Towards Quantum Superpositions of a Mirror

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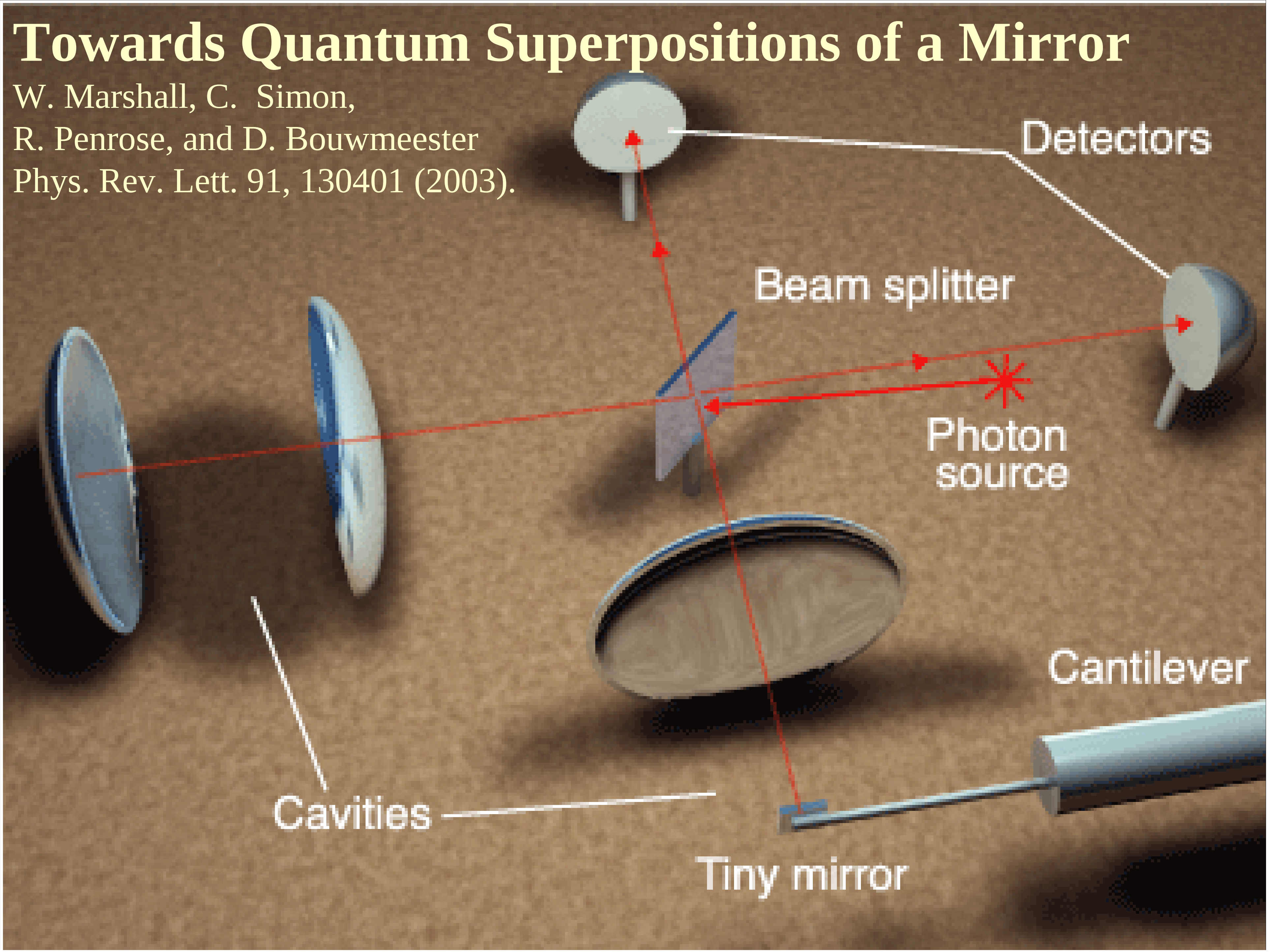
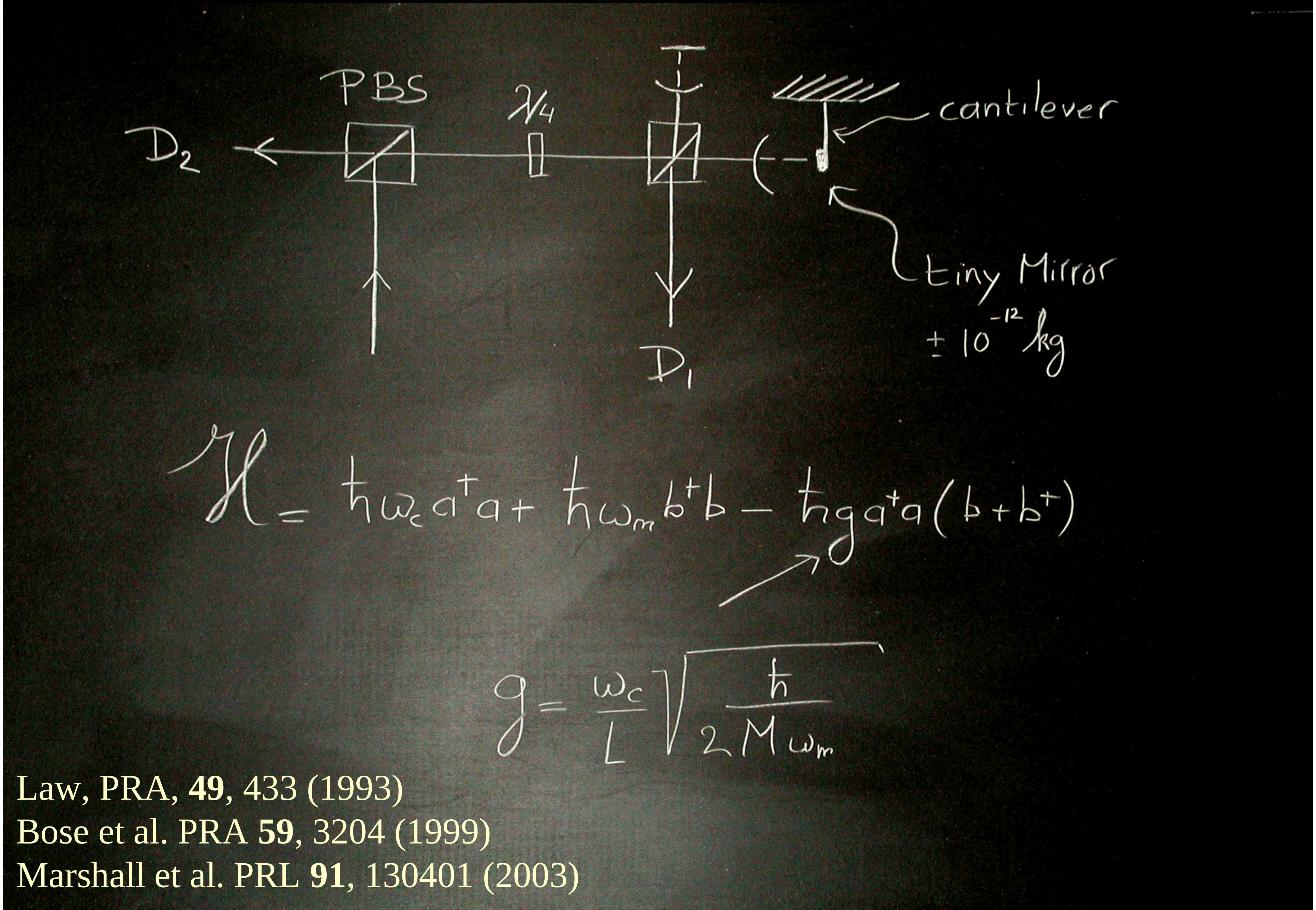




