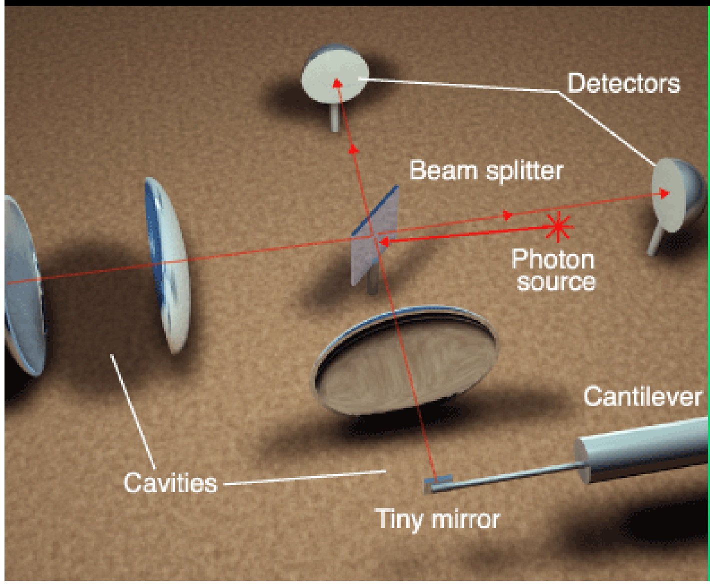
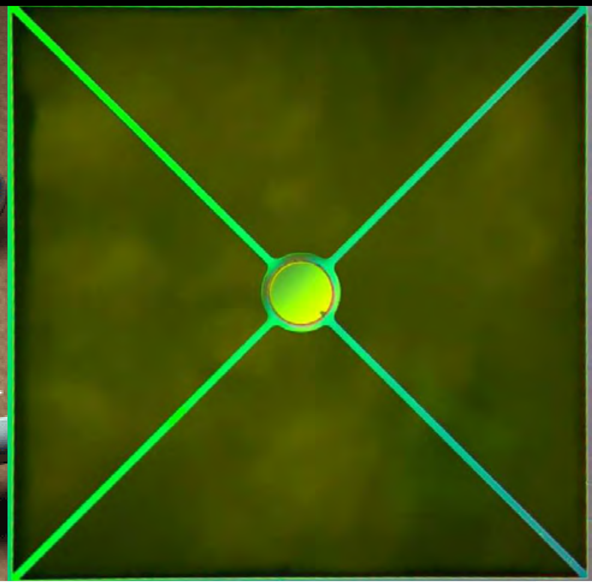

Dirk Bouwmeester
Glasgow, August 1, 2011

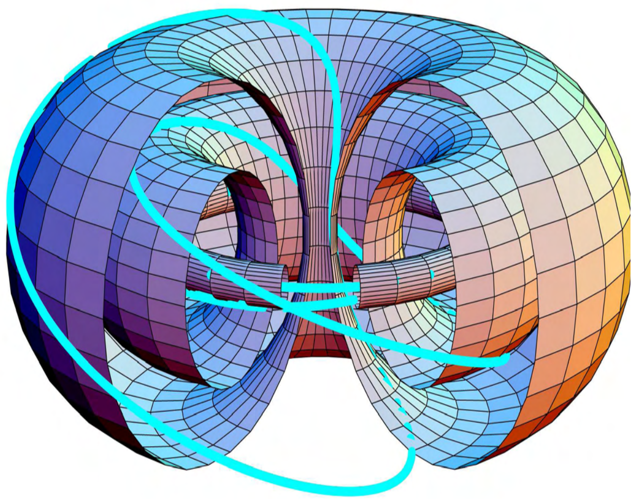
Macroscopic Quantum Superpositions

Lecture 2

Penrose's Arguments, Quantum Decoherence,
Optical Cooling, Knots of Light





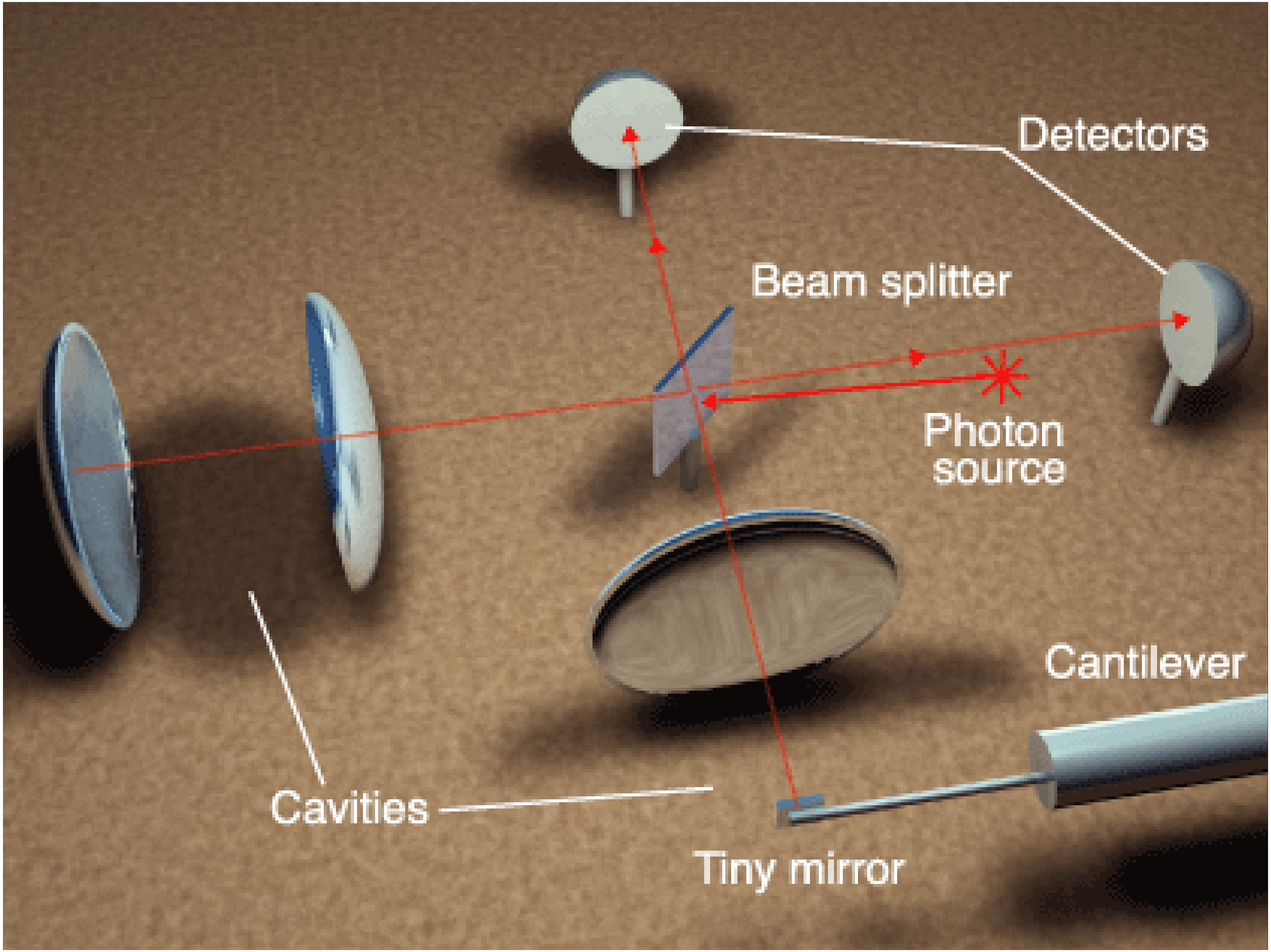


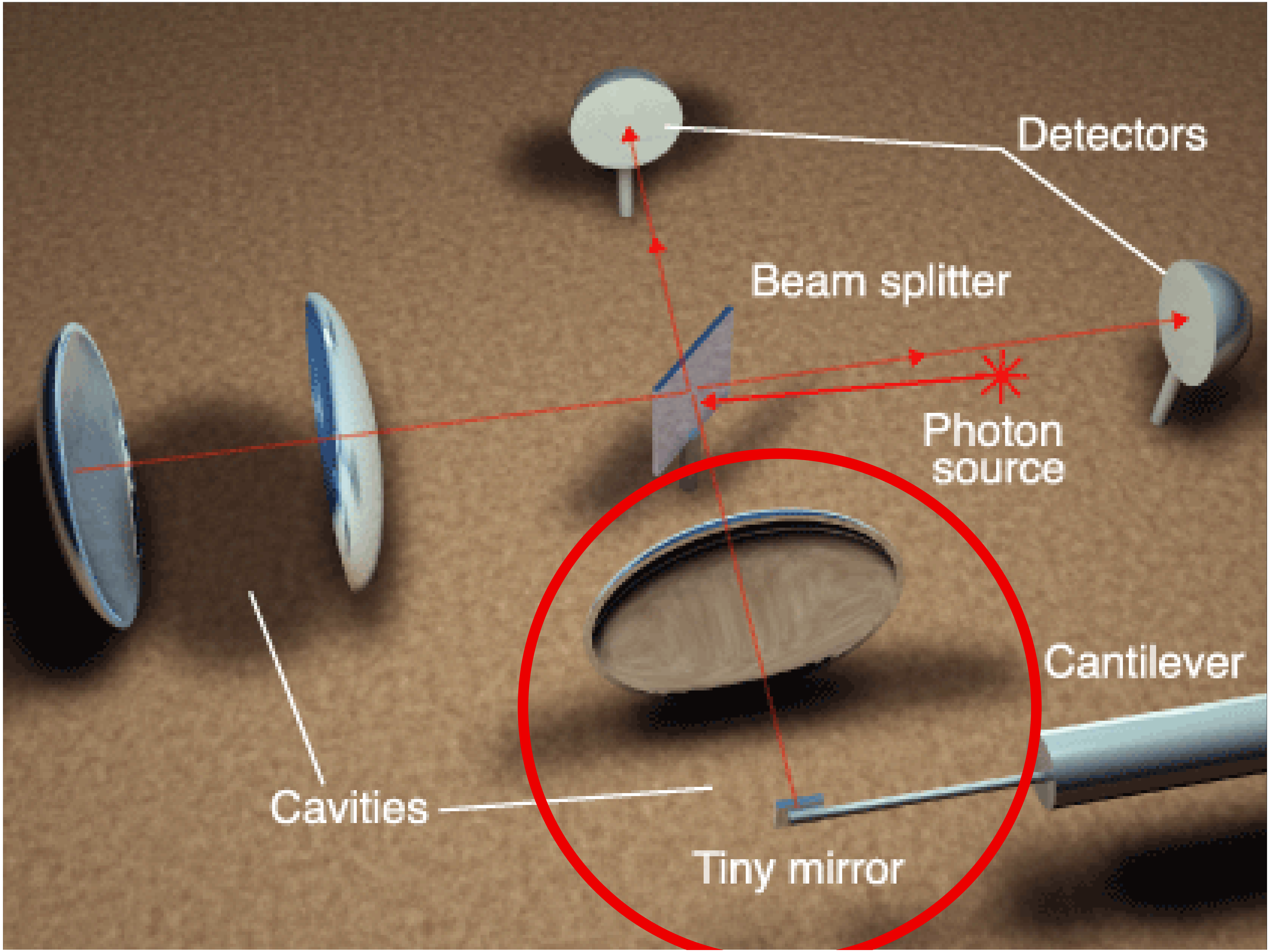

Oxford 2001

VOLUME 91, NUMBER 13 PHYSICAL REVIEW LETTERS week ending 26 SEPTEMBER 2003

Towards Quantum Superpositions of a Mirror

William Marshall,[1,2] Christoph Simon,[1] Roger Penrose,[3,4] and Dik Bouwmeester[1,2]


Detectors
Beam splitter
Photon source
Cantilever
Cavities
Tiny mirror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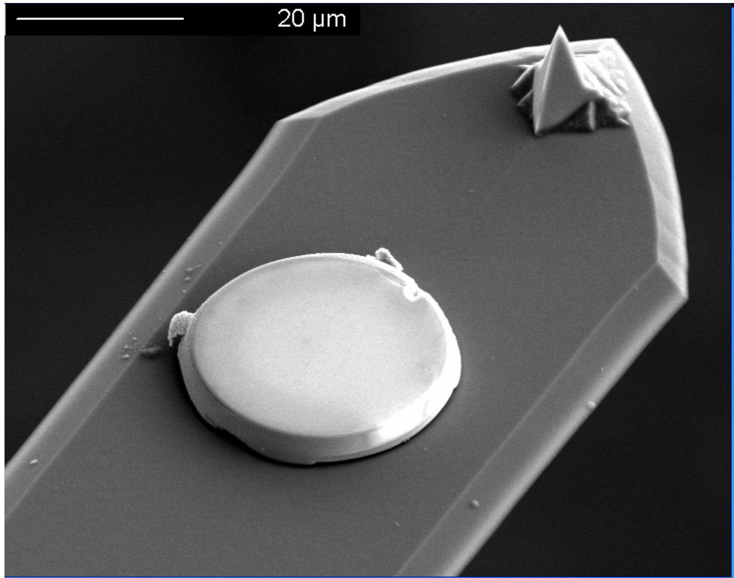


UCSB

Generation 1

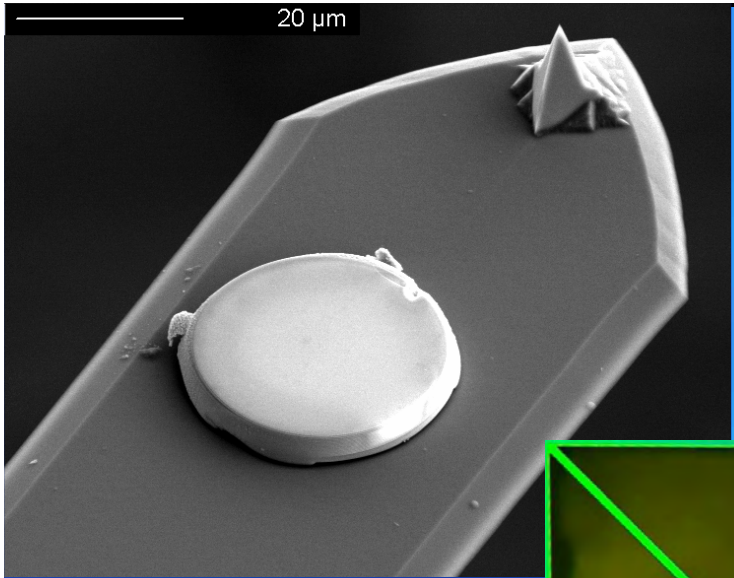
Finesse: 2100

Mechanical Q:130.000

2006

10-100kHz

Opto-mechanical system



UCSB

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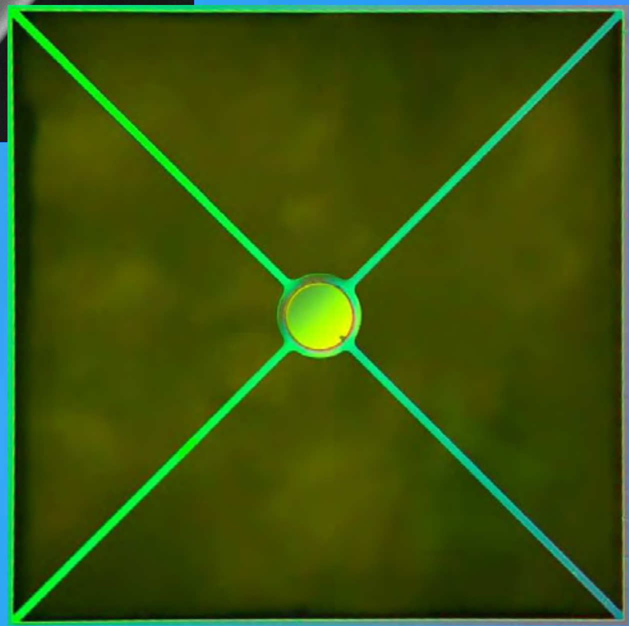


Generation 2
Finesse: 3000
Q: 400.000

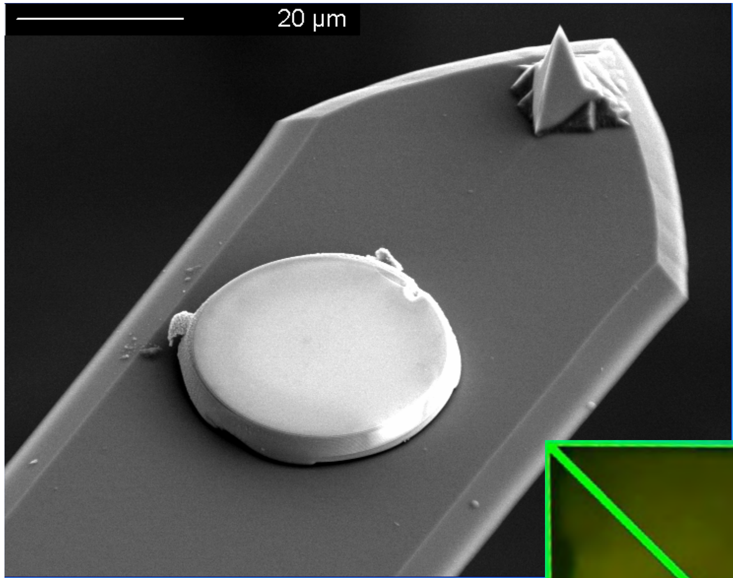
2009

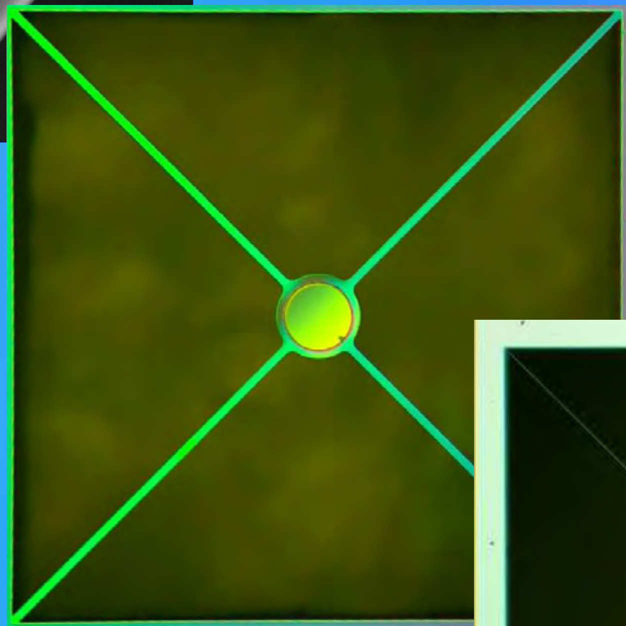
10-100kHz

Opto-mechanical system

UCSB

Generation 1
Finesse: 2100
Mechanical Q:130.000





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Q: 400.000

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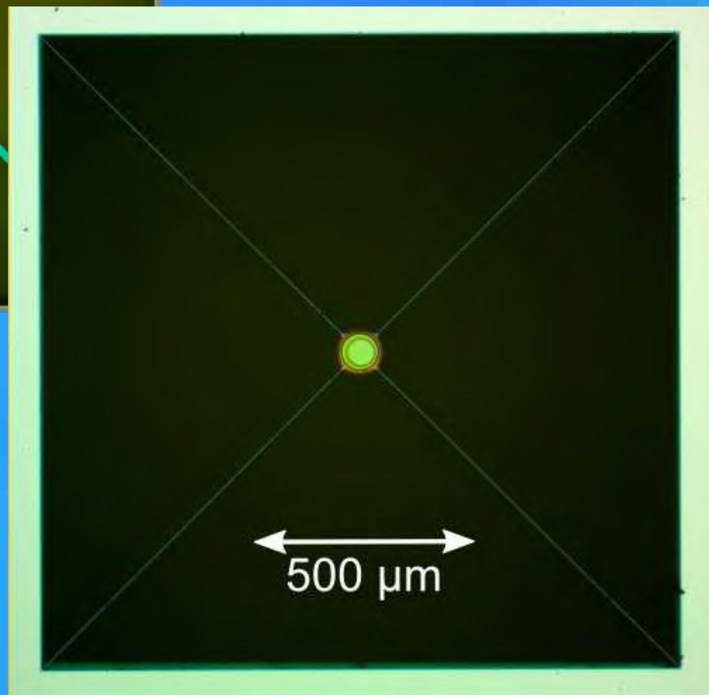
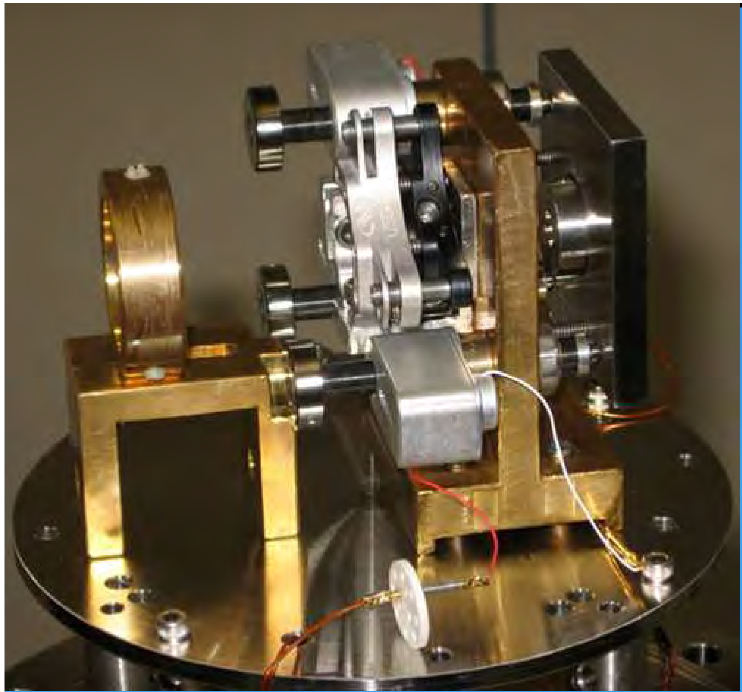


