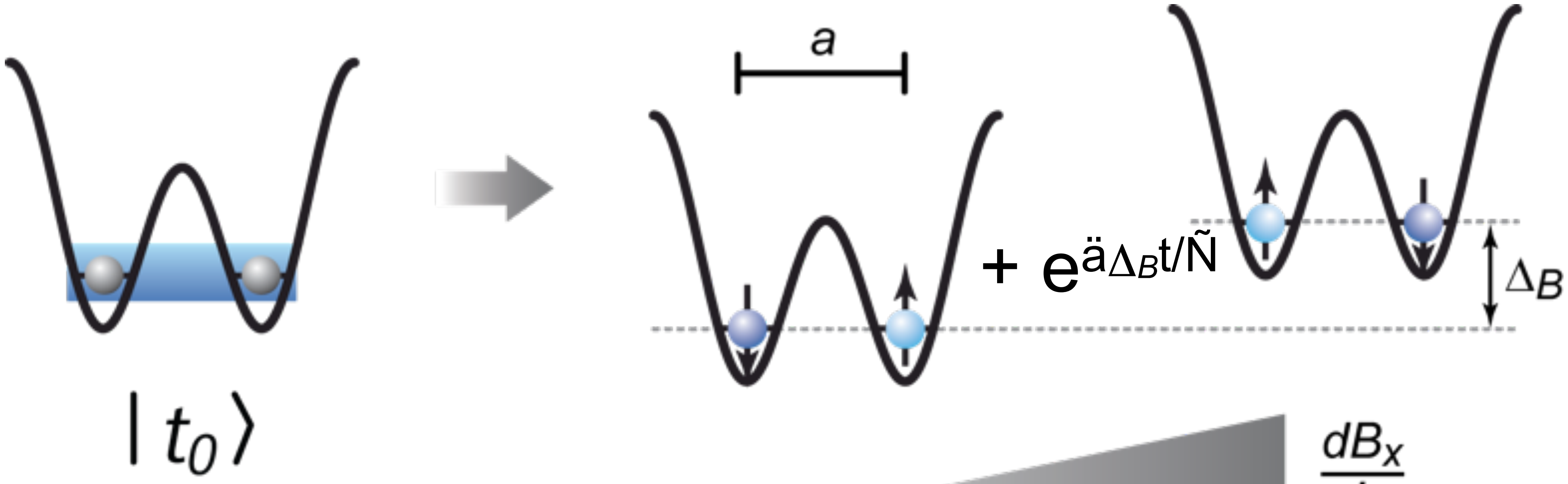
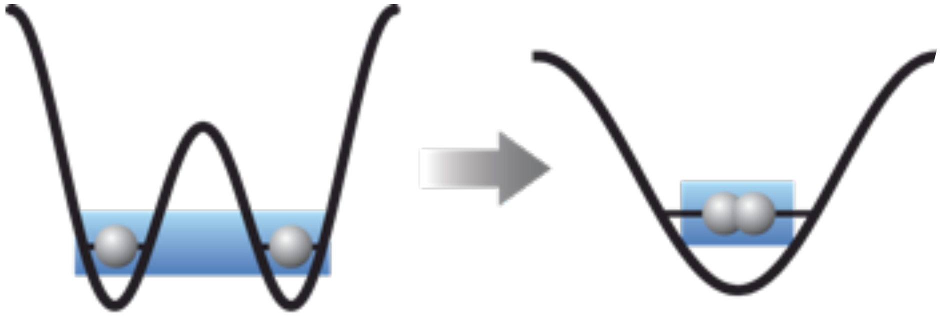


- **Triplet-Singlet oscillations** with frequency Δ_B / Ñ

$$\boxed{|t_0\rangle \quad \leftrightarrow \quad |s\rangle}$$

How to detect triplets and singlets

- Barrier lowered slowly to **merge** double-wells
  → **Triplet**: both atoms reach the **ground state**



How to detect triplets and singlets

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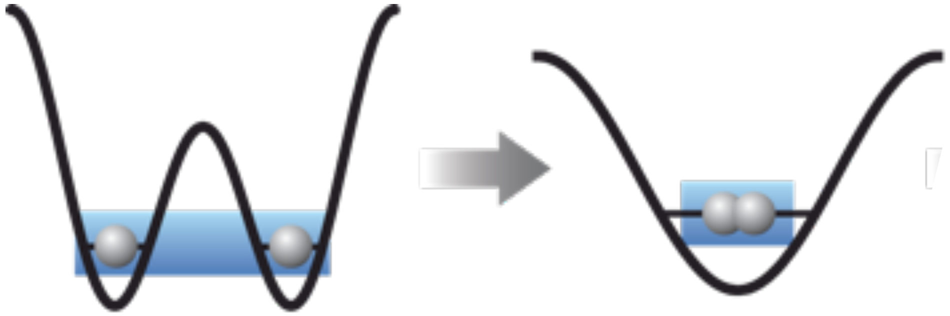
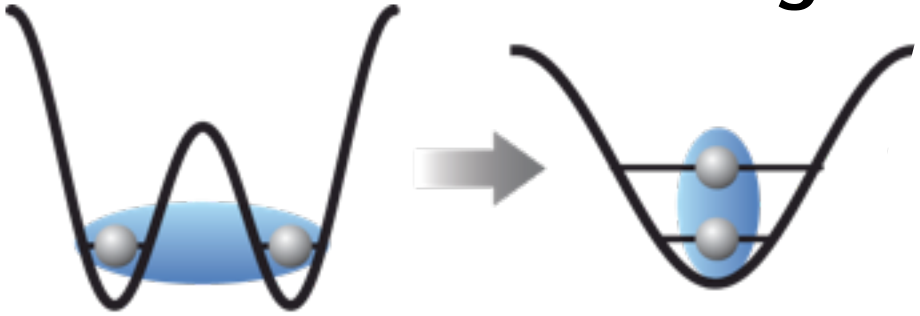