Law, PRA, **49**, 433 (1993)
Bose et al. PRA **59**, 3204 (1999)
Marshall et al. PRL **91**, 130401 (2003)

Optomechanical coupling κ =(g/ω$_c$)

Consider single cavity mode: $\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a}$ $\omega_a = \frac{n}{2}\omega_0$ $\omega_0 = 2\pi c/L$

If mirrors moves by *x:* $\hat{H} = \hbar \frac{n}{2} \frac{2\pi c}{L+x} \hat{a}^\dagger \hat{a}$

$$\hat{H} = \hbar \frac{n}{2}\omega_0 \frac{L}{L+x} \hat{a}^\dagger \hat{a}$$
$$\approx \hbar\omega_a (1 - \frac{x}{L}) \hat{a}^\dagger \hat{a}$$
$$= \hbar\omega_a \hat{a}^\dagger \hat{a} - \hbar \frac{\omega_a}{L} \hat{a}^\dagger \hat{a} x$$

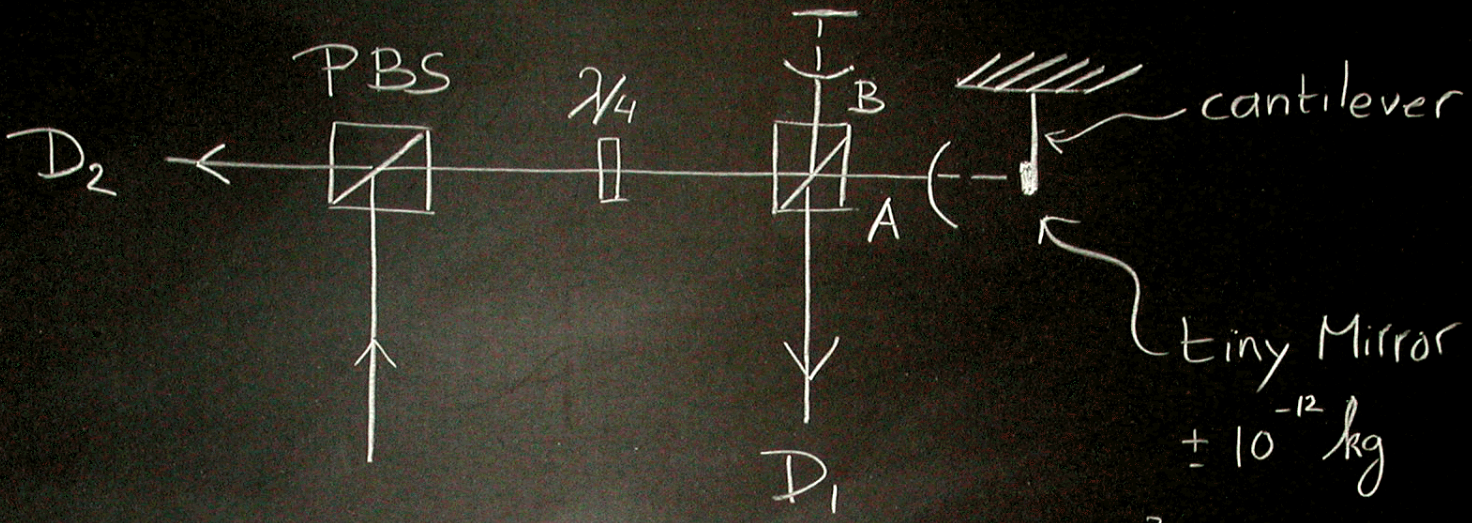
Quantize mirror position: $x \rightarrow \hat{x} = \sqrt{\hbar/2m\omega_c}(\hat{c} + \hat{c}^\dagger)$

Gives:

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} - \hbar\omega_c \frac{\omega_a}{\omega_c L} \sqrt{\frac{\hbar}{2m\omega_c}} (\hat{c} + \hat{c}^\dagger) \hat{a}^\dagger \hat{a}$$

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} - \hbar\omega_c \kappa \hat{a}^\dagger \hat{a} (\hat{c} + \hat{c}^\dagger) \qquad \kappa = \frac{\omega_a}{\omega_c} \frac{1}{L} \sqrt{\frac{\hbar}{2m\omega_c}}$$

*****Sorry on this his page ω_c is the cantilever frequency*****



mirror in coherent state $|\beta\rangle = e^{-\frac{|\beta|^2}{2}} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} |n\rangle$

Initial state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B\right)|\beta\rangle$