2010

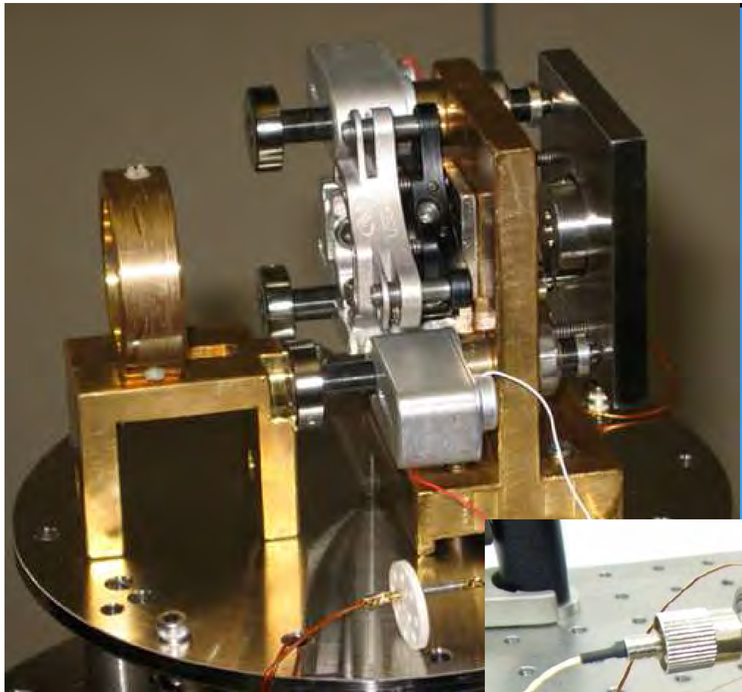
Generation 3
Finesse: 40.000
Q: 900.000



UCSB

Generation 1
Room temperature
vacuum
2006

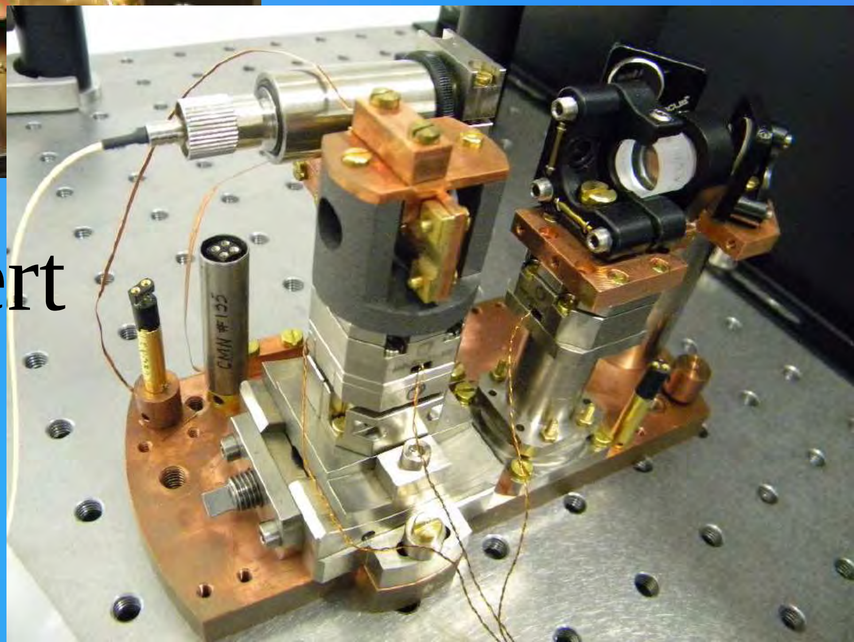
Optical insert



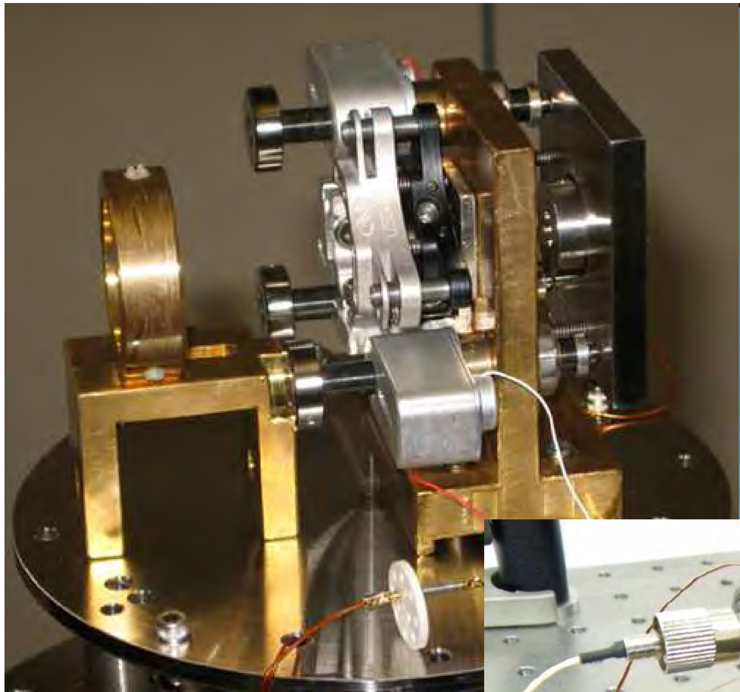
Generation 1
Room temperature
vacuum
2006

UCSB
&
Leiden

Optical insert

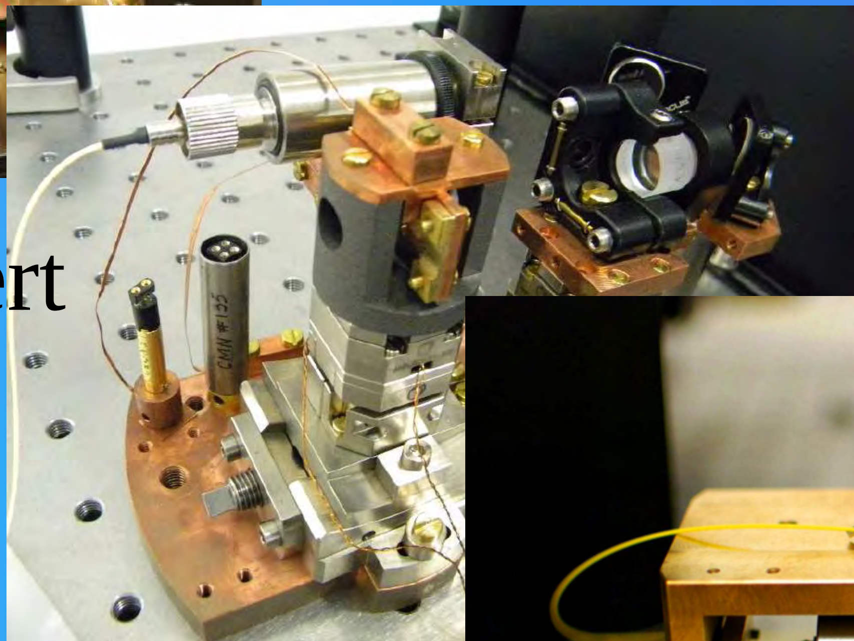


Generation 2
Low temperature
vacuum
2009



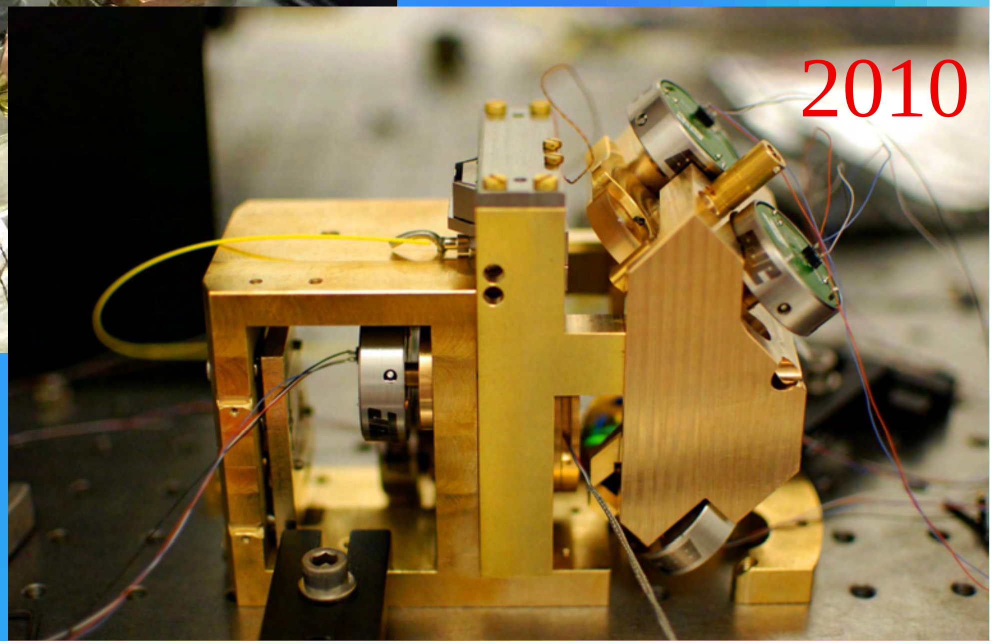
Generation 1
Room temperature
vacuum
2006

UCSB
&
Leiden



Generation 2
Low temperature
vacuum

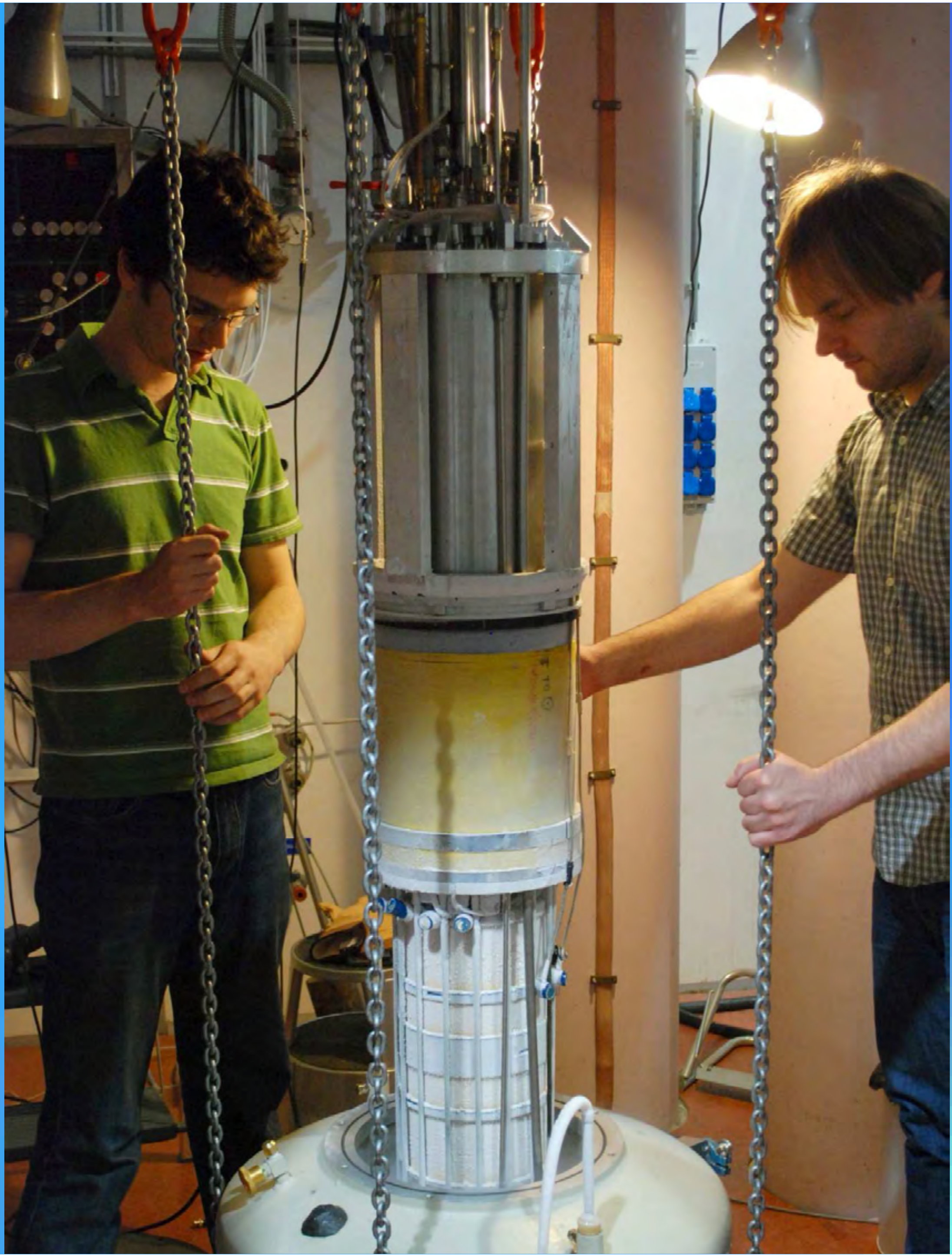
Optical insert



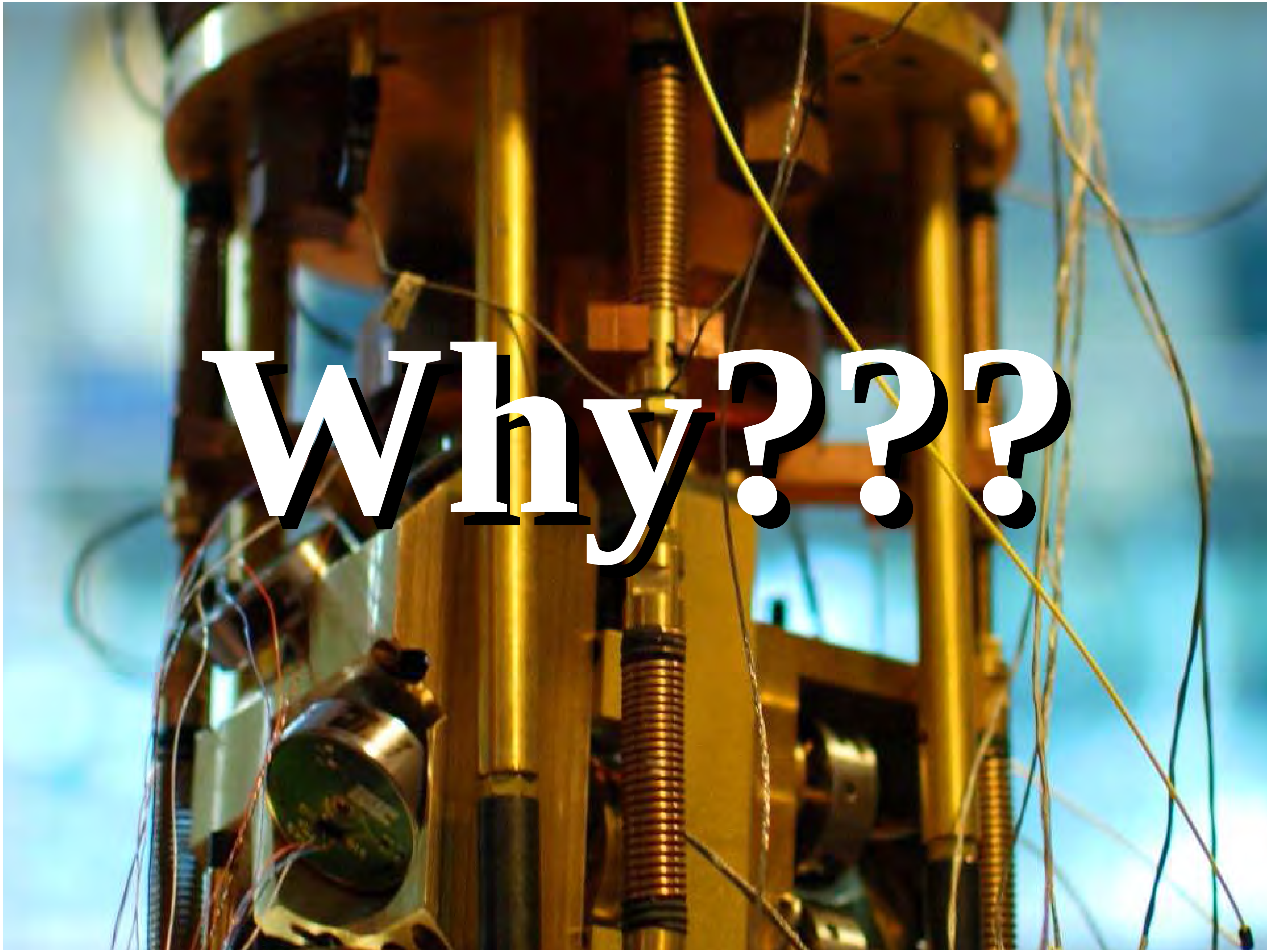
2010

Generation 3
Low temperature vacuum
& stable

D. Kleckner
B. Pepper
UCSB



P. Sonin
E. Jeffrey
Leiden

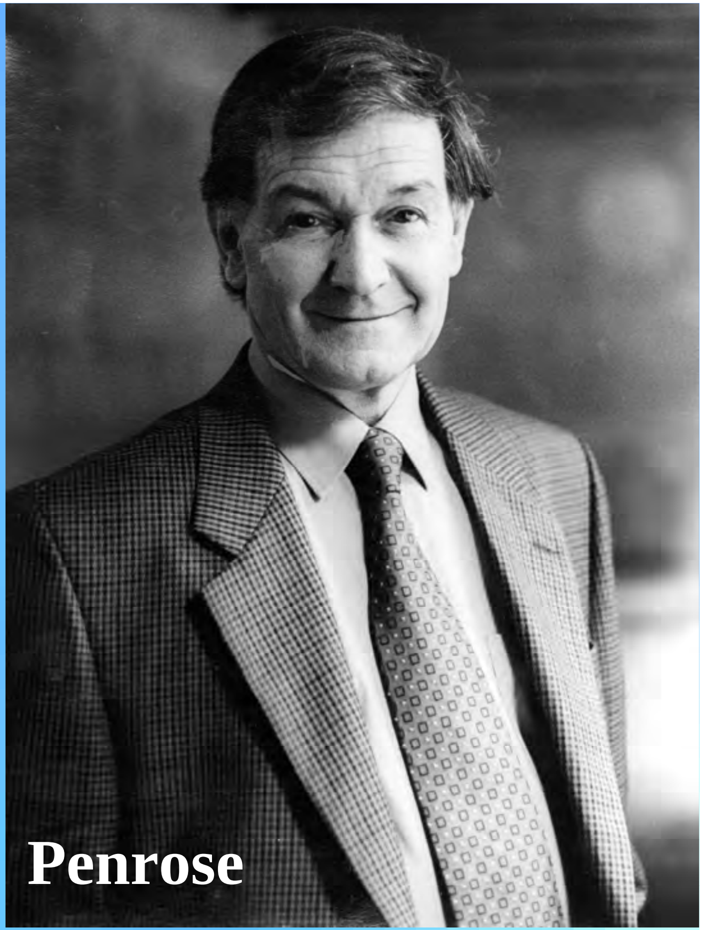

Why???