→ **Singlet**: needs anti-symm. spatial wavefunction (Bosons)

One atom transferred to **higher vibrational band**



JG|U

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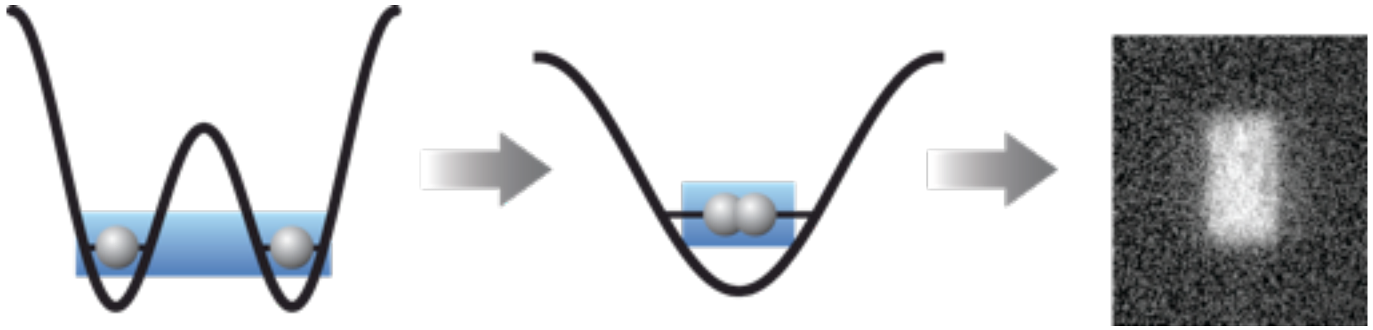
LMU

How to detect triplets and singlets

- Barrier lowered slowly to **merge** double-wells

→ **Triplet**: both atoms reach the **ground state**



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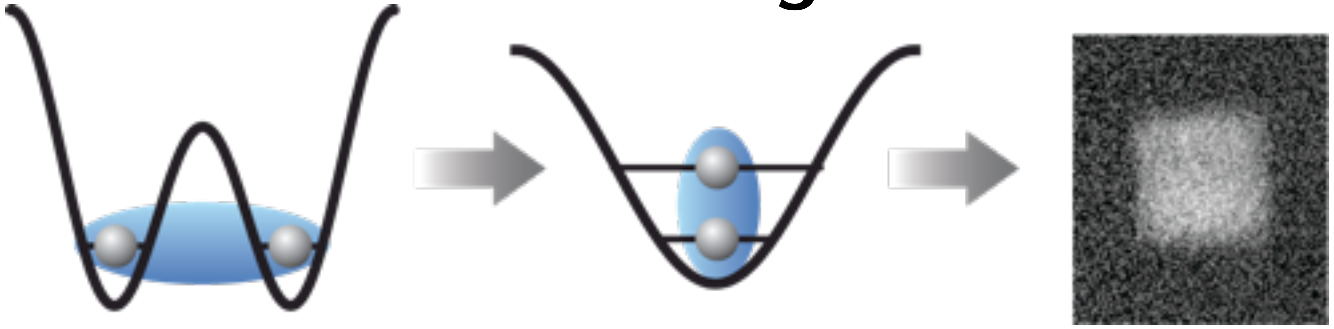
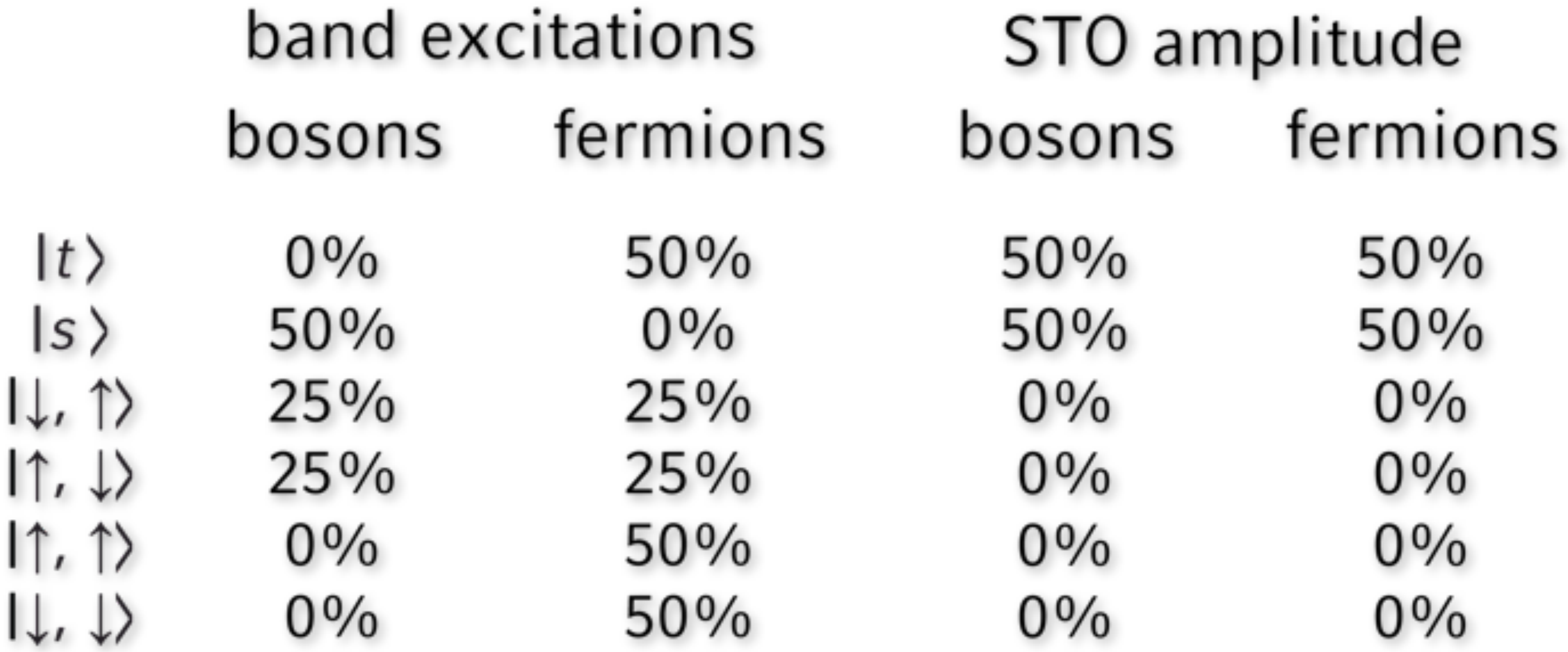