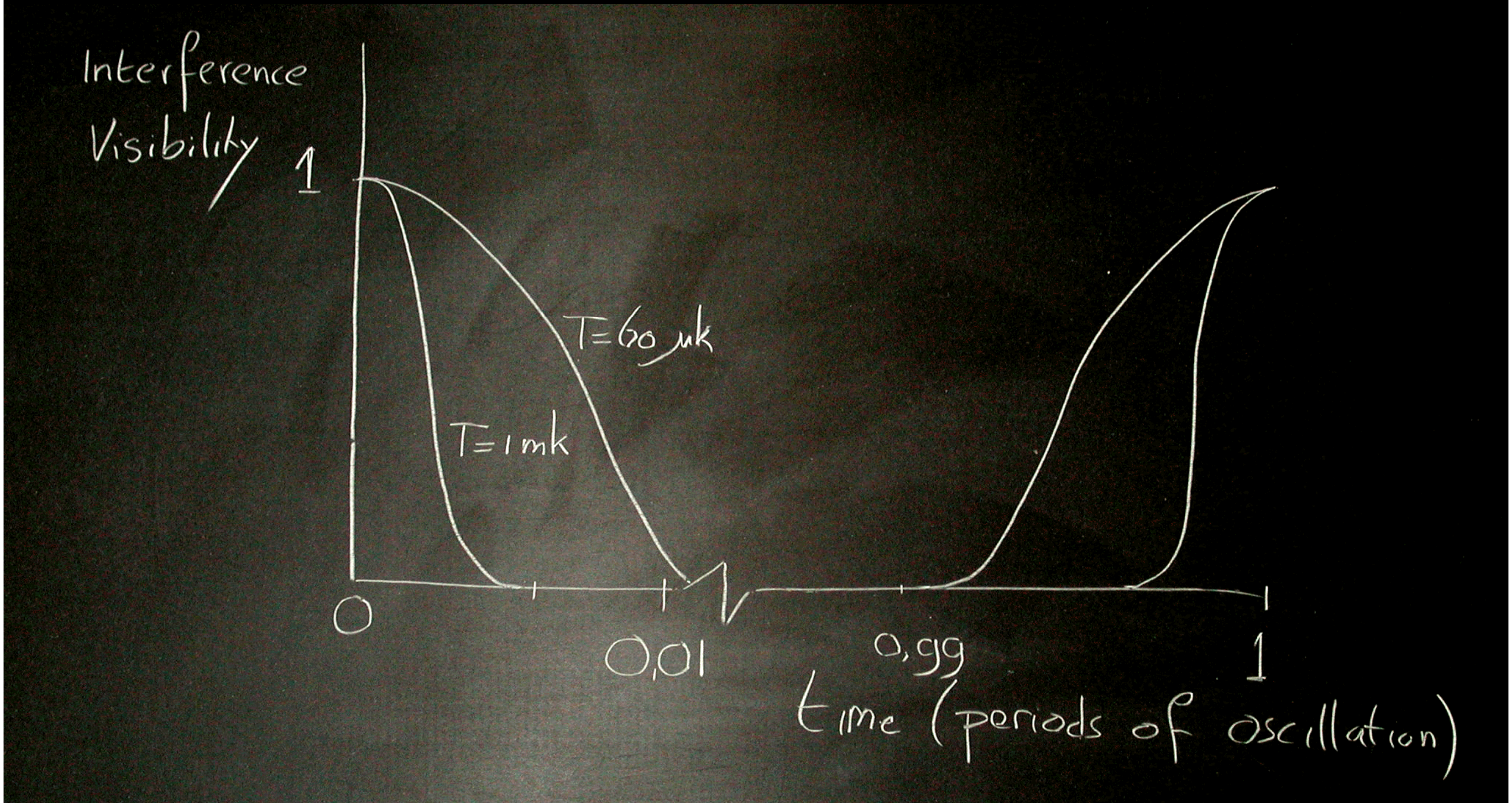
$|\psi(t)\rangle =$ entangled state of mirror and photon
except after full period of oscillation