Three levels of reasoning

1: Twistor theory

2: Conformal symmetry
& space time singularities

3: Gravitational induced
quantum state reduction



Level 1: (Twistor) Spinor Theory

$V^a \in M$ (Minkowski space) with components (V^0, V^1, V^2, V^3)

$$\Rightarrow V^{AA'} = \begin{bmatrix} V^{00'} & V^{01'} \\ V^{10'} & V^{11'} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} V^0 + V^3 & V^1 + iV^2 \\ V^1 - iV^2 & V^0 - V^3 \end{bmatrix}, \quad \text{Det} = 4-\text{interval}$$

$$V^{AA'} \to \tilde{V}^{AA'} = t^A{}_B V^{BB'} \overline{t}_{B'}{}^{A'}, \text{ where } \begin{bmatrix} t^0{}_{0'} & t^0{}_{1'} \\ t^1{}_{0'} & t^1{}_{1'} \end{bmatrix} \in SL(2,C), \text{and } \overline{t}_{B'}{}^{A'} = \overline{t^A{}_B}$$

$SL(2,C) \;\to\; L_+^\uparrow$ (Lorentz group) is 2-1 isomorphism.

example 1: Lorentz boost in z-direction: $t = \begin{bmatrix} e^{\frac{\varphi}{2}} & 0 \\ 0 & e^{-\frac{\varphi}{2}} \end{bmatrix}$

example 2: Rotation through φ in the x-y plane: $t = \begin{bmatrix} e^{\frac{i\varphi}{2}} & 0 \\ 0 & e^{-\frac{i\varphi}{2}} \end{bmatrix}$

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Note: rotation by 2π gives -I in $SL(2,C)$, rotation by 4π gives I in $SL(2,C)$,

(Twistor) Spinor Theory

if $\det V^{AA'} = 0$, $\begin{bmatrix} V^{00'} & V^{01'} \\ V^{10'} & V^{11'} \end{bmatrix} = \begin{bmatrix} \alpha^0 \bar{\alpha}^{0'} & \alpha^0 \bar{\alpha}^{1'} \\ \alpha^1 \bar{\alpha}^{0'} & \alpha^1 \bar{\alpha}^{1'} \end{bmatrix} = \alpha^A \bar{\alpha}^{A'}$

Note:

spinor α^A determined up to phase factor by this construction

$\Rightarrow$ intrinsic quantum mechanical features

Note:

SL(2,C) acts on spinor α^A

$\Rightarrow$ intrinsic fermionic properties

Twistor (Spinor) Theory

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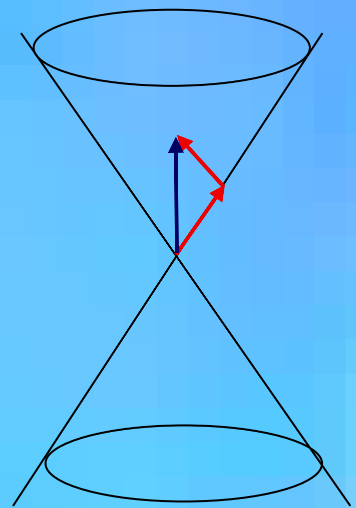
Use two spinors to define a point within the light cone.



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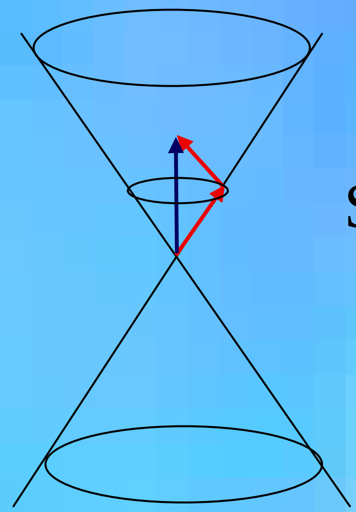

SU(2) gauge symmetry

(Twistor) Spinor Theory

if $\det V^{AA'} = 0$, $\begin{bmatrix} V^{00'} & V^{01'} \\ V^{10'} & V^{11'} \end{bmatrix} = \begin{bmatrix} \alpha^0\bar{\alpha}^{0'} & \alpha^0\bar{\alpha}^{1'} \\ \alpha^1\bar{\alpha}^{0'} & \alpha^1\bar{\alpha}^{1'} \end{bmatrix} = \alpha^A\bar{\alpha}^{A'}$

Note:

spinor α^A determined up to phase factor by this construction

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Note:

SL(2,C) acts on spinor α^A

SU(2) gauge symmetry

6 dim. Lorentz group

10 dim. Poincare group

15 dim. Conformal group

Twistor Theory

A twistor $Z^{\alpha} = \left(\omega^{\mathrm{A}}, \pi_{A'}\right) \in$ T(4-complex dimensional twistor space, SU(2,2) acts on twistors and is a 4 to 1 representation of the conformal group) .

There is no direct relation to space-time points (not even to points on the light cone) but there are planes in (complexified campactified) M defined by the twistor incidence relation:

$$\omega^{\mathrm{A}} = i x^{AA'} \pi_{A'}$$

with $x^{AA'} = \begin{pmatrix} t+z & x+iy \\ x-iy & t-z \end{pmatrix}$

If a plane intersects with real M the resulting line is a null geodesic and Z^{α} is called null .

To define a point inside the light cone of (real) M we need a pair of twistors.

This point is now automatically (nonlocally) related to all geodesics through this point and its identification has a gauge freedom closely related to the gauge symmetries of the standard model.

Twistor Theory

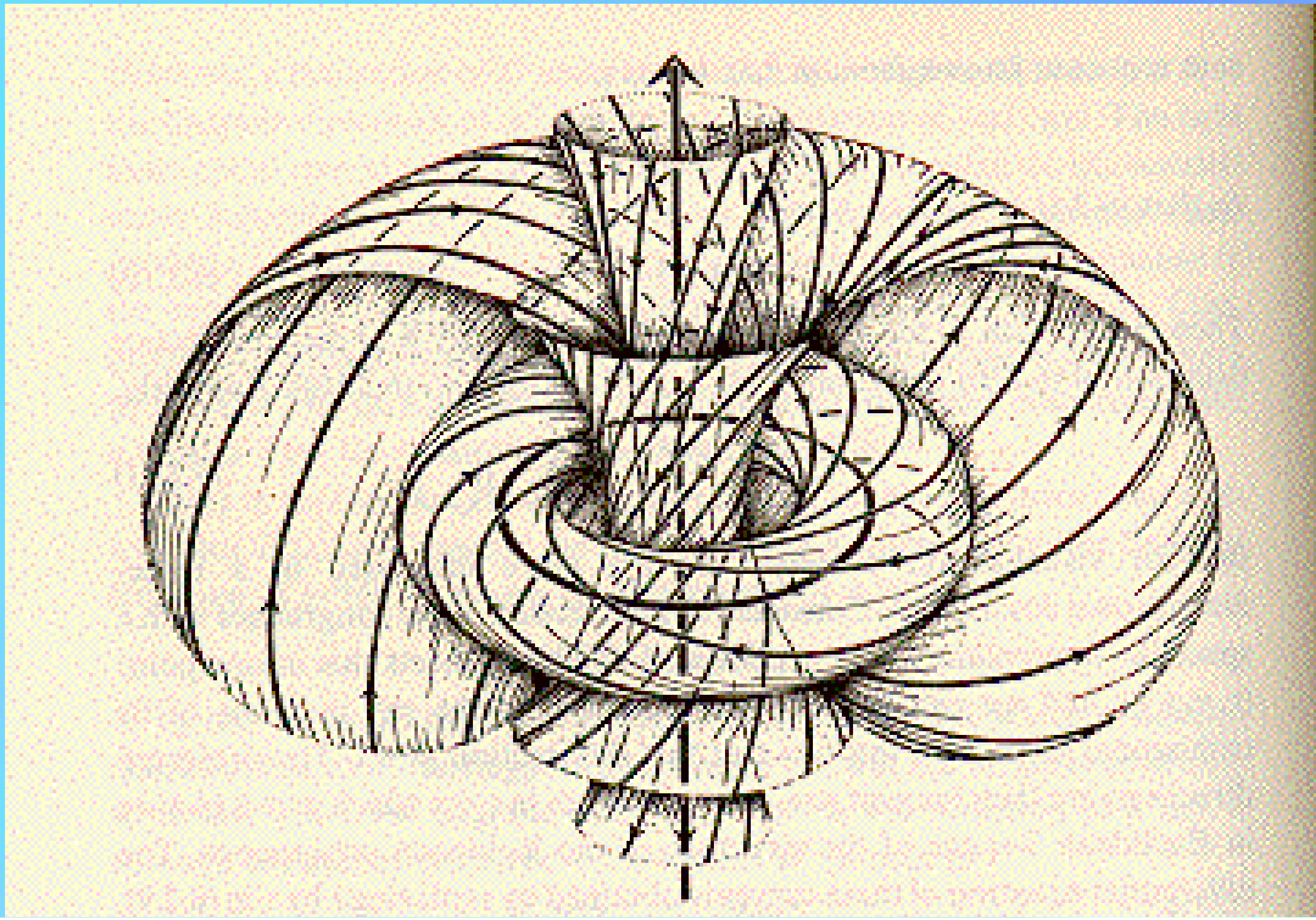
Natural objects in twistor theory are also non-null twistors:

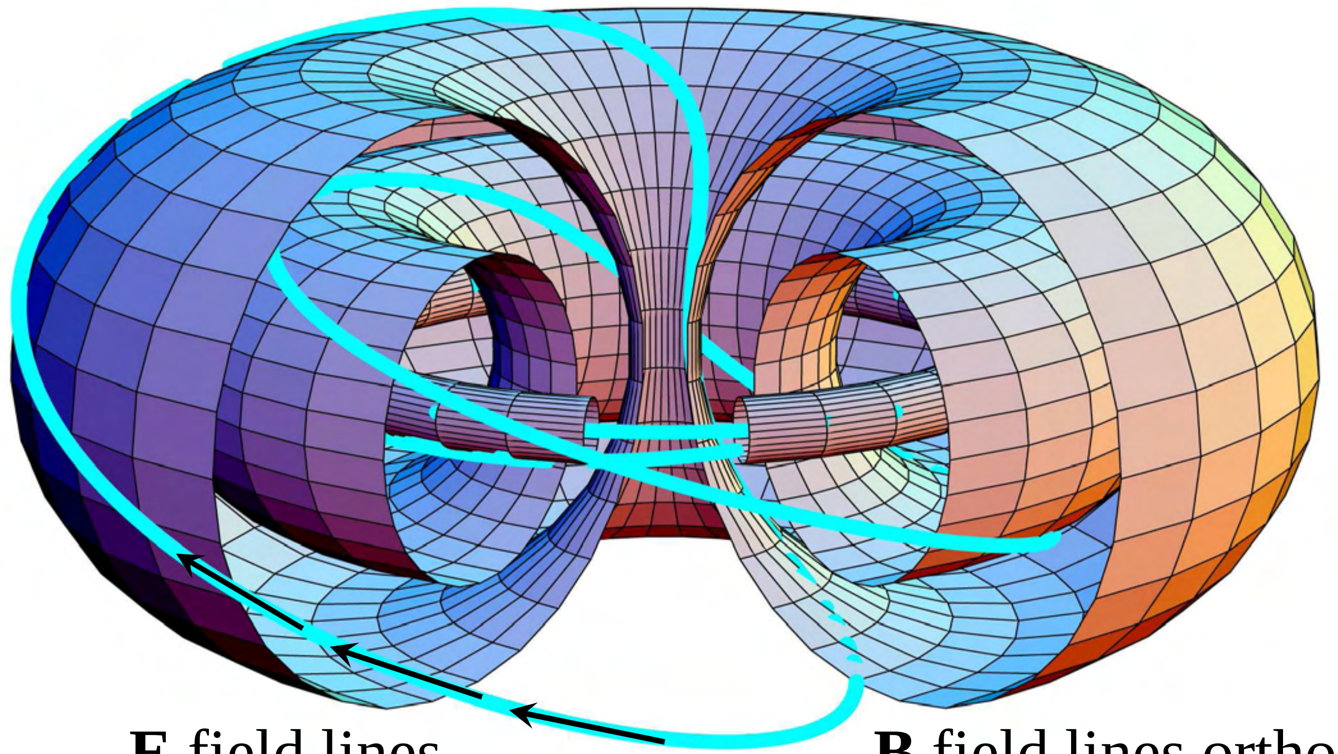
For Z^α non-null we can get a visualization of the twistor by drawing the null geodesics corresponding to null twistors that are "orthogonal" to it $\Rightarrow$ Robinson congruence of null geodesics.

$Z^\alpha = (\omega^A, \pi_{A'}) = (Z^0, Z^1, Z^2, Z^3)$, and $\bar{Z}_\alpha = (\bar{\pi}_{A'}, \bar{\omega}^A) = (\bar{Z}^2, \bar{Z}^3, \bar{Z}^0, \bar{Z}^1)$.

inner product $= Z^\alpha \bar{Z}_\alpha$

Robinson congruence: visualization of a (non-null) twistor:





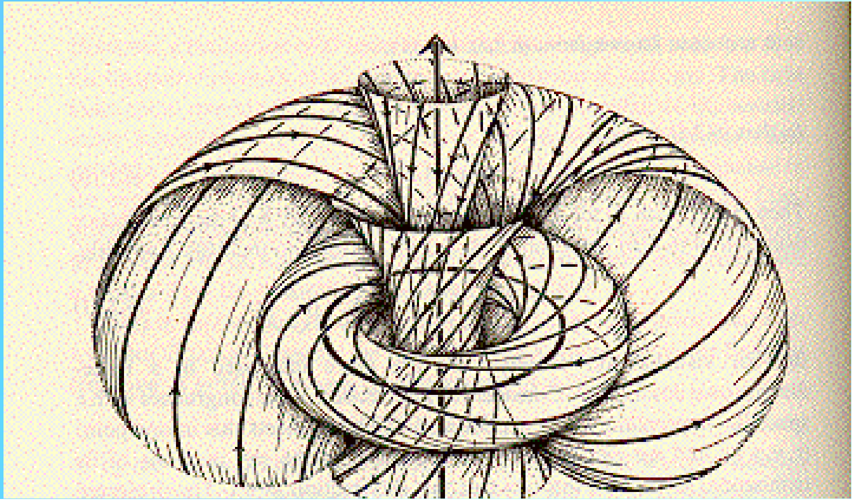
KNOTS OF LIGHT

A.F. Ranada and J.L. Trueba, Phys. Lett. 232 A, 25 (1997).
William Irvine and Dirk Bouwmeester, Nature Physics, September 2008
J.W. Dalhuisen and Dirk Bouwmeester, submitted

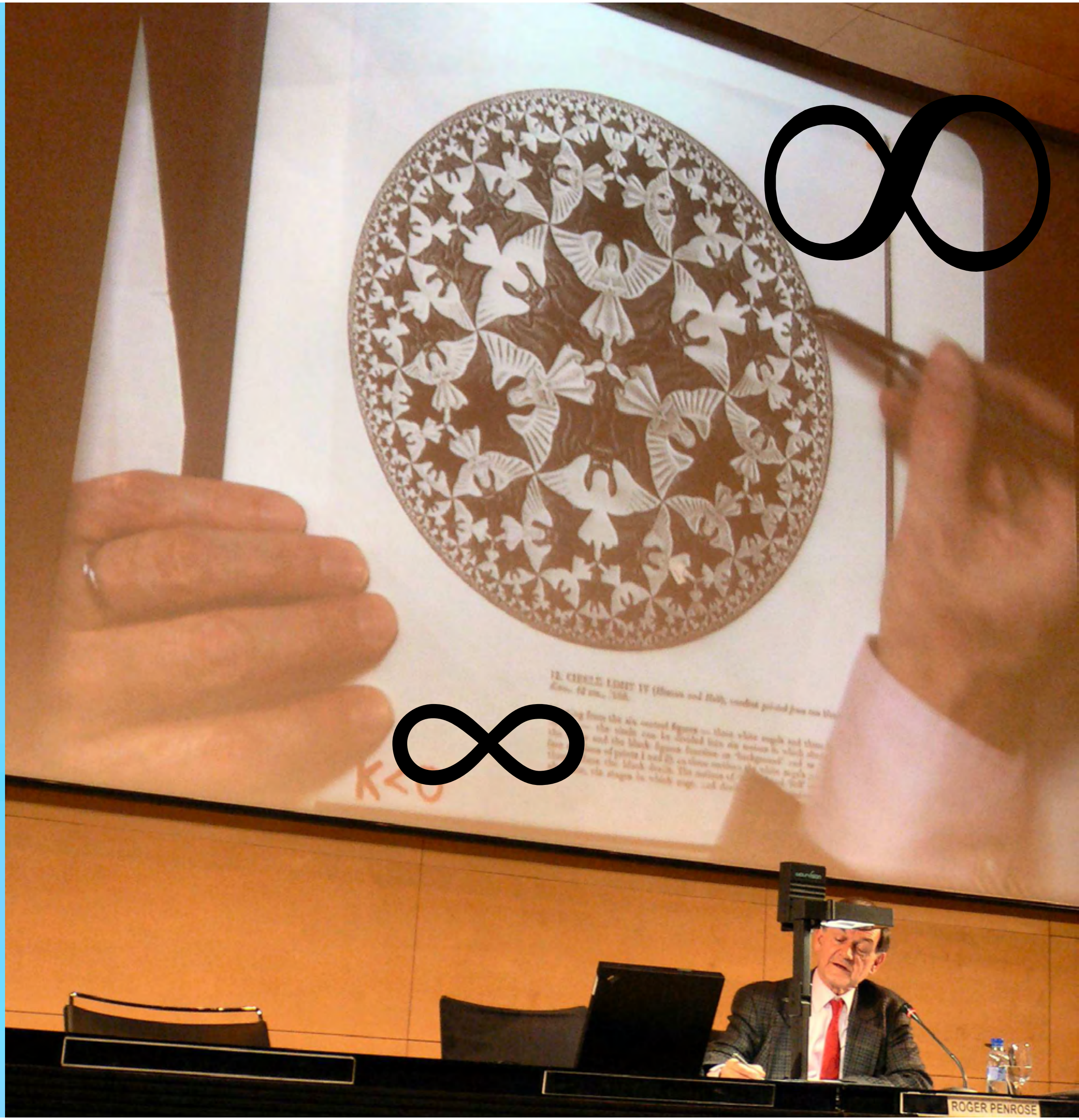
Twistor theory

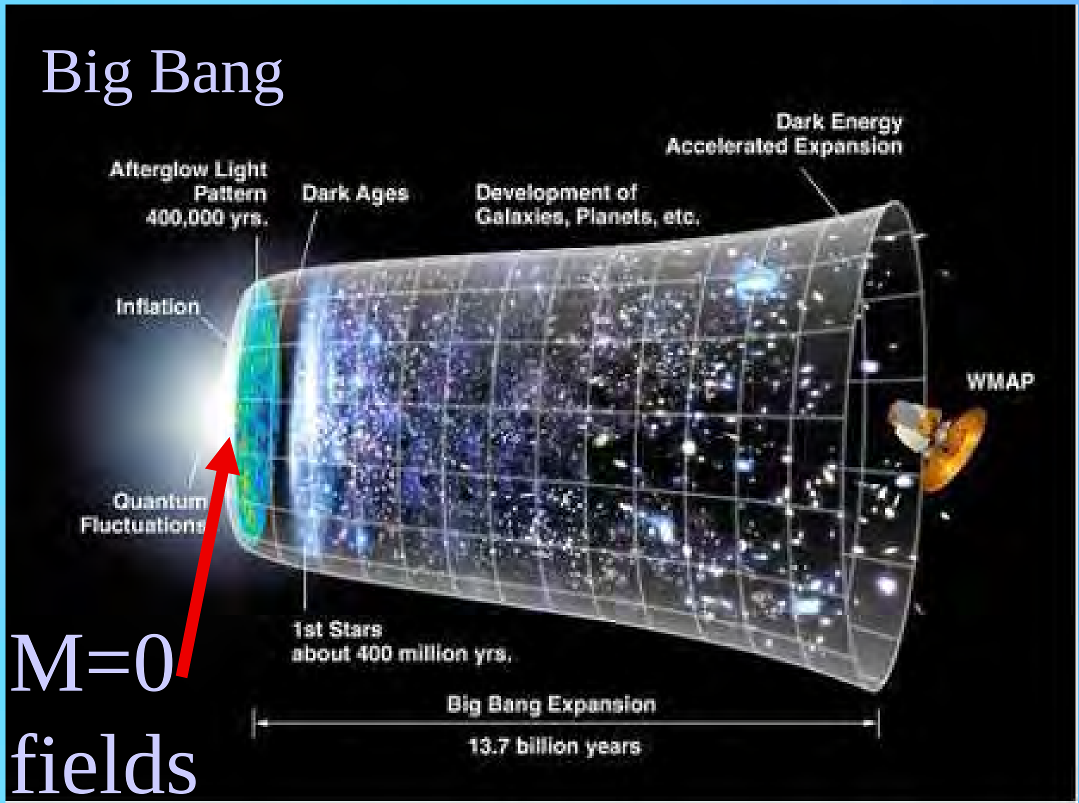
Space-time is a secondary concept and has:

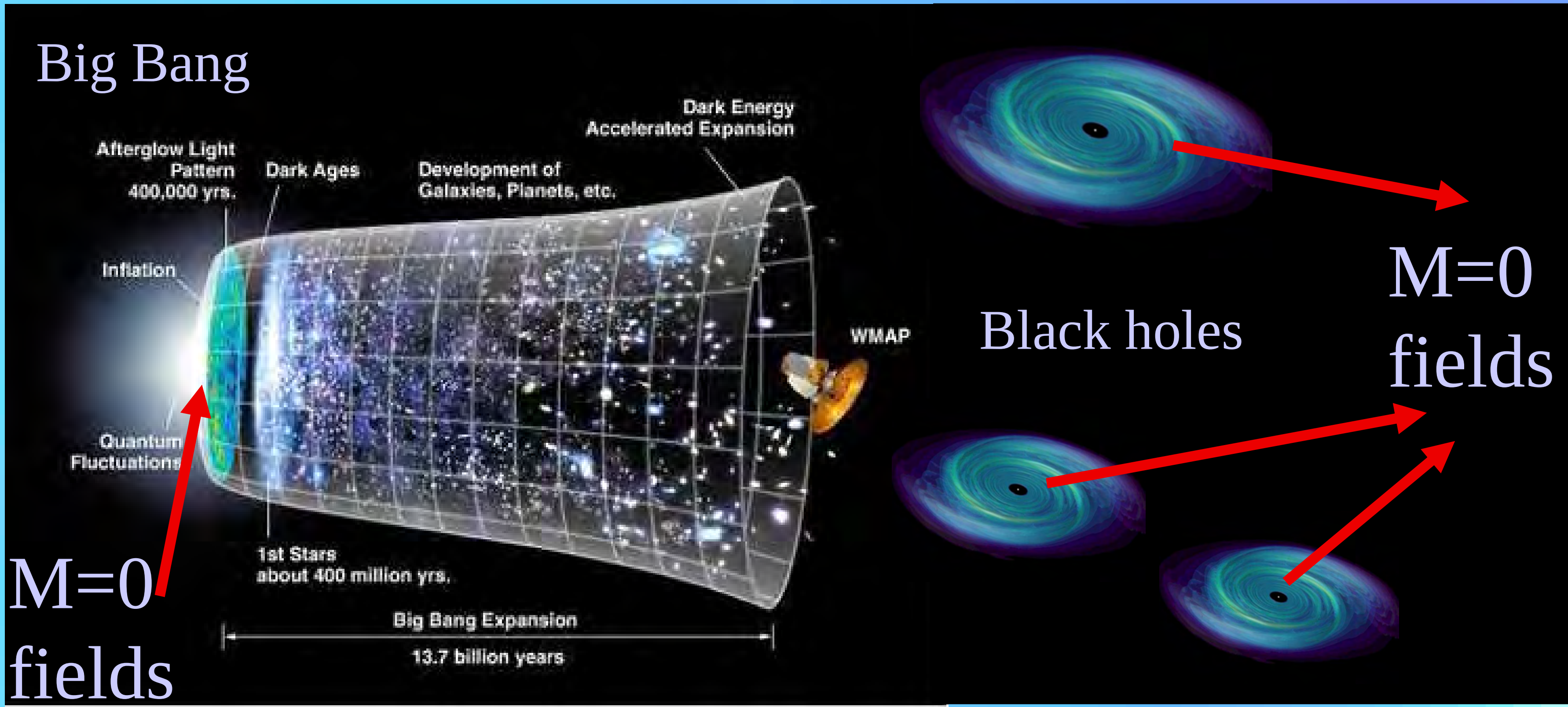
- **naturally 1 time- 3 space coordinates**
- **intrinsic fermionic properties**
- **intrinsic quantum mechanical properties**
- **conformal invariant**
- **intrinsic gauge freedom related to standard model**
- **intrinsic nonlocal**
- **has elementary solutions to wave equations that are related to Robinson congruences**



Level 2: conformal symmetry & space time singularities




Big Bang
Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
M=0
fields
Big Bang Expansion
13.7 billion years



Big Bang
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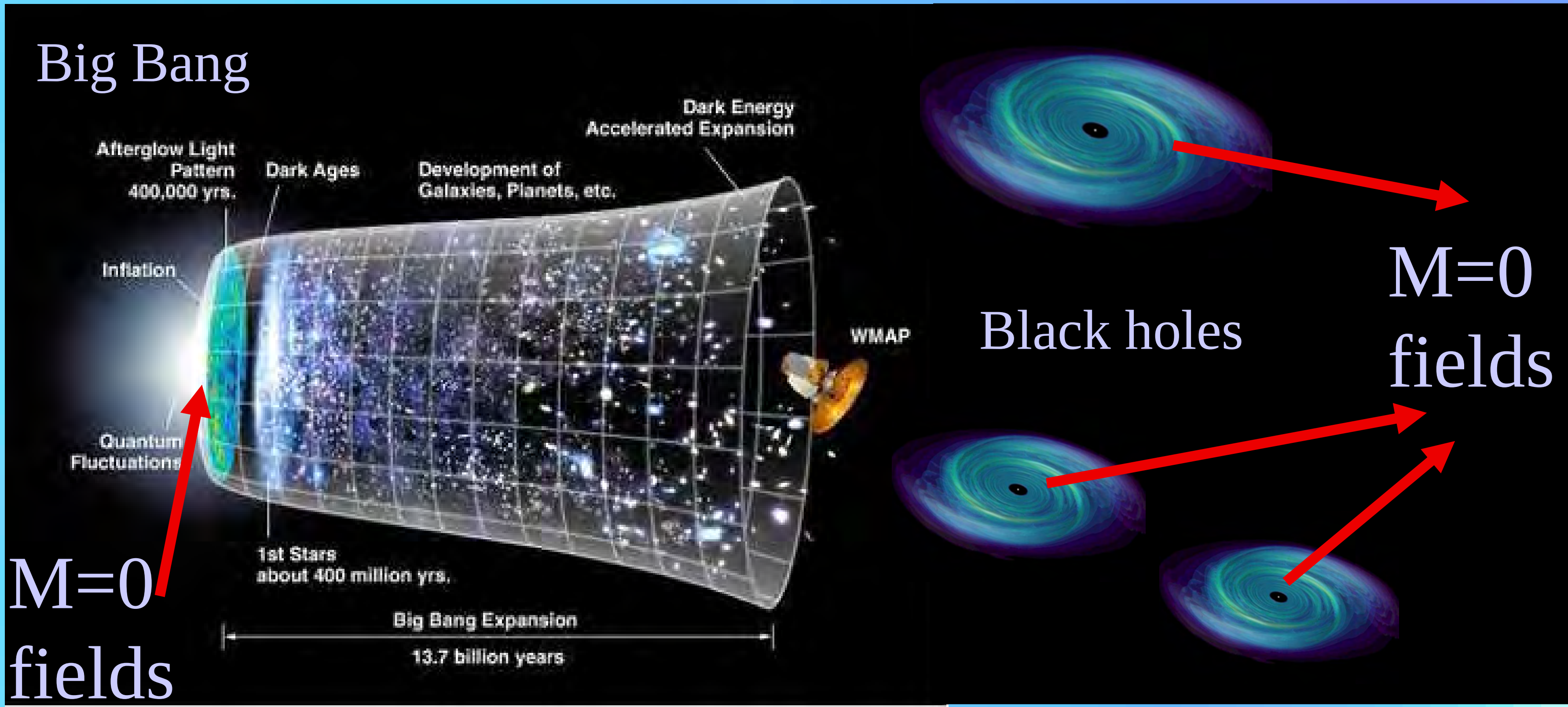
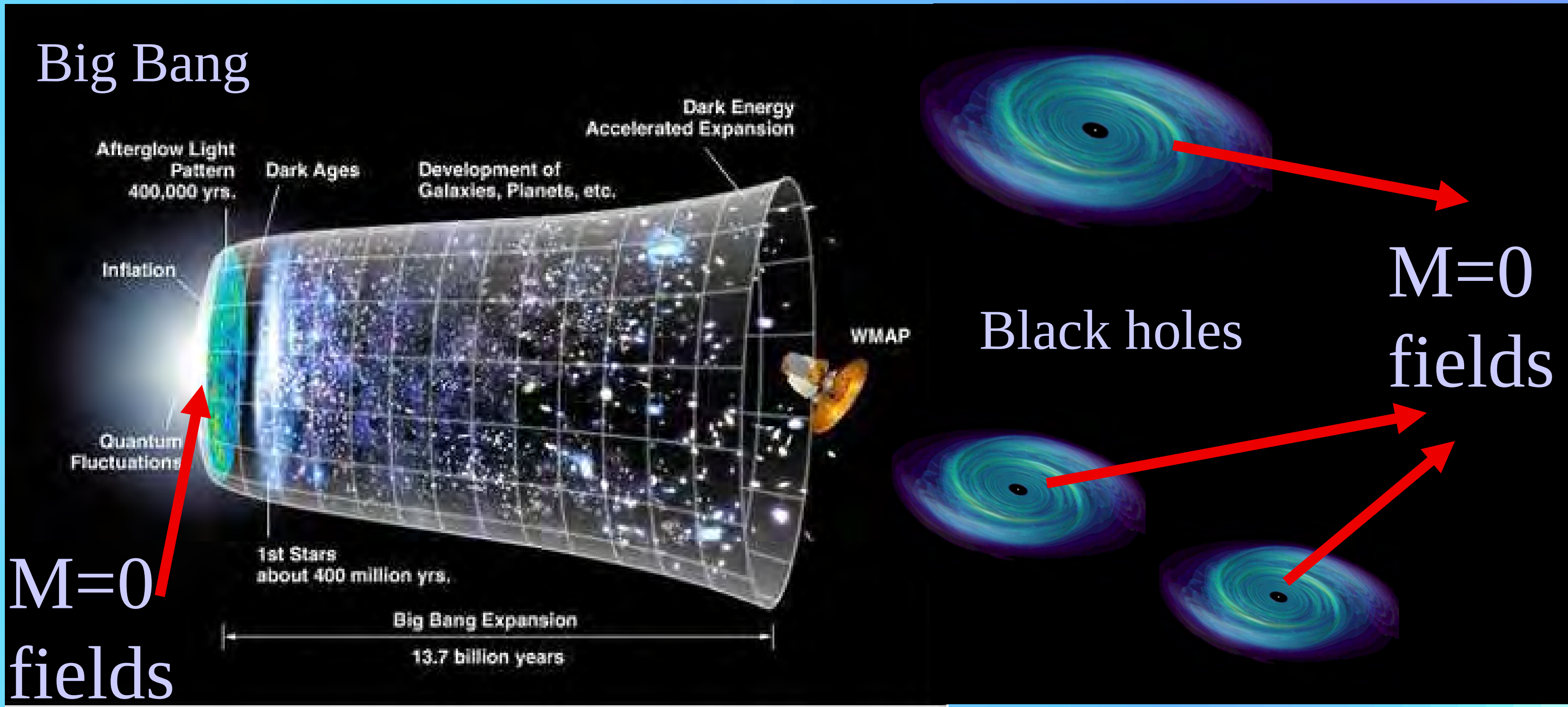


$(ct)^2 - x^2 - y^2 - z^2 = 0,\qquad g_{\mu\nu}x^\mu x^\nu = 0,$ zero-mass wave equation $g_{\mu\nu}\partial^\mu\partial^\nu\Psi = 0$

Lorentz symmetry: $\tilde{g}_{\mu\nu} = g_{\mu\nu}$



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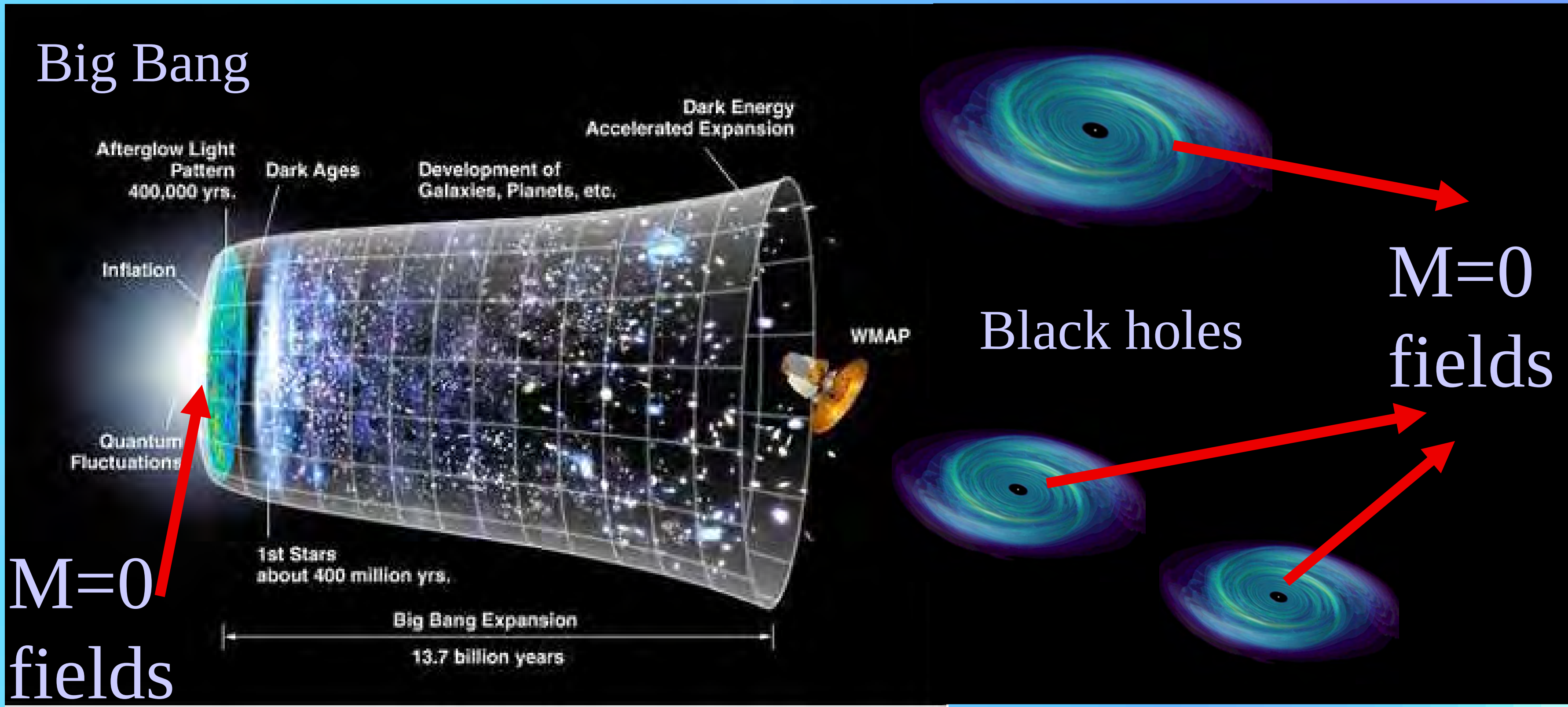
Lorentz symmetry: $\tilde{g}_{\mu\nu} = g_{\mu\nu}$

Conformal symmetry: $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ angle preserving transformations



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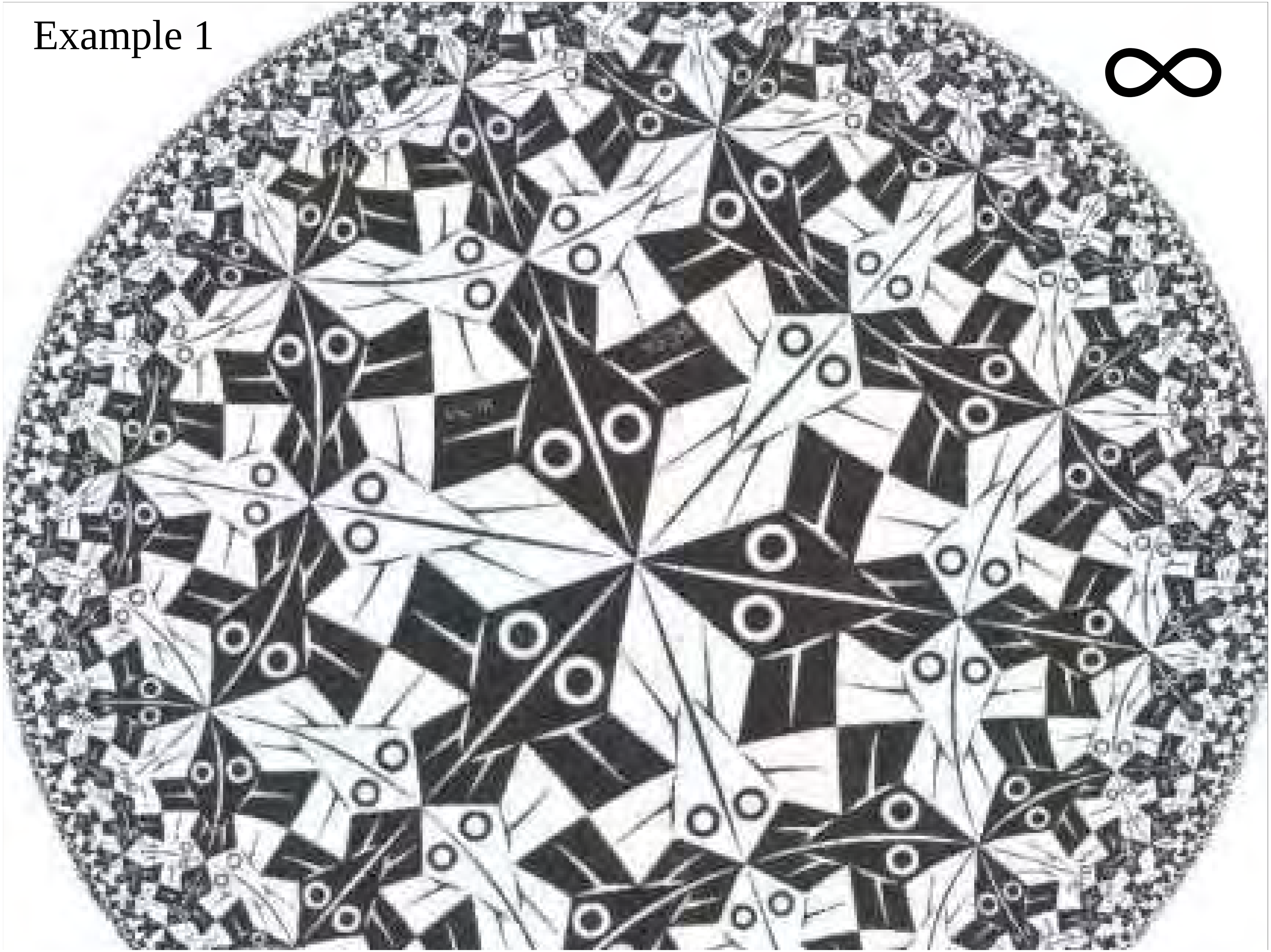
Lorentz symmetry: $\tilde{g}_{\mu\nu} = g_{\mu\nu}$