Band-mapping reveals **singlet-contribution**
in higher Brillouin-Zone

A sensitive probe of next-neighbor spin-correlations in Mott-insulator type many-body systems

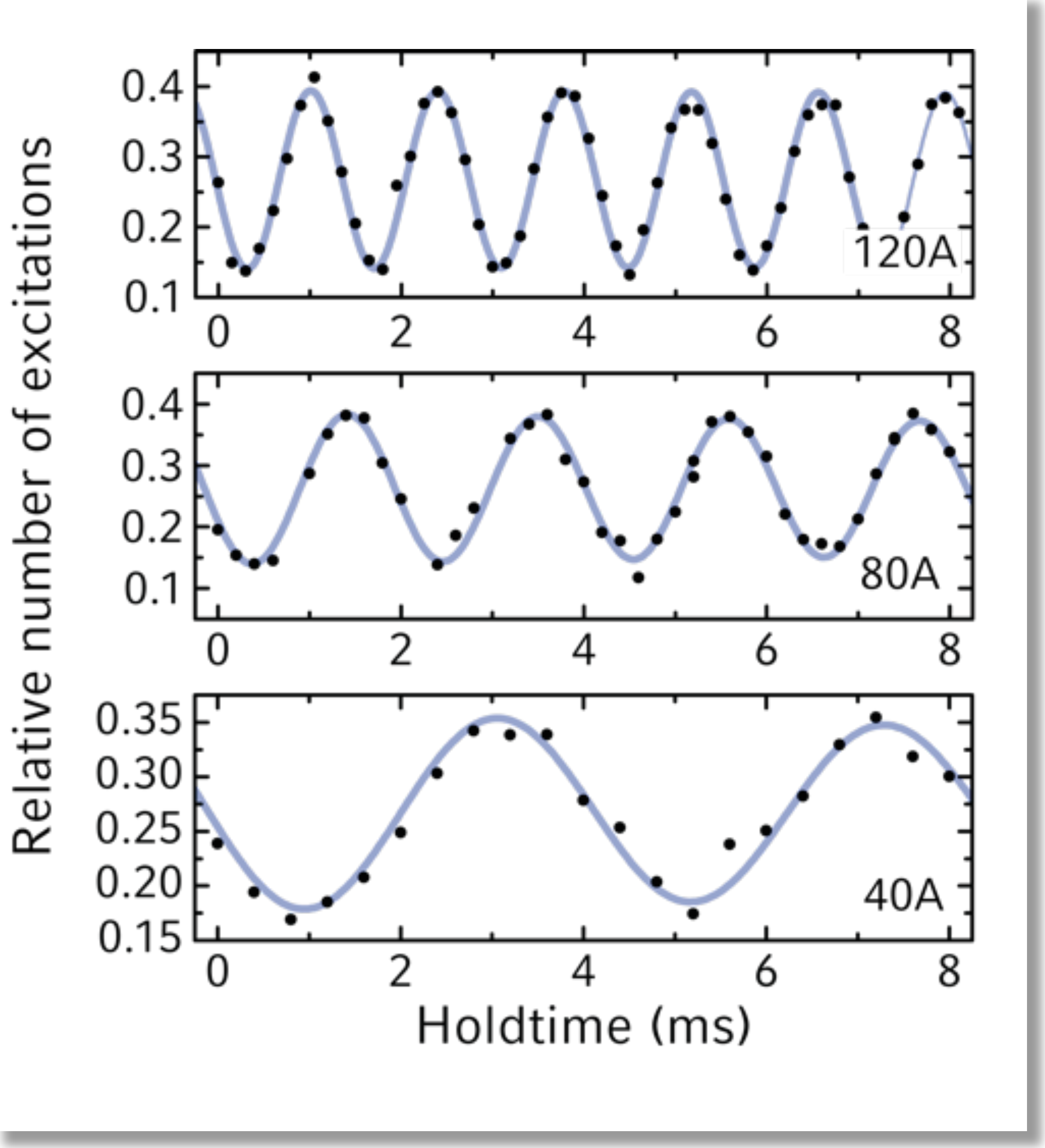


	band excitations		STO amplitude	
	bosons	fermions	bosons	fermions
$\vert t\rangle$	0%	50%	50%	50%
$\vert s\rangle$	50%	0%	50%	50%
$\vert\downarrow,\uparrow\rangle$	25%	25%	0%	0%
$\vert\uparrow,\downarrow\rangle$	25%	25%	0%	0%
$\vert\uparrow,\uparrow\rangle$	0%	50%	0%	0%
$\vert\downarrow,\downarrow\rangle$	0%	50%	0%	0%