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar}$$

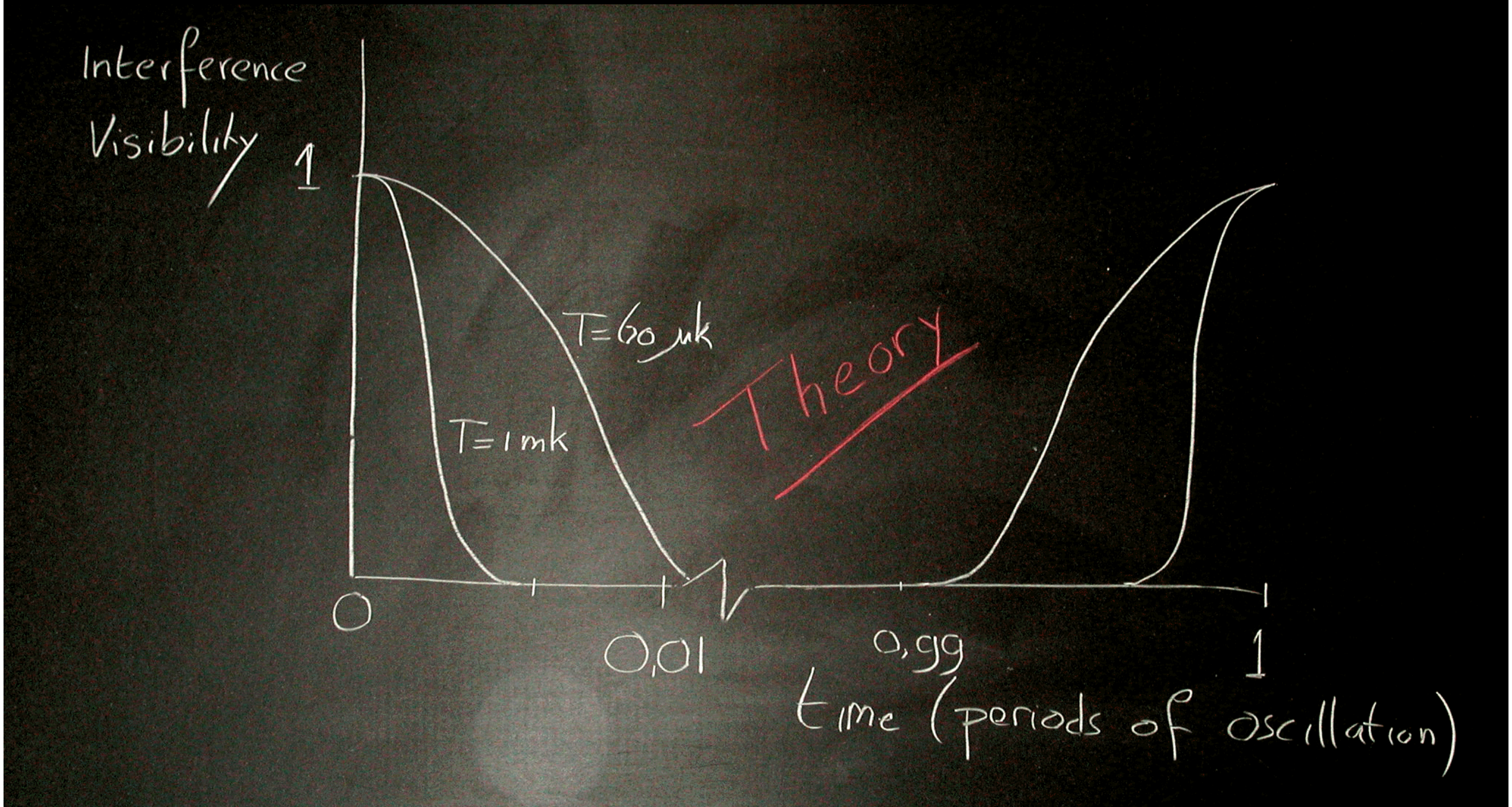
$$\hat{U}(t) = e^{-i\omega_a \hat{a}^\dagger \hat{a} t} e^{i\kappa^2 \left(\hat{a}^\dagger \hat{a}\right)^2 (\omega_c t - \sin\omega_c t)} e^{\kappa \hat{a}^\dagger \hat{a}\left[\hat{c}^\dagger \left(1 - e^{-i\omega_c t}\right) - \hat{c}\left(1 - e^{i\omega_c t}\right)\right]} e^{-i\omega_c \hat{c}^\dagger \hat{c} t}$$

$$\hat{U}|n\rangle|\beta\rangle = e^{-in\omega_a t} e^{i\kappa^2 n^2 (\omega_c t - \sin\omega_c t)} e^{i\kappa n \mathrm{Im}[\beta(1-e^{-i\omega_c t})]} |n\rangle \boxed{|\beta e^{-i\omega_c t} + \kappa n(1 - e^{-i\omega_c t})\rangle}$$

Note that after a full mechanical oscillation the cantilever is back in the same state as it was originally



T is effective temperature of the fundamental resonance of cantilever



T is effective temperature of fundamental resonance of cantilever

Overlap of two components of cantilever wave-function

$$|\langle \Phi_0(t) \,|\, \Phi_1(t) \rangle| = e^{-\kappa^2 (1 - cos\omega_c t)}$$

0 after full oscillation and maximal after half periods

$$t \;=\; \pi/\omega_c \qquad |\langle \Phi_0(\pi/\omega_c) \,|\, \Phi_1(\pi/\omega_c) \rangle| \;=\; e^{-2\kappa^2}$$

Requirement for creating superposition: $\kappa \gtrsim 1/\sqrt{2}$