Conformal symmetry: $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ angle preserving transformations

(massive wave equation $g_{\mu\nu}\partial^\mu\partial^\nu\Psi = \mathrm{m}\Psi$, breaks conformal symmetry)

Example 1

∞



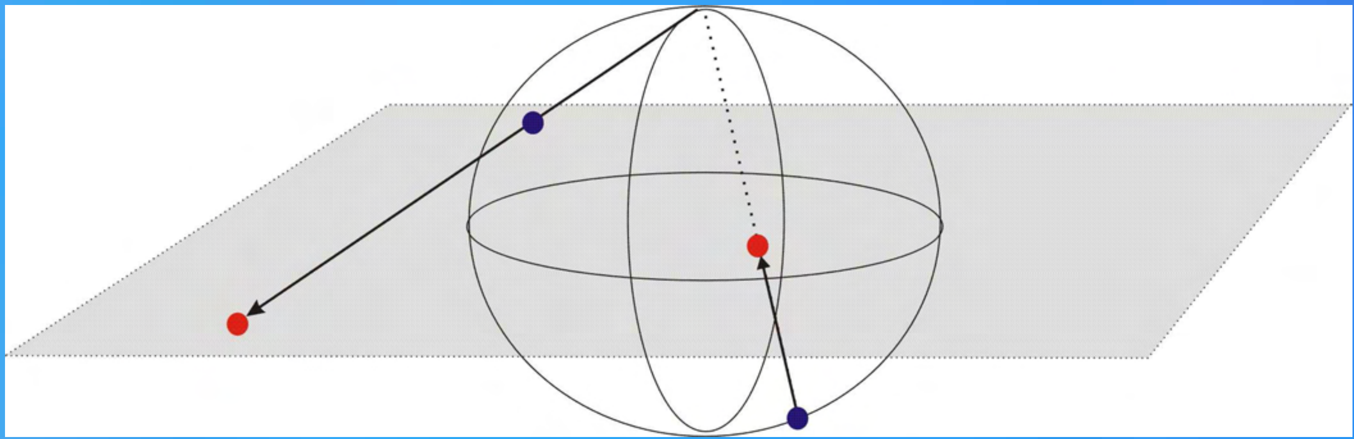
Example 2

Stereographic projection

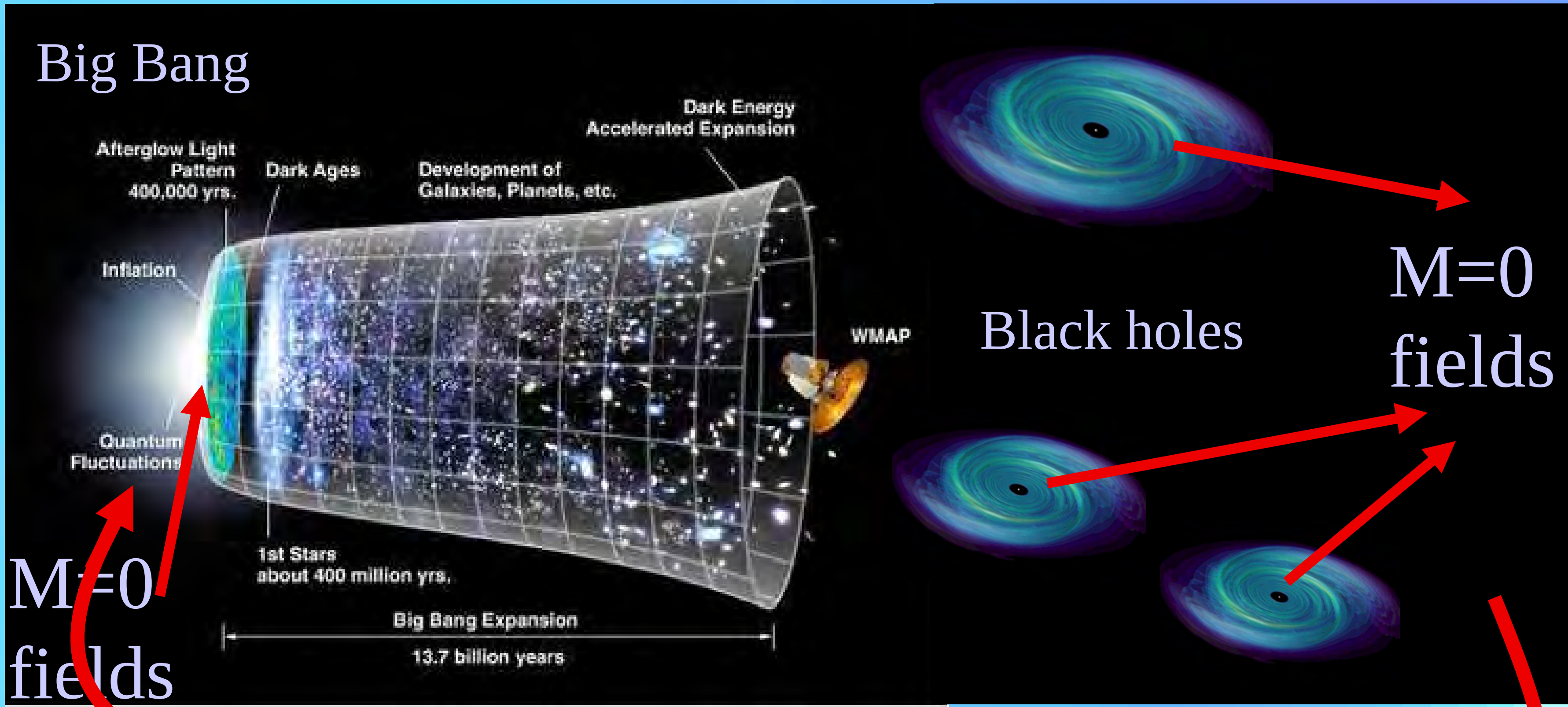
$ST^2 : S^2 \rightarrow R^2 \cup \infty$

or

$S^2 \rightarrow C \cup \infty$

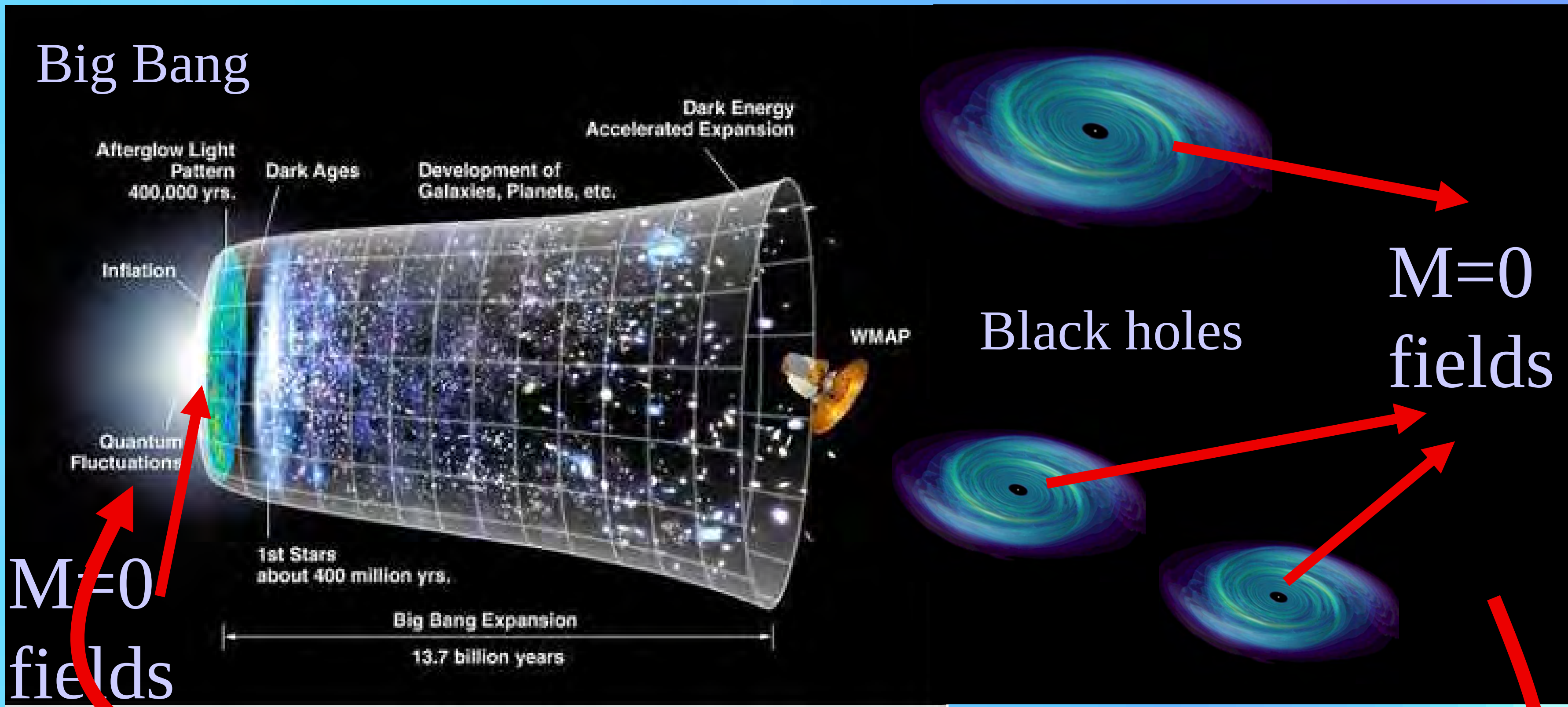


Conformal transformation:
circles remain circles, and
local angles remain unchanged



Increase in entropy
counter balanced by state reduction process

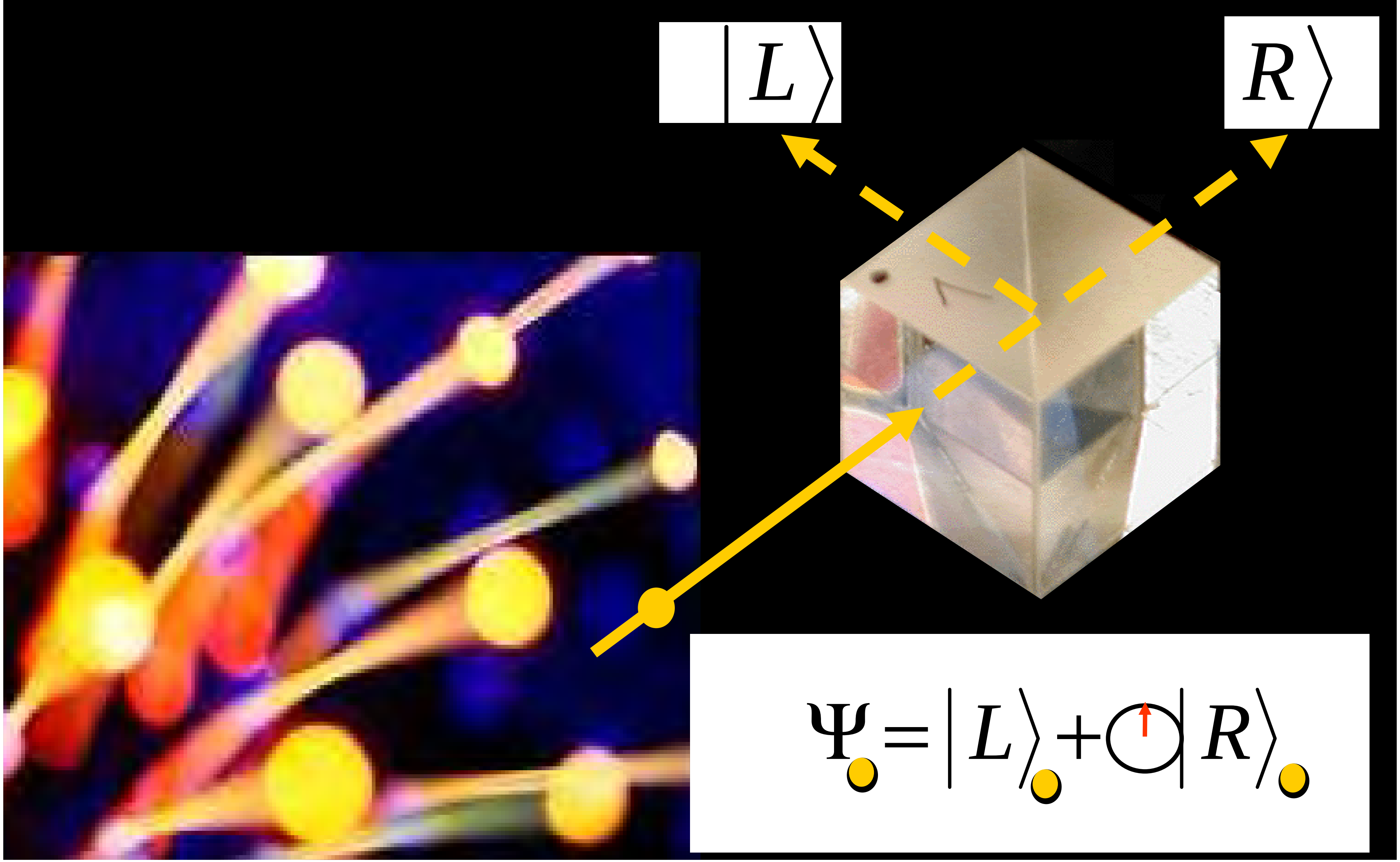
Note: cosmic microwave background radiation shows thermal equilibrium (maximum entropy state for massless fields)



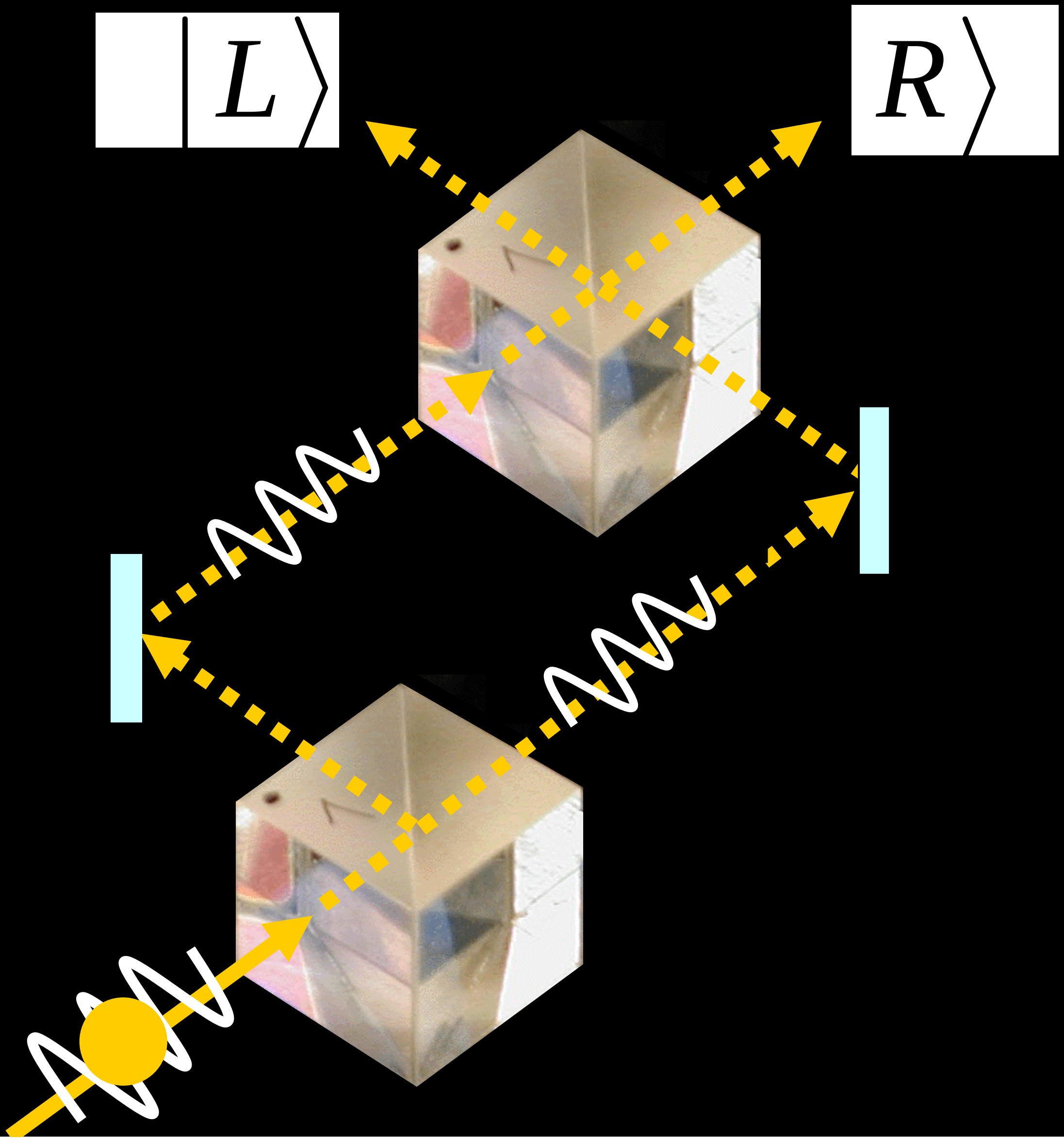
Increase in entropy
counter balanced by state reduction process