→ Capable of probing spin-order in strongly correlated phases at low temperatures

Band-mapping reveals **singlet-contribution in higher Brillouin-Zone**

Singlet-Triplet oscillations



- **Load** system and create **spin pairs**
- Split pairs into **triplets**
- Induce **STO** via gradient
- **Merging** and **band-mapping** for detection

→ **Traces of STO** versus holdtime with gradient

- Vary gradient coil current

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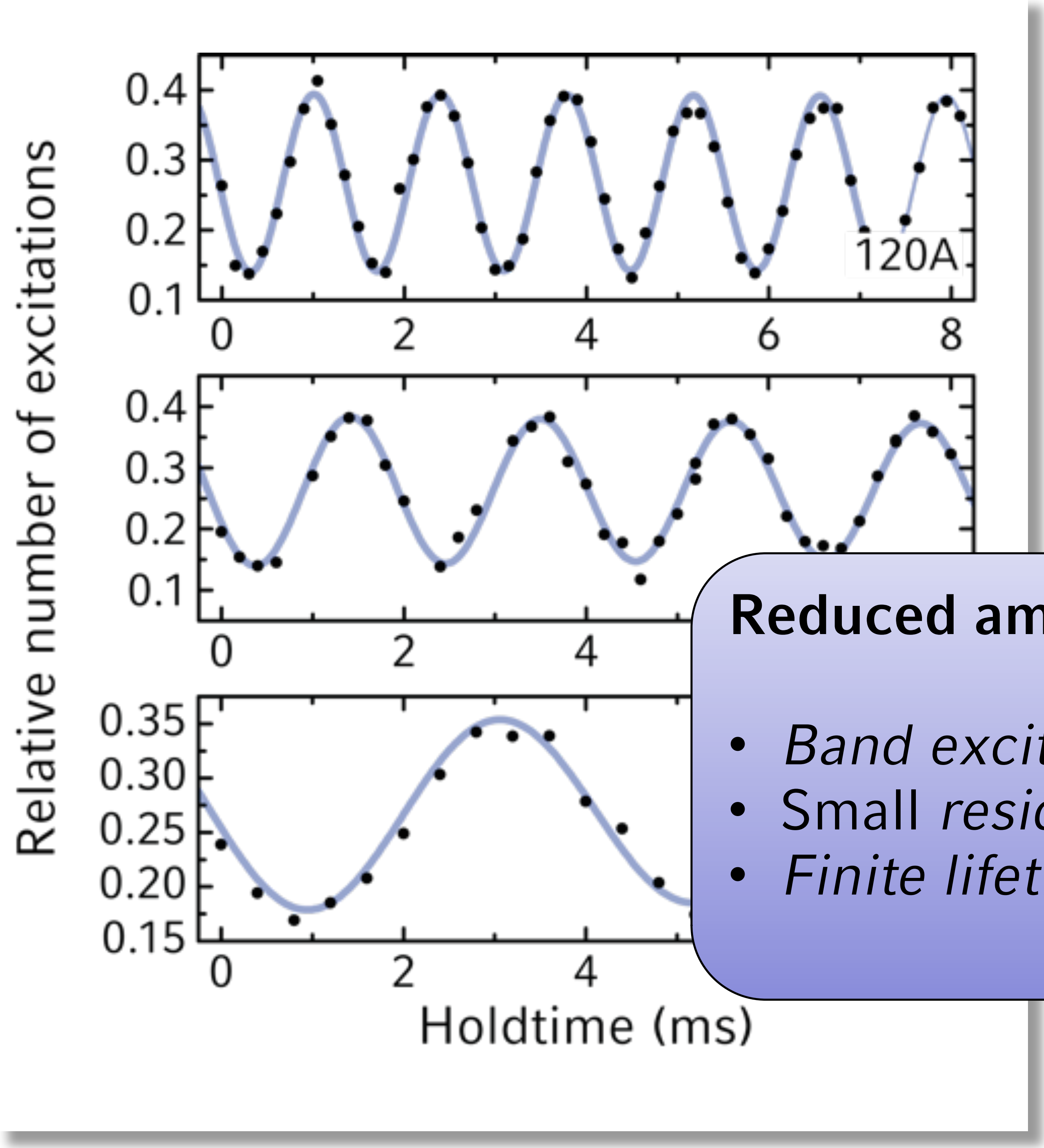
Singlet-Triplet oscillations