Quantum projection postulate

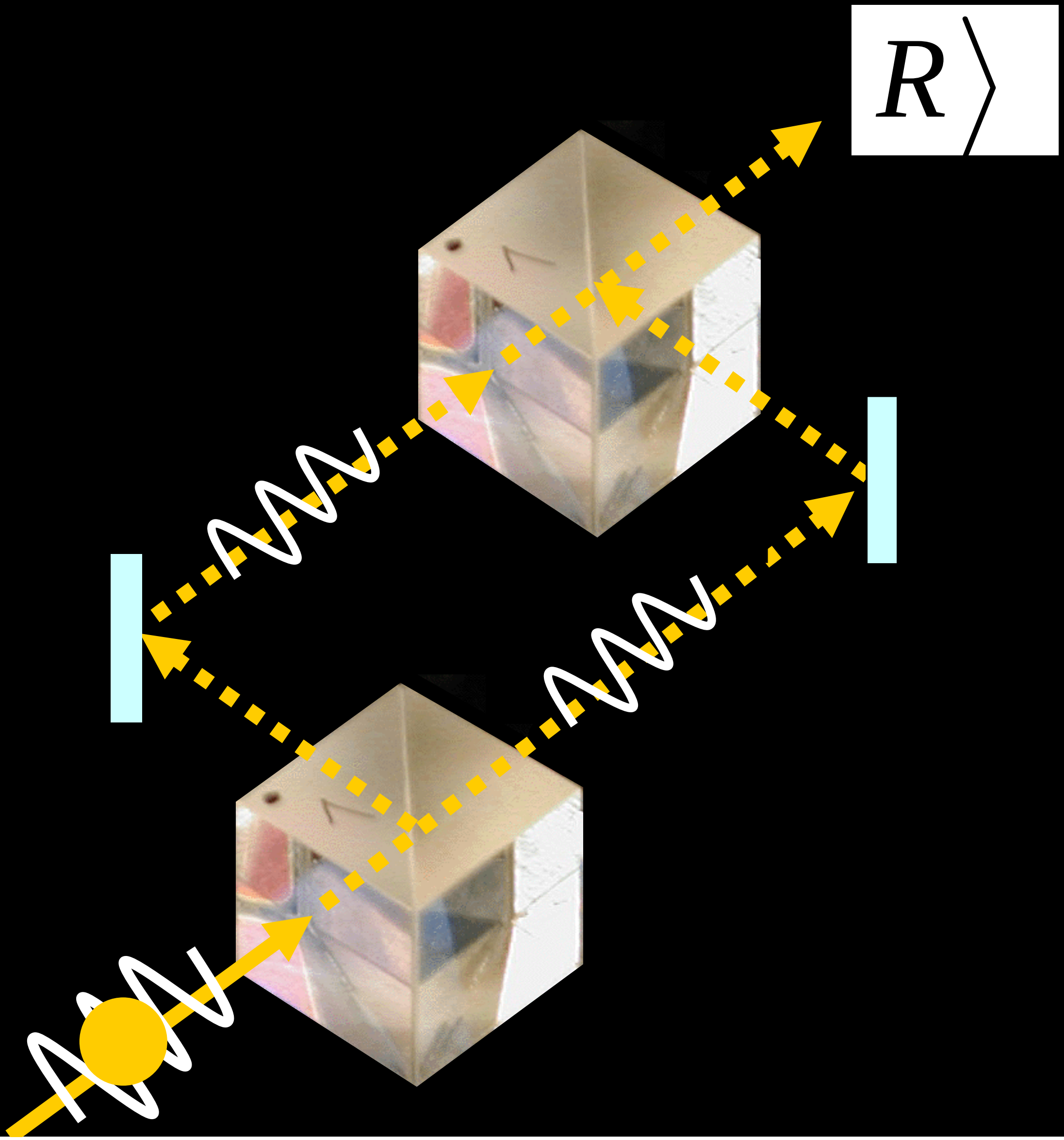
Quantum superposition



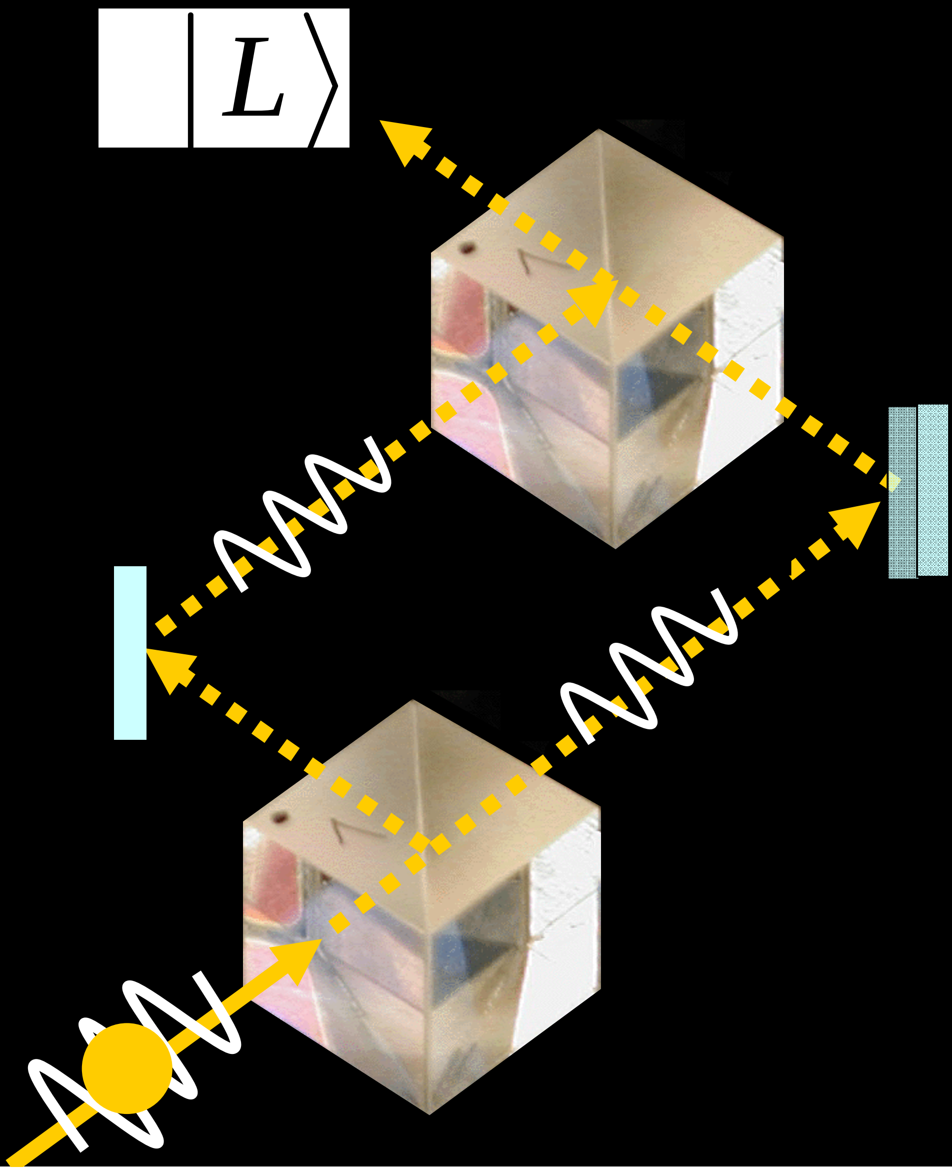
Quantum interference



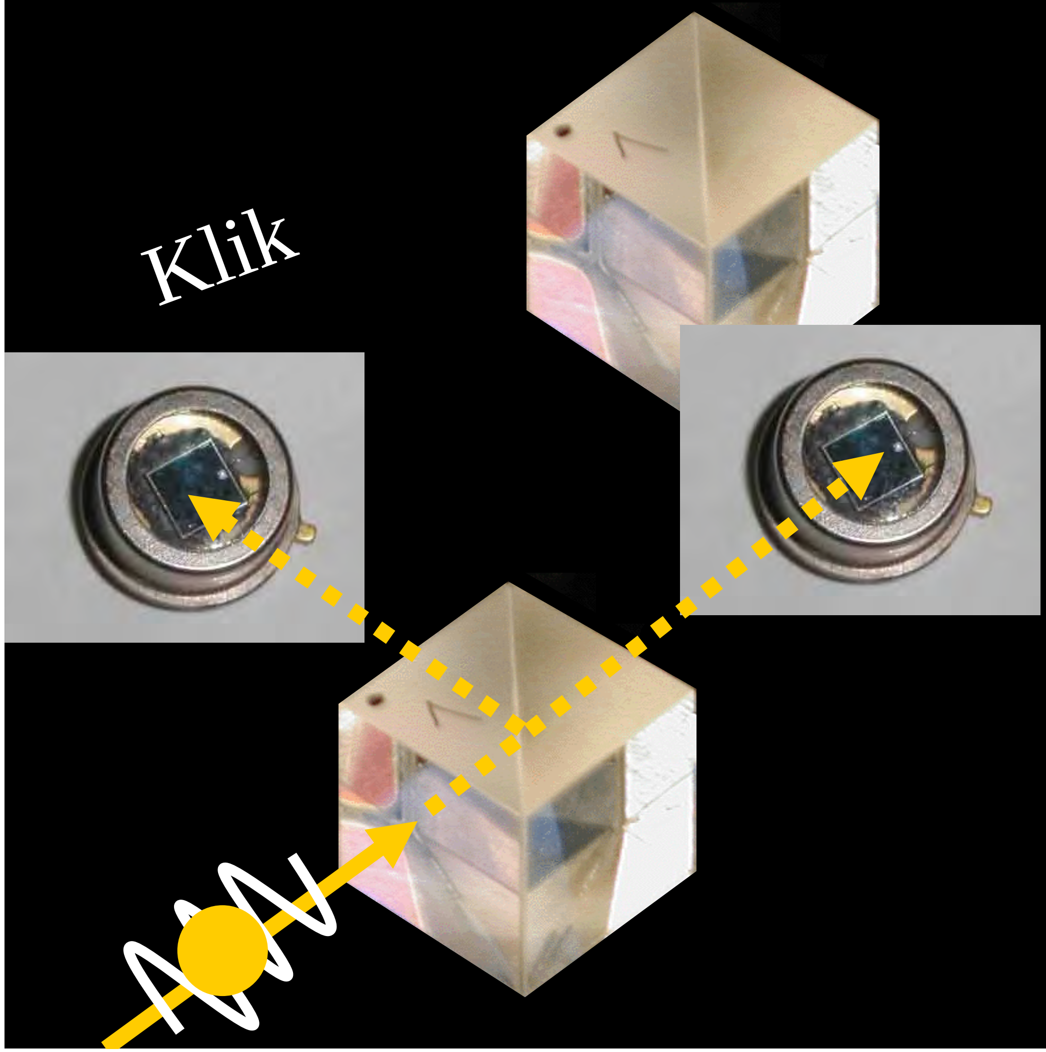
Quantum interference



Quantum interference

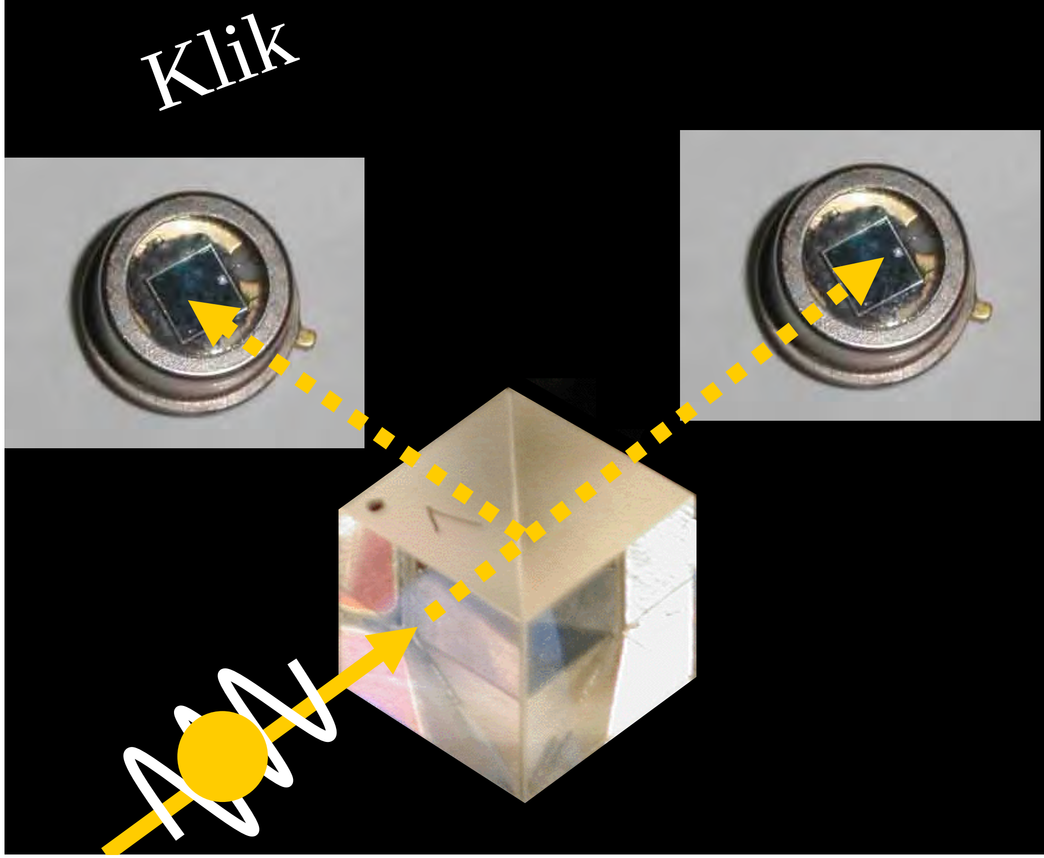


Quantum measurement

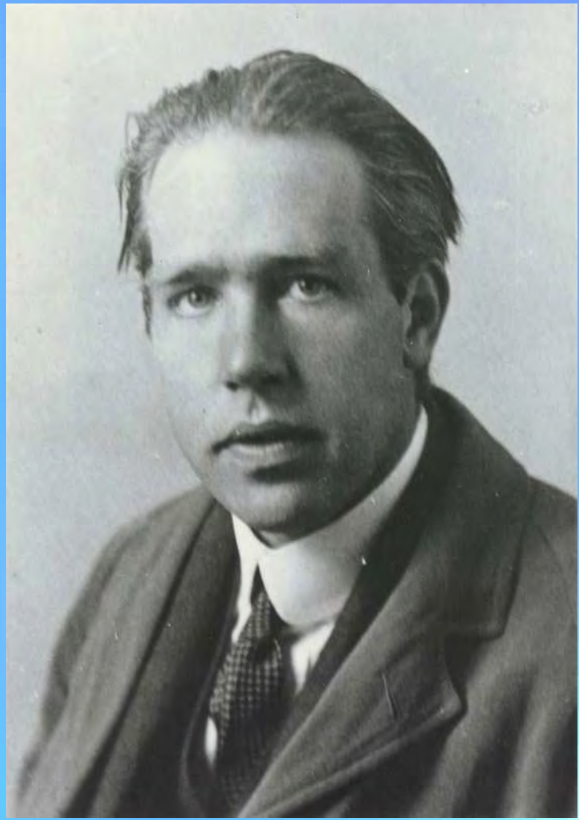


Quantum measurement

Probabilities seem fundamental



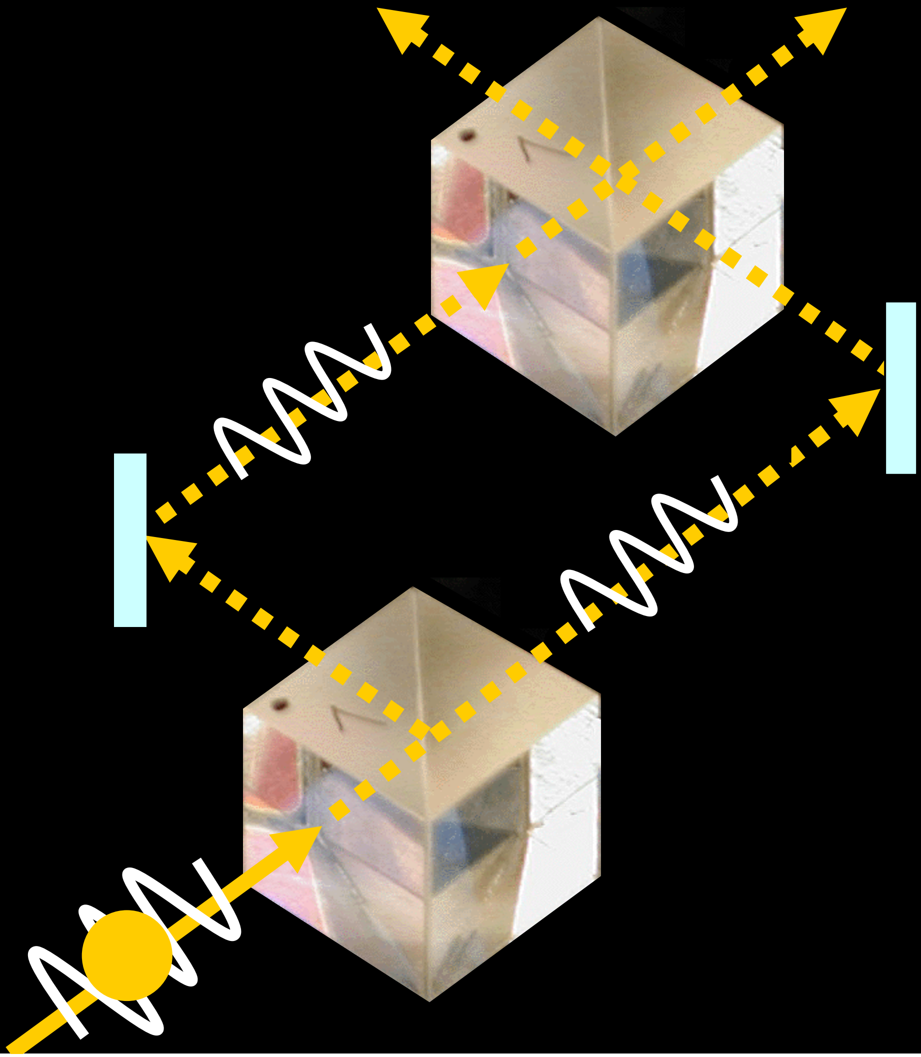
Quantum Mechanics



Niels Bohr
Copenhagen interpretation:

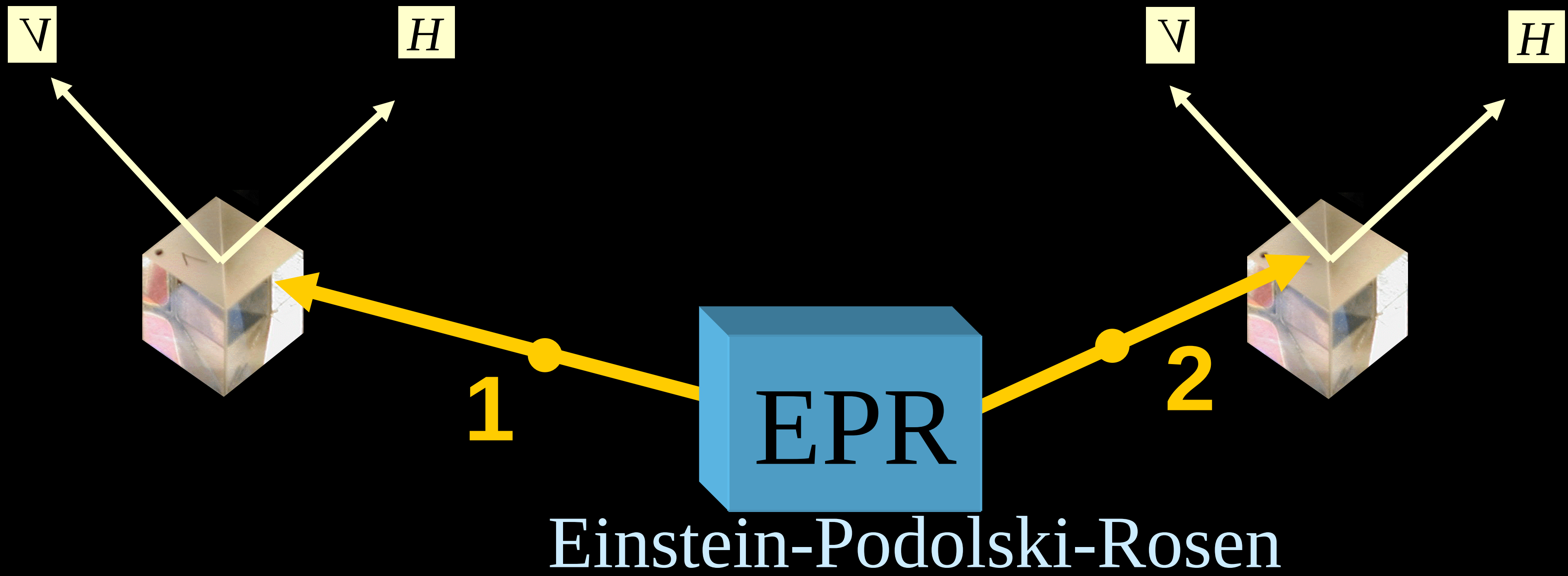
The wavefunction $\left|\Psi\right\rangle$ is not to be taken seriously as describing a quantum level physical reality, but is to be regarded as merely referring to our knowledge of the system.

Quantum interference



Surfing electron?!?

Quantum entanglement



$$\Psi_{12} = |H\rangle_1 |V\rangle_2 + \oslash |V\rangle_1 |H\rangle_2$$

Quantum Measurements

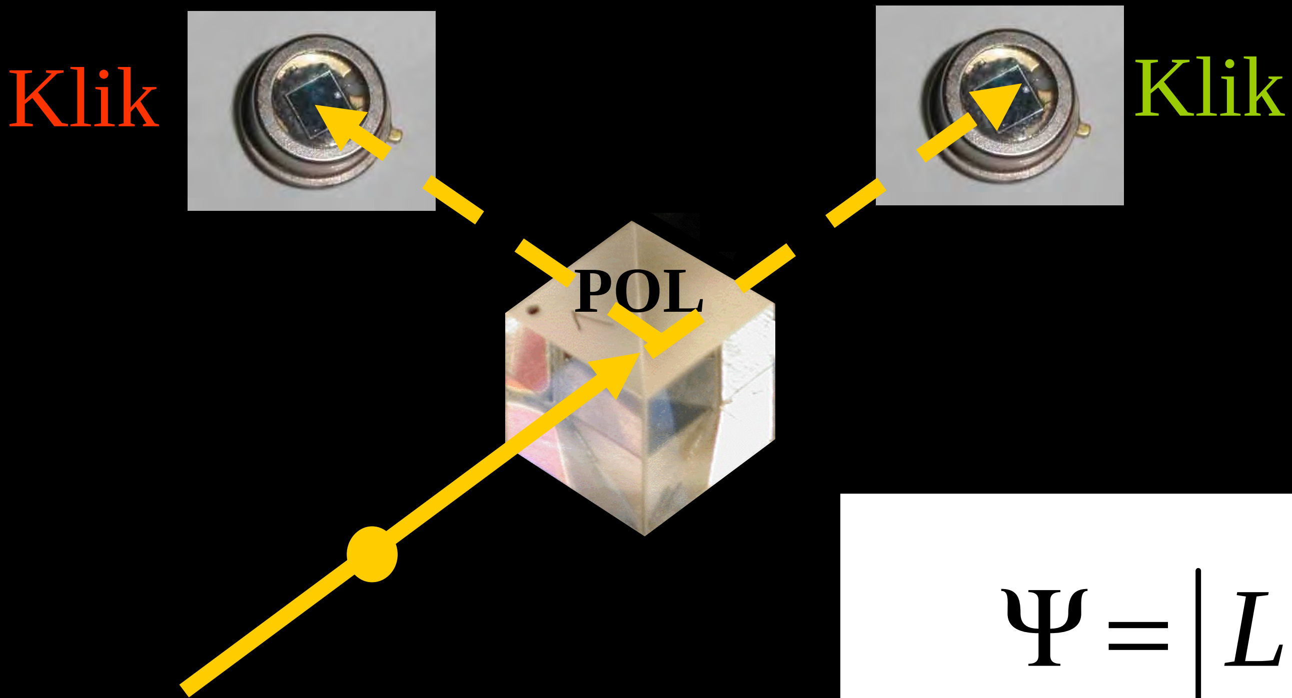
Zurek (and others):

Environment Induced Decoherence

Caldeira-Leggett model (and others) assumes a linear coupling between the position of the system and a bath of harmonic oscillators

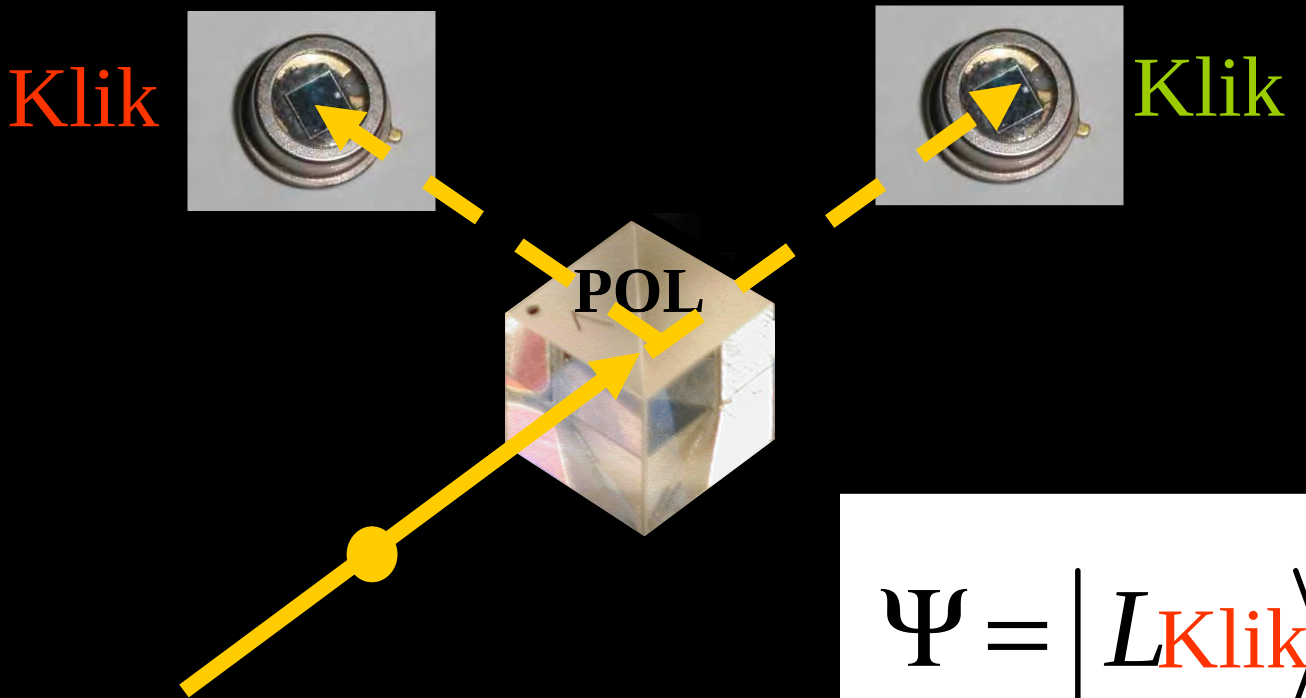
Stamp (and others) considers coupling to spin bath

Quantum measurement



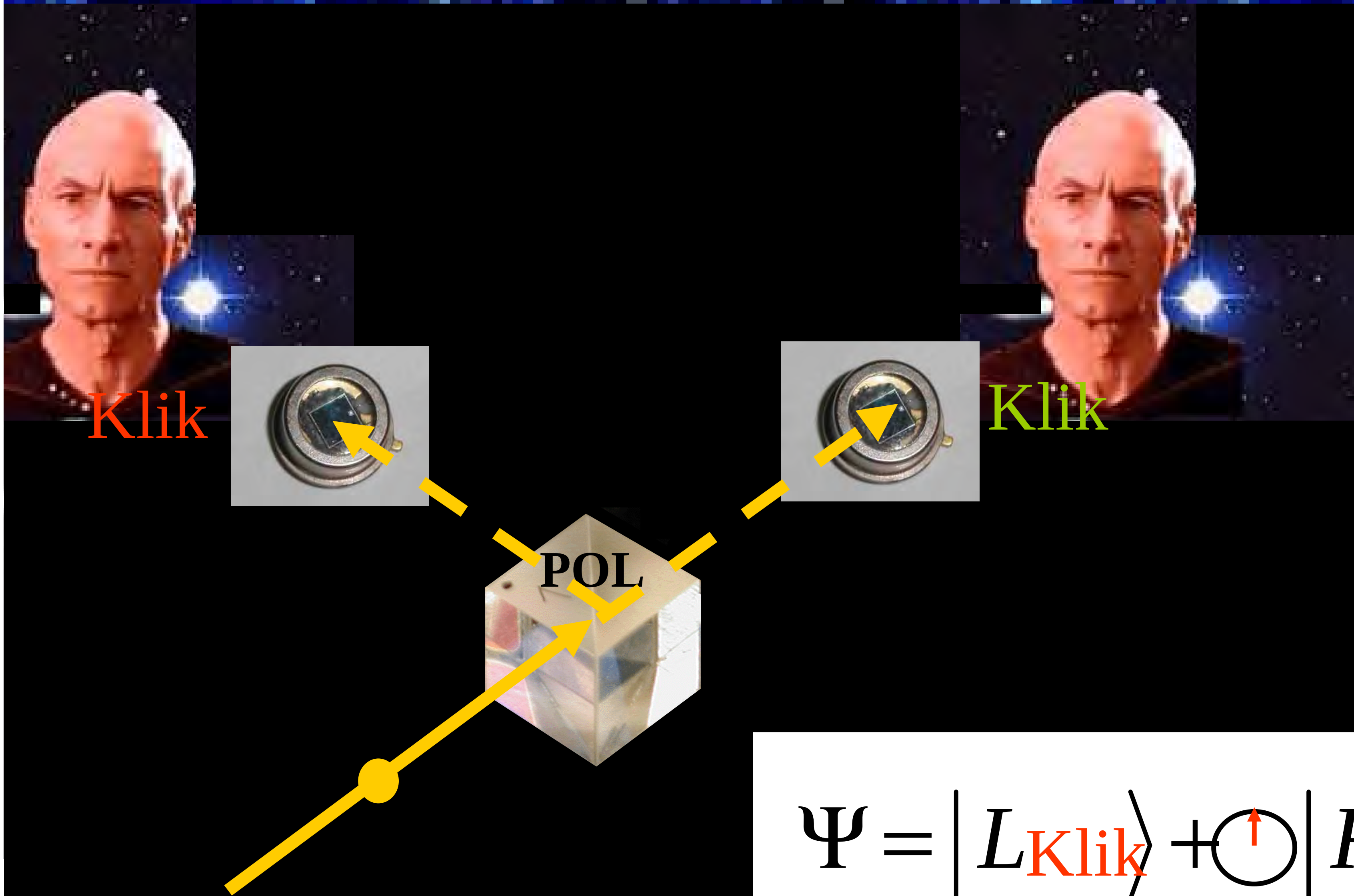
$$\Psi = \left| L \right\rangle + \bigcirc \left| R \right\rangle$$

Quantum measurement



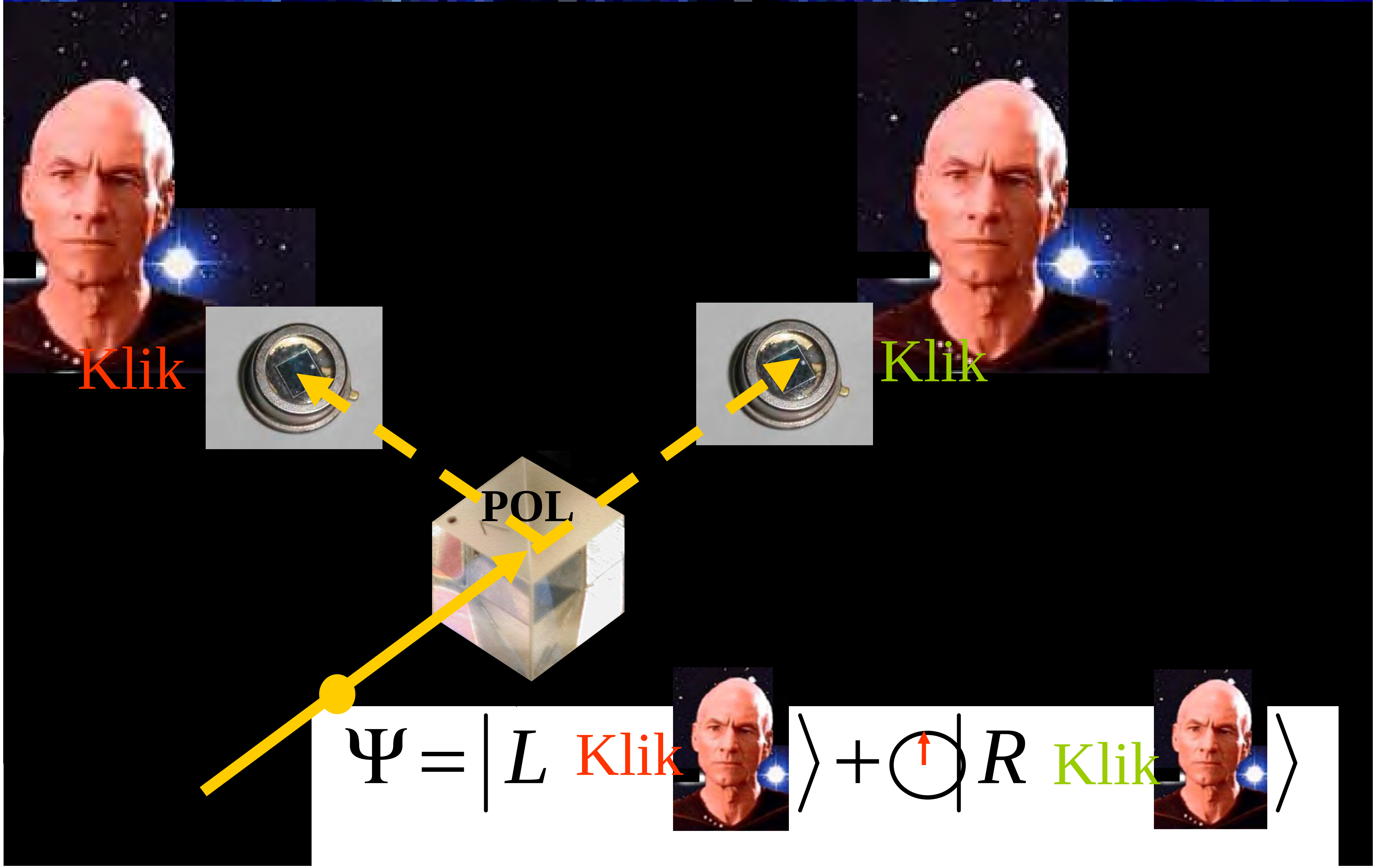
$$\Psi = \left| L\,\text{Klik} \right\rangle + \left| R\,\text{Klik} \right\rangle$$

Quantum measurement

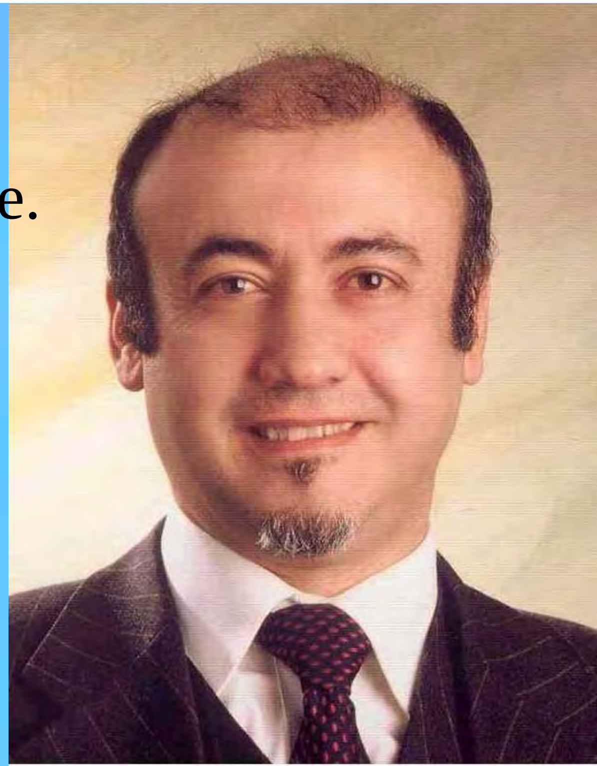


$$\Psi = \left| L_{\text{Klik}} \right\rangle + \circlearrowleft \left| R_{\text{Klik}} \right\rangle$$

Quantum measurement



The wavefunction $|\Psi\rangle$ is a representation of a *real* physical state.