- **Load** system and create **spin pairs**
- Split pairs into **triplets**
- Induce **STO** via gradient
- **Merging** and **band-mapping** for detection

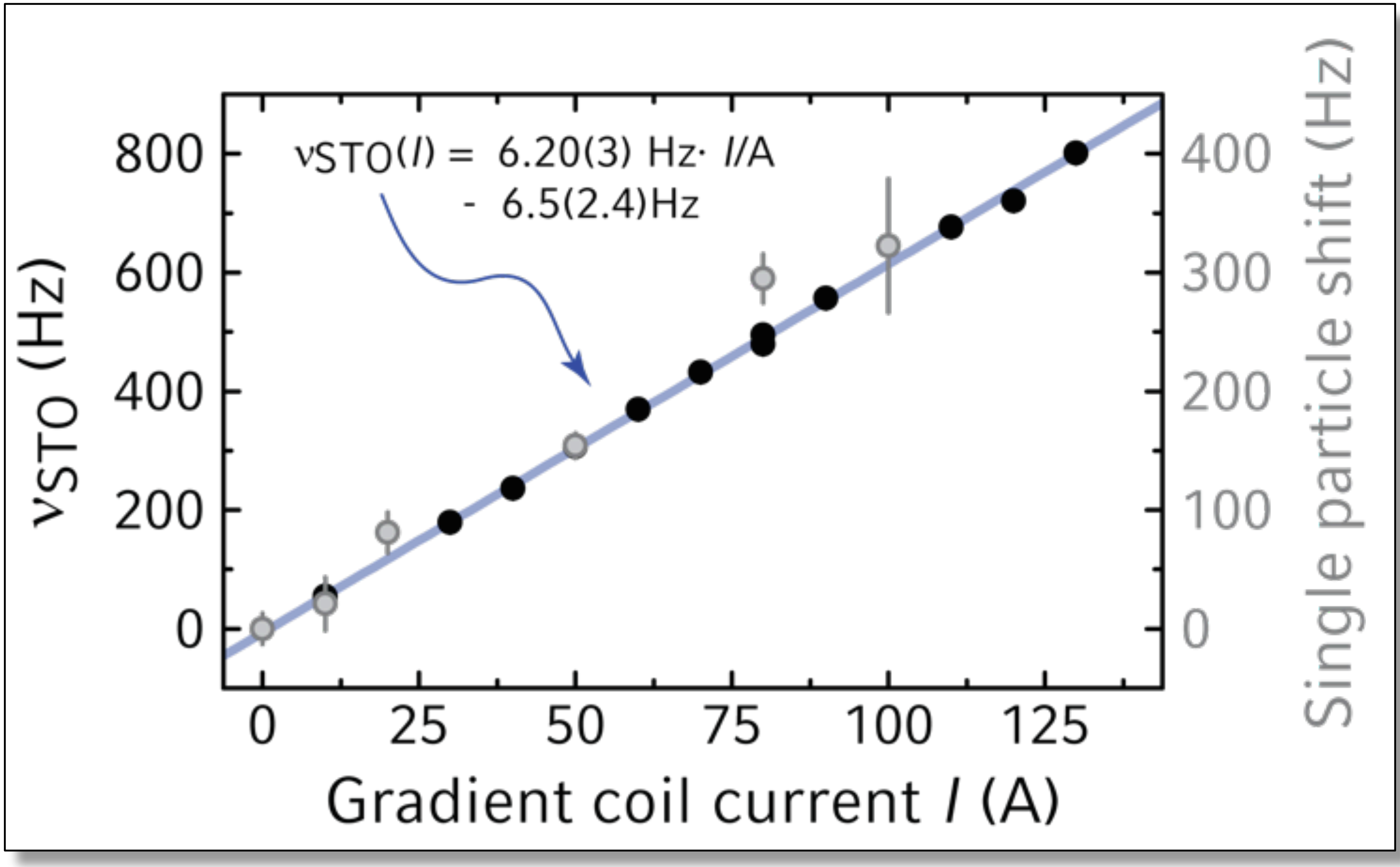
nt

Reduced amplitude due to:

- *Band excitations* from loading
- Small *residual gradient* during splitting
- *Finite lifetime* of triplets & singlets

Singlet-Triplet oscillations

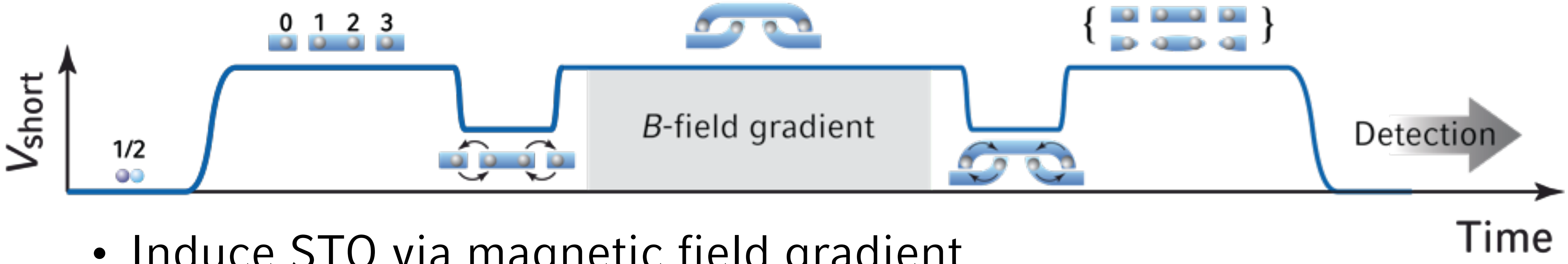
- **Linear increase** in Frequency with gradient strength
- Frequency = **2x single particle shift** (independently meas.)
  → confirms **2-particle nature** of oscillations



One (and only one) step beyond

- Coupling neighboring triplets with the superlattice
- Lower barrier to induce superexchange oscillations (**SWAP**)

*Stretched triplets: P. Barmettler et al., PRA **78** (2008)*



- Induce STO via magnetic field gradient
- SWAP back again and read-out as before

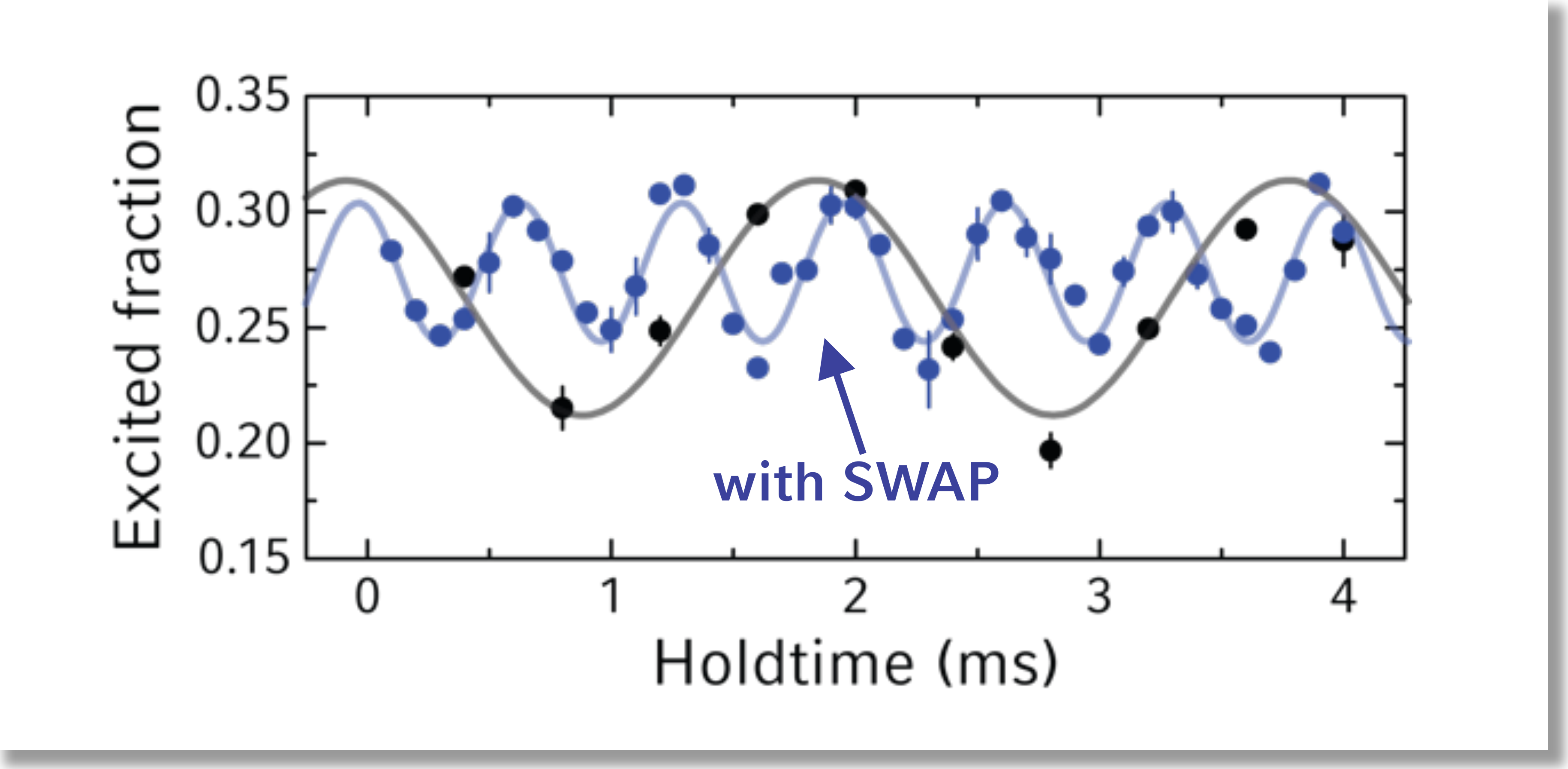
$\sqrt{\text{SWAP}}$ entangles neighboring triplets