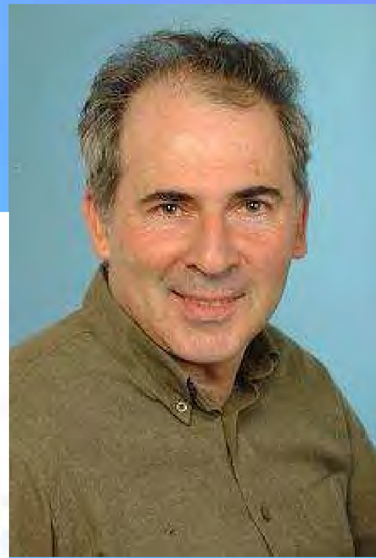


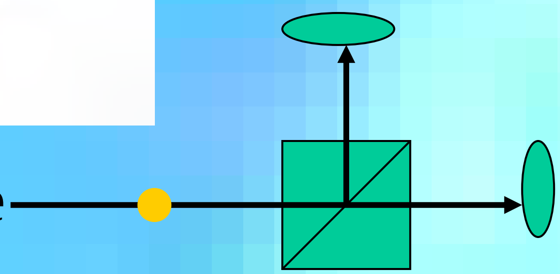
Everett

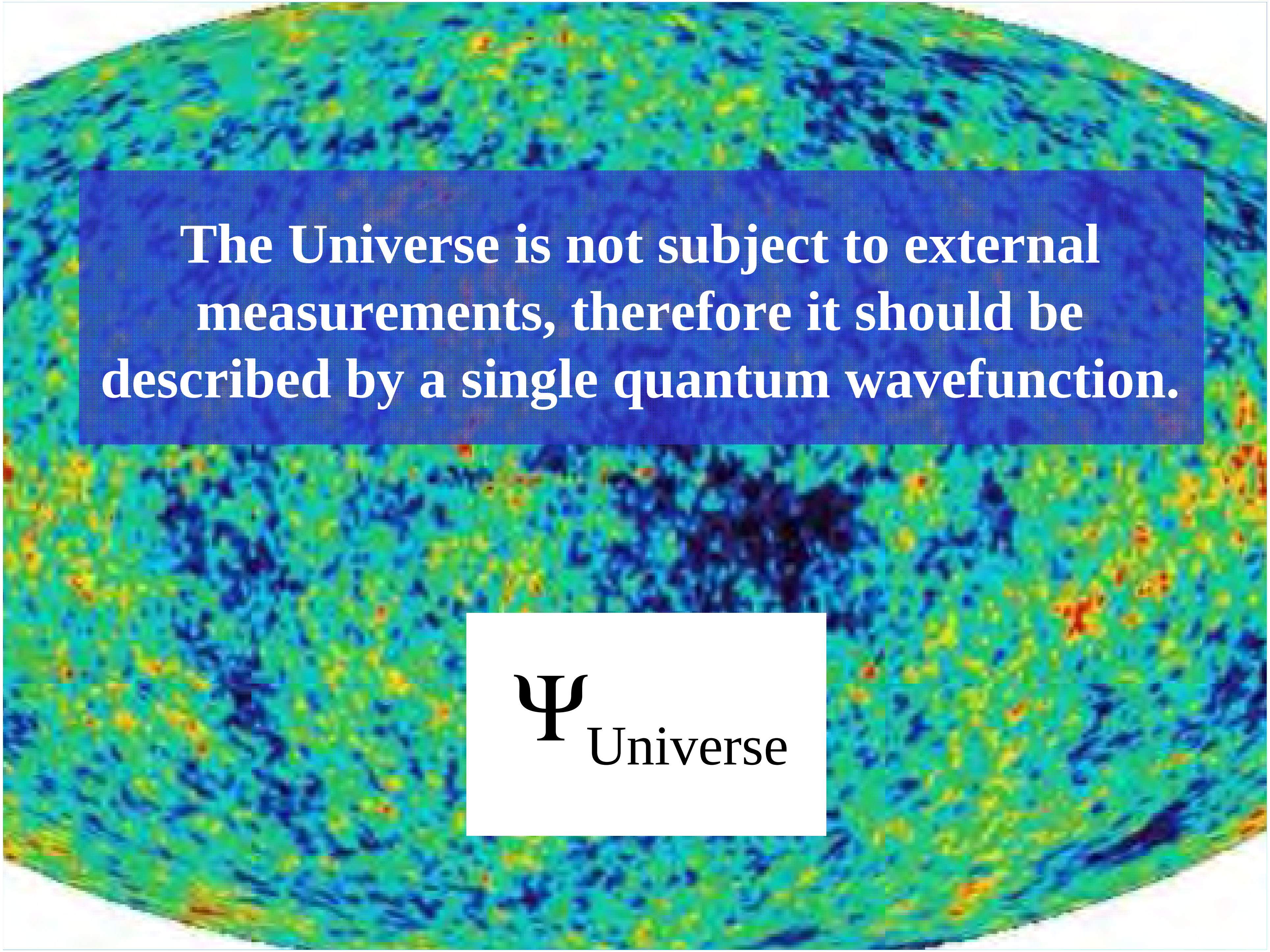
Many Worlds Interpretation

Deutsch

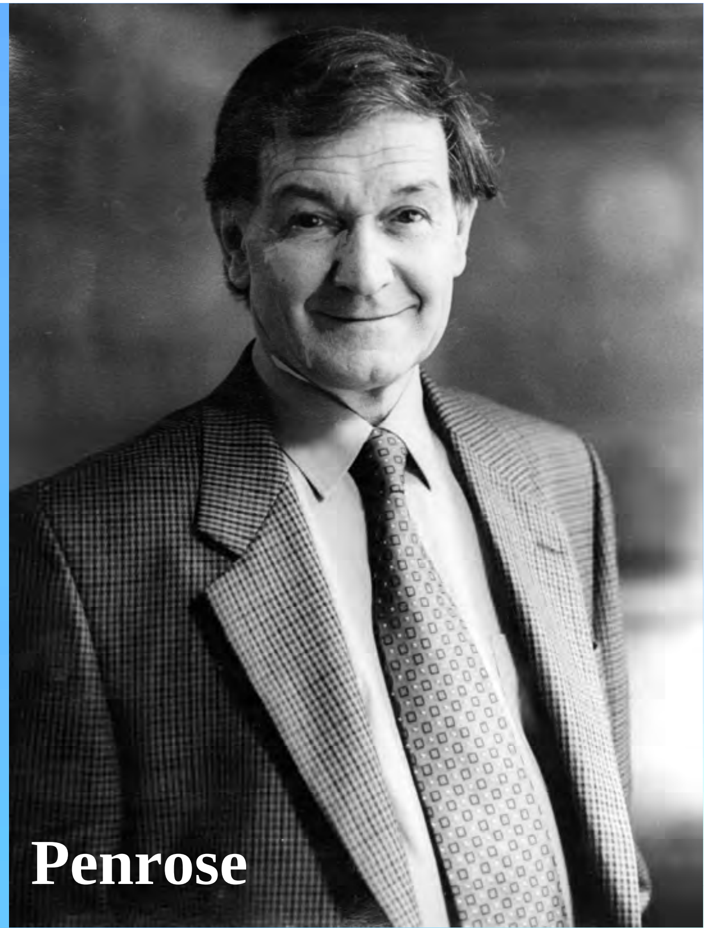
Vaidman's watch






The Universe is not subject to external measurements, therefore it should be described by a single quantum wavefunction.
Ψ_{Universe}

Level 3: Gravitational induced quantum state reduction.



There is a conflict between Einstein's general covariance principle and the quantum superposition principle.

R. Penrose, **Wavefunction Collapse as a Real Gravitational Effect**
General Relativity and Gravitation 28, 581 (1996).

Two alternative locations of a massive object will each have stationary states, and have wavefunctions $|\Psi\rangle$ and $|\Phi\rangle$, that are eigenstates of the $\frac{\partial}{\partial t}$ operator with eigenvalues related to the energy.

$$\frac{\partial}{\partial t}|\Psi\rangle = -i\hbar E_{\Psi}|\Psi\rangle$$

$$\frac{\partial}{\partial t'}|\Phi\rangle = -i\hbar E_{\Phi}|\Phi\rangle$$

But how to deal with superpositions

$$\left.\frac{\partial}{\partial t}\right)\left(\alpha|\Psi\rangle + \beta|\Phi\rangle\right) = ???$$

Consider an equal superposition $\frac{1}{\sqrt{2}}\left(|\Psi\rangle + |\Phi\rangle\right)$

f and **f'** are the acceleration 3-vectors of the free-fall motion in the two space-times (**f** and **f'** are gravitational forces per unit test mass).

Penrose postulate: at each point the scalar (|**f-f'**|)^2 is a measure of incompatibility of the identification. The total measure of incompatibility (or "uncertainty) Δ at time *t* is:

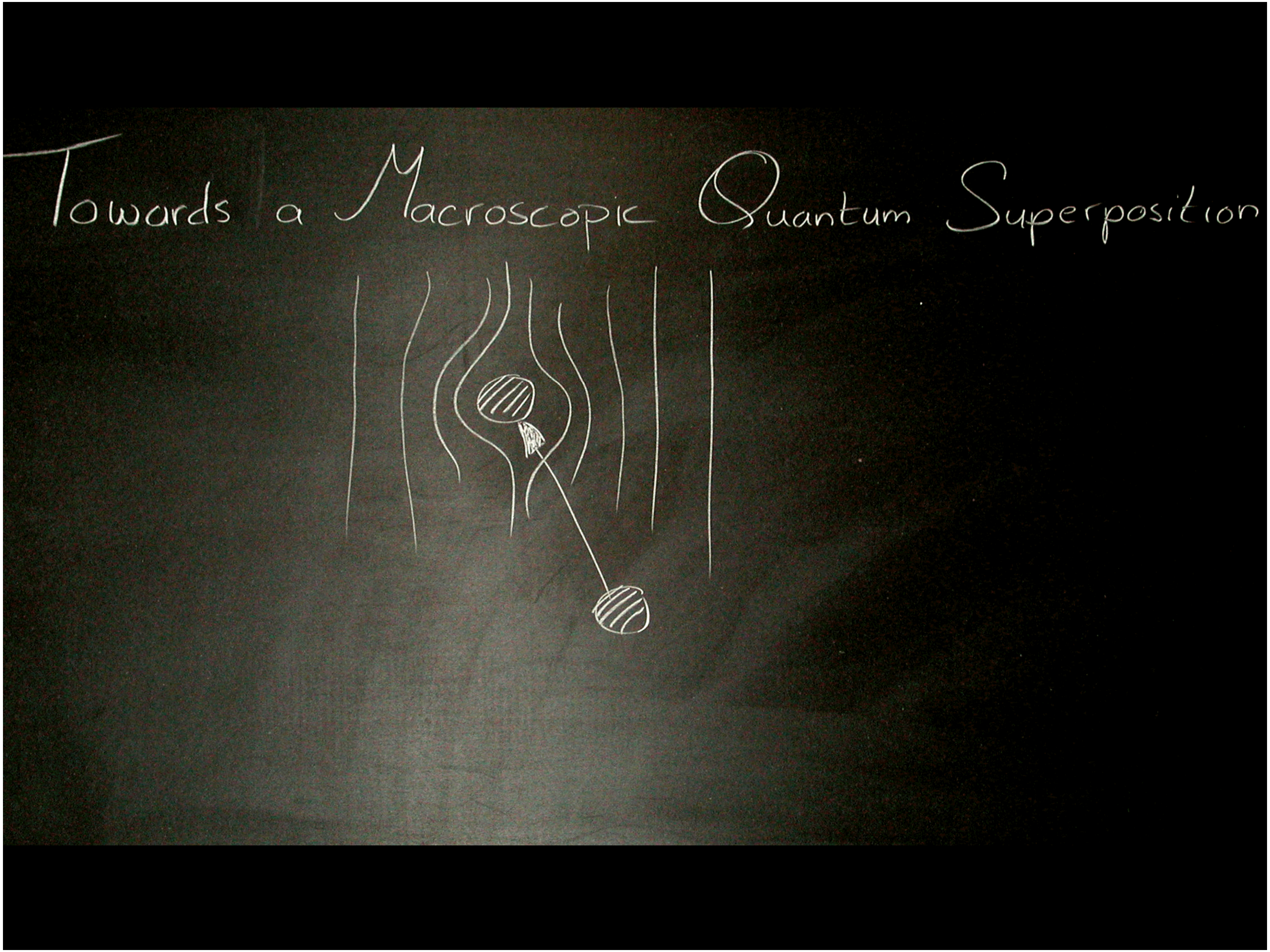
$$\Delta = \frac{1}{4\pi G}\int \left(\text{f-f}'\right)^2 \mathrm{d}^3 x$$

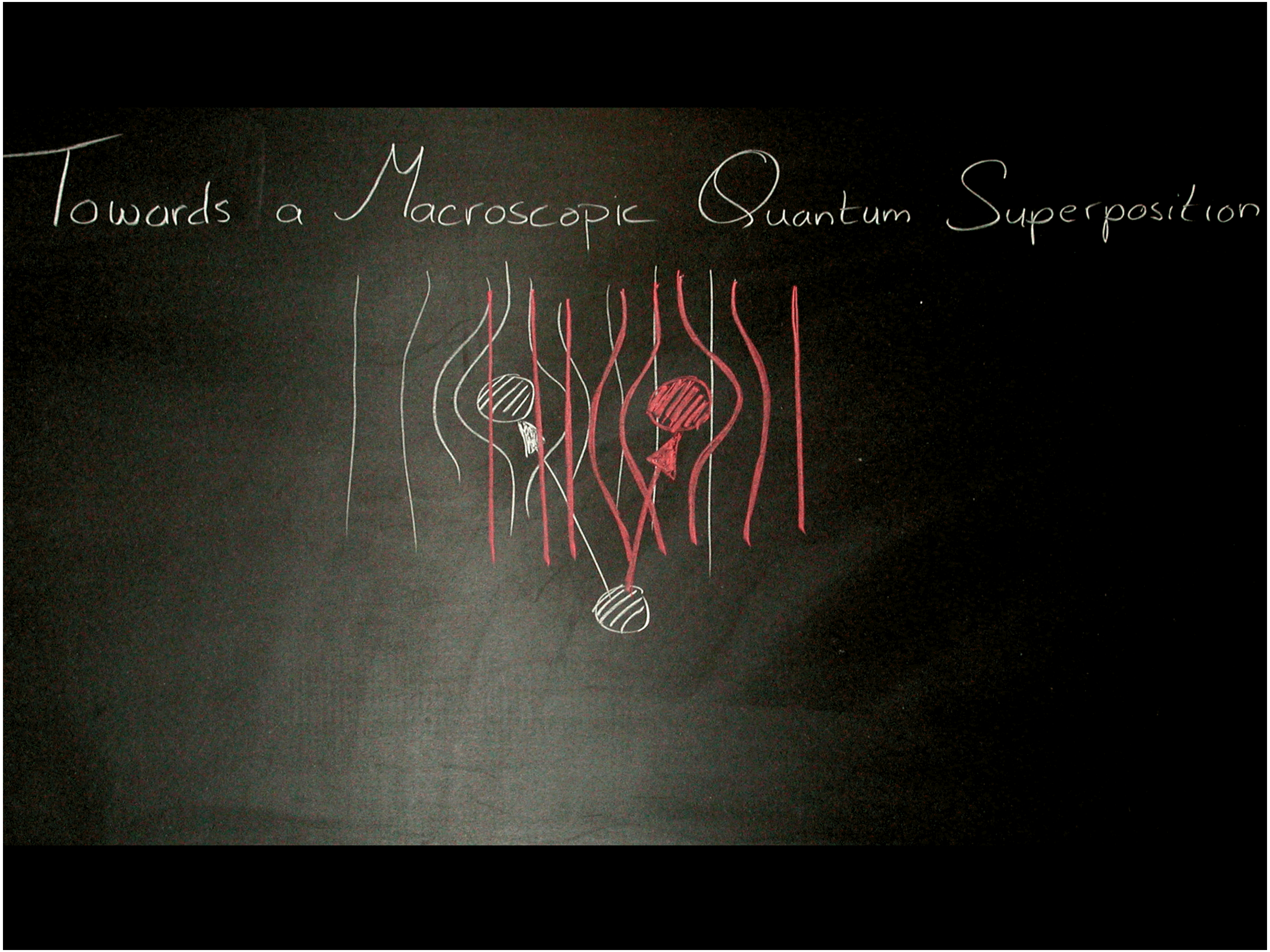
$$\equiv E_G$$

This is the gravitational self energy of the difference between the mass distributions of each of the two lump locations.

Prediction: The superposition state is unstable and has a lifetime of the order of $\frac{\hbar}{E_G}$

(see also GRW, Diosi, others)


Towards a Macroscopic Quantum Superposition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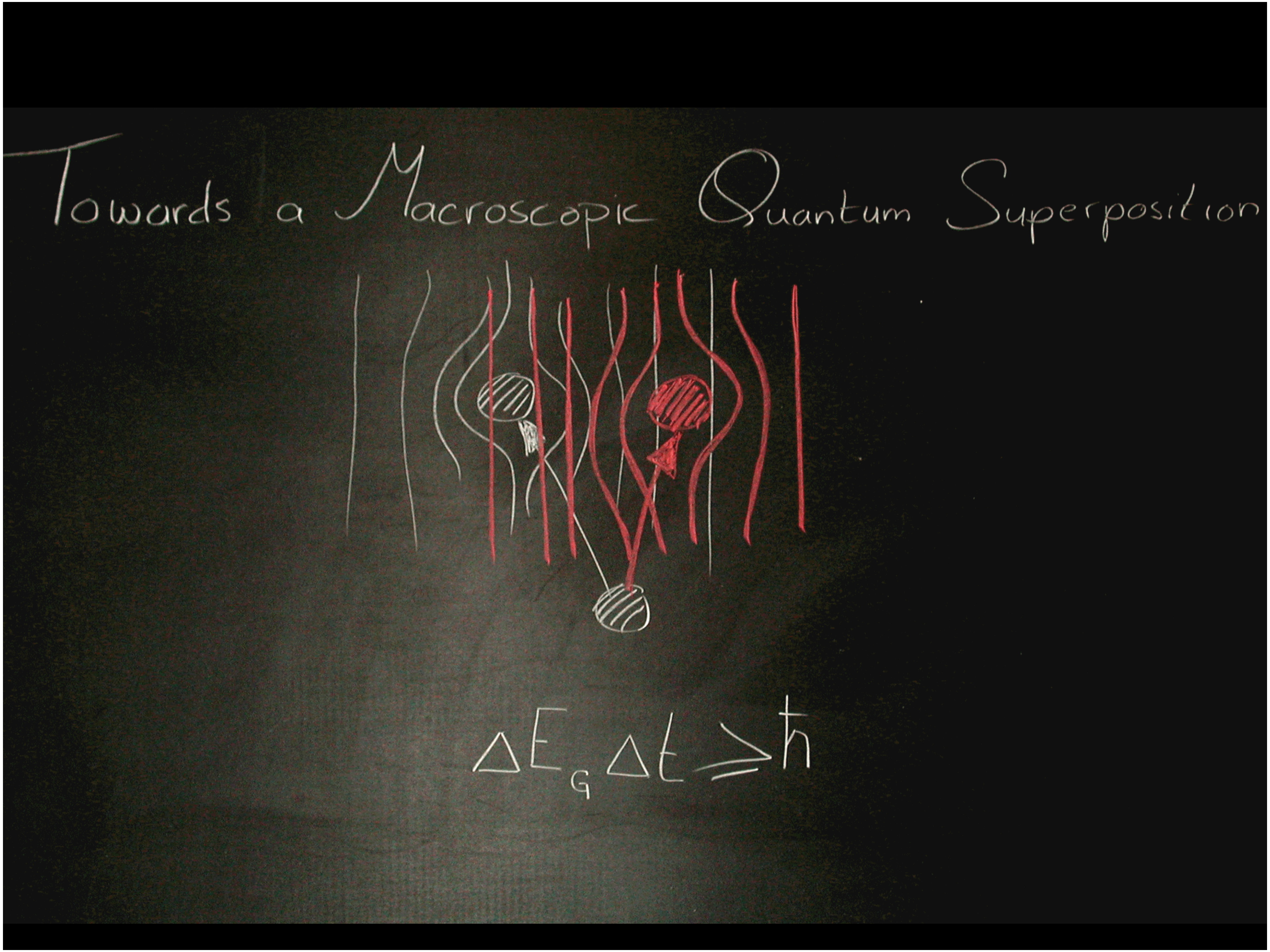


Towards a Macroscopic Quantum Superposition
$\Delta E_G \Delta t \geq \hbar$

$$E_{i,j} = -G \int\int \mathrm{d}\vec{r}_1 \mathrm{d}\vec{r}_2 \frac{\rho_i(\vec{r}_1)\rho_j(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|},$$

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2},$$

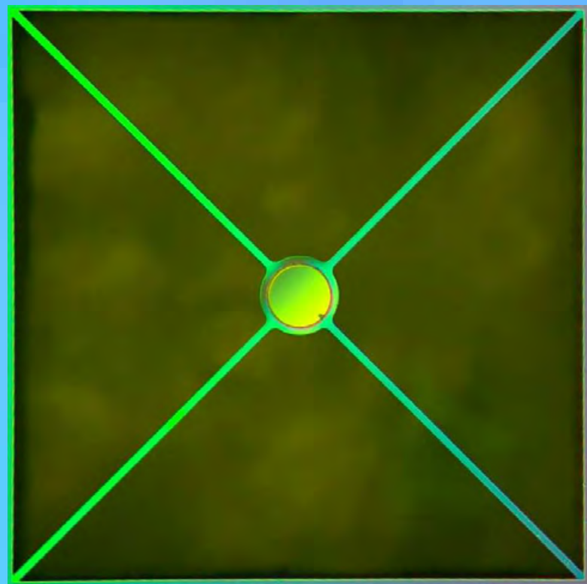
$$\Delta E = 2Gmm_1\left(\frac{6}{5a} - \frac{1}{\Delta x}\right), \qquad (\text{given}: \Delta x \geq 2a)$$

$m \sim 10^{-12}$kg,

$\omega_c \sim 1$-10kHz

$\kappa \sim 1$

$m_1 = 4.7 \times 10^{-26}$kg (Silicon nuclear mass)



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Small problem: what is mass and what is the mass distribution of a piece of material?

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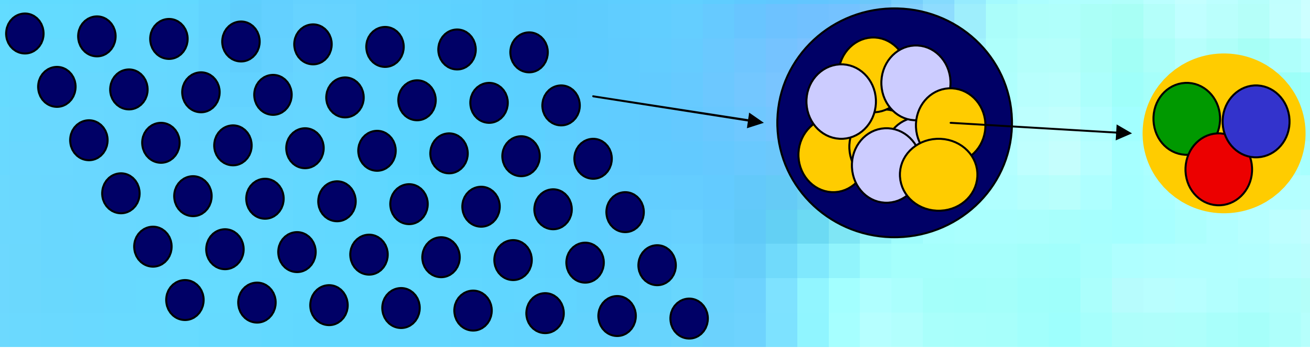

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Take, $a \sim 10^{-15}$m size of nucleus, or take $a \sim 10^{-13}$m size of ground-state wave function

Decoherence time ~1 ms, or ~0.1-1s

Compare: For C_{60} experiments decoherence time is 10^{10}s