→ **Many-(!)-particle entanglement** by a single step

*On-site SWAP: M. Anderlini et al., Nature **448** (2007)*
*Superexchange: J.J. Garcia-Ripoll et al., NJP **5** (2003)*
*L.-M. Duan et al., PRL **91** (2003), S.T. et al., Science **319** (2008)*

Stretched Triplet Pairs

- Oscillations with SWAP **3x faster**
  → confirms succesful ***stretching* of triplets**



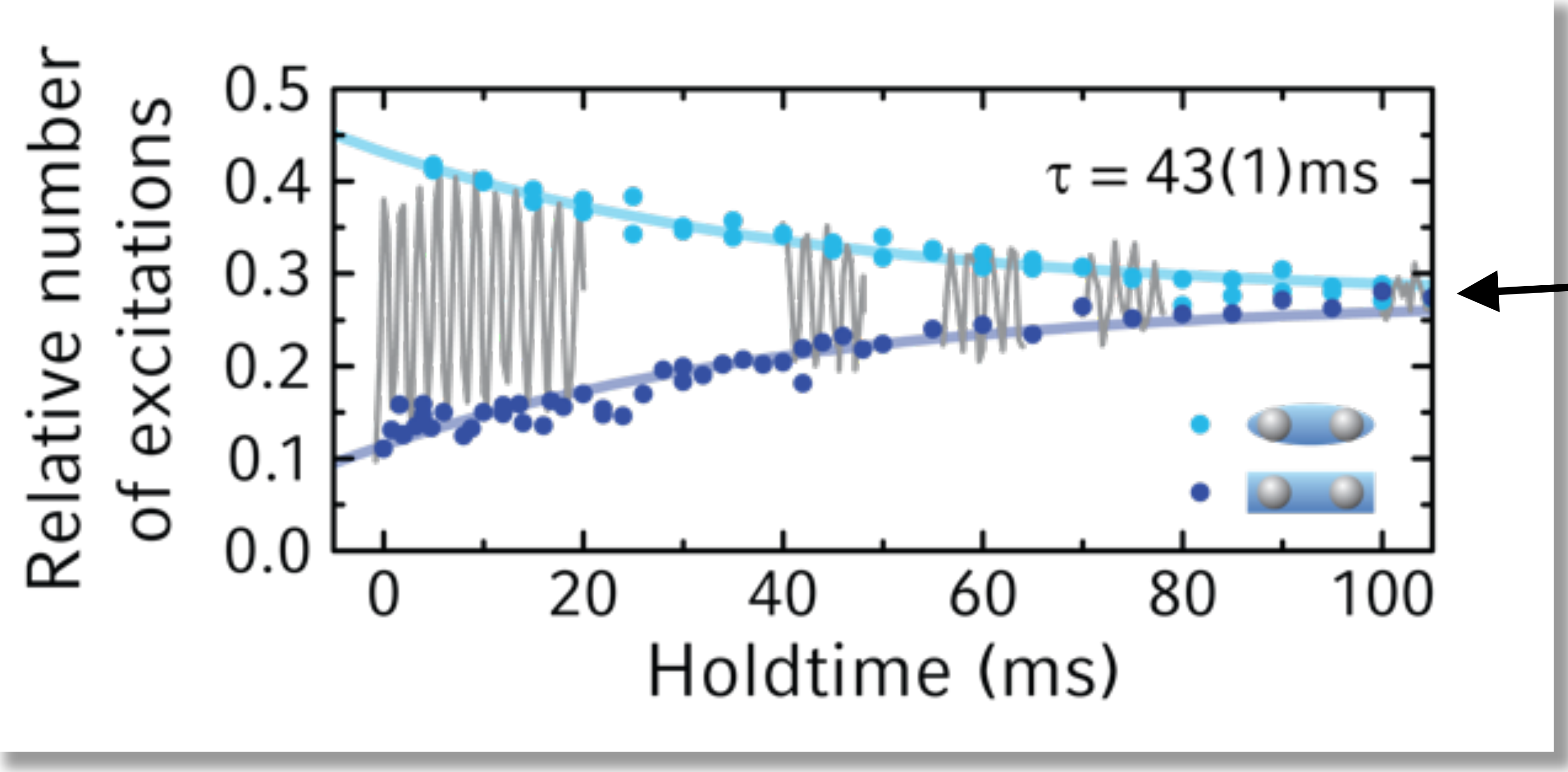
- Reduced amplitude due to defects in the system
- Note: no signal observable for $\sqrt{\text{SWAP}}$
  → alternative probes?

Damping of the oscillations

- Loading Triplets / Singlets and holding before merging

  → **Equal lifetime** of triplets and singlets

  → **Lifetime = Damping time** of oscillations



~25% excited,
no STO:
$|\uparrow,\downarrow\rangle = |t_0\rangle + |s\rangle$

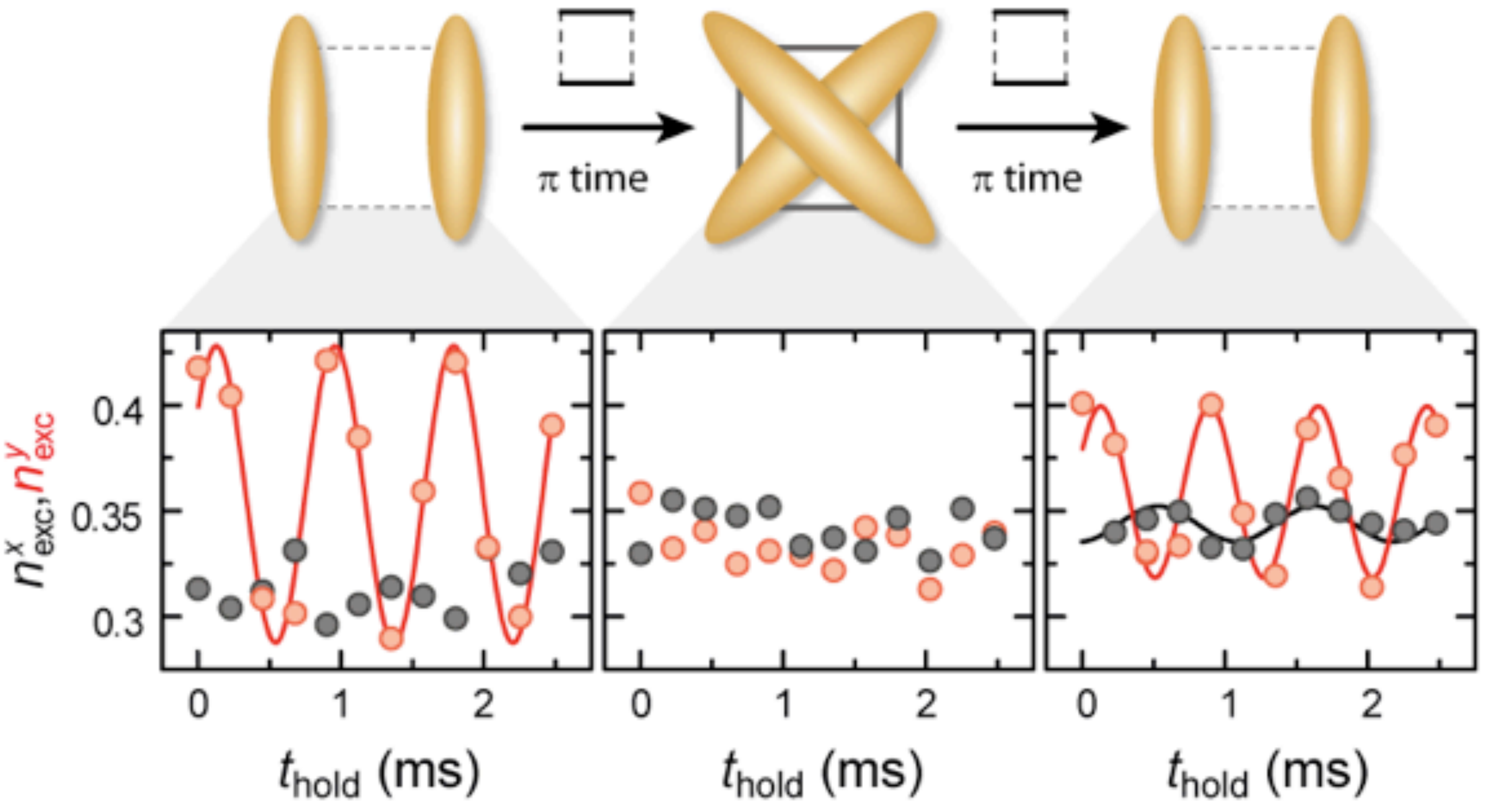
- Any „measurement“ of spin destroys triplets / singlets (e.g. scattering of lattice photons)

Preparation of the *d*-wave RVB state

Procedure:

- prepare valence bond states along y
- switch on J_x, with J_y=0
- wait for the π time of the evolution

Preparation of the *d*-wave RVB state



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Kasztelan, Ulrich Schollwöck

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