

## Quantum information processing with trapped ions



Trapped ion experiments:

- Exploring quantum physics
- Elements of quantum computing
- Quantum simulations
- Precision spectroscopy with entangled states

Christian Roos Institute for Quantum Optics and Quantum Information Innsbruck, Austria

## Quantum physics with trapped ions

A single trapped ion: Realization of a quantum harmonic oscillator

Motional degrees of freedom



A single trapped ion: Realization of a quantum bit

Internal degrees of freedom



 $H\propto\sigma_x,\,H\propto\sigma_y$ 

Strings of trapped ion: Entangled quantum bits



 $\Psi \propto |\!\downarrow\rangle|\!\downarrow\rangle + |\!\uparrow\rangle|\!\uparrow\rangle$ 

### **1952: Experiments with single atoms ?**

In the first place it is fair to state that we are not *experimenting* with single particles, anymore than we can raise Ichtyosauria in the zoo.

..., this is the obvious way of registering the fact, that we *never* experiment with just *one* electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this envariably entails ridiculous consequences.

British Journal of the Philosophy of Science III (10), (1952)

E. Schrödinger



### **1953: Invention of the Paul trap**



Dr. Wolfgang Paul und Dr. Helmut Steinwedel, Bonn sind als Erfinder genannt worden

Dr.=Jng. Wolfgang Paul, Bonn

Verfahren zur Trennung bzw. zum getrennten Nachweis von Ionen verschiedener spezifischer Ladung

Patentiert im Gebiet der Bundesrepublik Deutschland vom 24. Dezember 1953 an Patentanmeldung bekanntgemacht am 5. Januar 1956 Patenterteilung bekanntgemacht am 7. Juni 1956

#### W. Paul

### **1978: Observation of single trapped ions**



W. Neuhauser et al., PRL 41, 233 (1978), PRA (1980)

### **Fluorescence detection**



### **Detection of single absorption/emission events**



Experiments: Dehmelt, Toschek, Blatt, Wineland (1986)

## Quantum physics with a single trapped ion



## Quantum physics with a single trapped ion

#### Important tools:

- Traps in UHV systems  $\longrightarrow$  Isolation + long storage times
- Narrow-band lasers  $\longrightarrow$  Laser cooling
- Photomultipliers  $\longrightarrow$  Efficient quantum state detection



Areas of physics:

- Quantum optics: a single ion interacting with single photons
- Tests of quantum physics
- Single-ion optical clocks → Alastair Sinclair's lectures

### **Quantum physics with ion strings**

### **Ion crystals**

At low temperatures:

Equibrium positions determined by trapping forces and mutual Coulomb repulsion







Boulder, USA: Hg<sup>+</sup>



Aarhus, Denmark: <sup>40</sup>Ca<sup>+</sup> (red) and <sup>24</sup>Mg<sup>+</sup> (blue)

### **Quantum physics with ion strings**

### **Ion crystals**

At low temperatures:

Equibrium positions determined by trapping forces and mutual Coulomb repulsion



FIG. 3. Crystalline structure of seven  $^{24}\text{Mg}^+$  ions observed

VOLUME 59, NUMBER 26

#### PHYSICAL REVIEW LETTERS

28 DECEMBER 1987

#### **Observation of a Phase Transition of Stored Laser-Cooled Ions**

F. Diedrich, E. Peik, J. M. Chen, W. Quint, and H. Walther

Max-Planck-Institut für Quantenoptik and Sektion Physik, Universität München, 8046 Garching, Federal Republic of Germany (Received 8 July 1987)

## **Quantum physics and information processing**

Journal of Statistical Physics, Vol. 22, No. 5, 1980

The Computer as a Physical System: A Microscopic Quantum Mechanical as Represented by Tur

Paul Benioff<sup>1,2</sup>

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

#### **Simulating Physics with Computers**

**Richard P. Feynman** 

Proc. R. Soc. Lond. A 425, 73-90 (1989)

Quantum computational networks

By D. Deutsch

arXiv:quant-ph/9508027v2 25 Jan 1996

Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer<sup>\*</sup>

Peter W. Shor<sup>†</sup>

## **Classical vs. quantum information processing**



Physical system with two distinct states 0 or 1

Logic gates

Boolean logic operation

 $0 \rightarrow 1$ 

 $1 \rightarrow 0$ 

 $(\epsilon_1, \epsilon_2) \rightarrow \epsilon_1 \oplus \epsilon_2$ 

#### Quantum bit:

Two-level quantum system with state

 $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ 

#### Quantum logic gate

Unitary transformation

single qubit gate

 $-U-ert \psi
angle 
ightarrow Uert \psi
angle$ 

two-qubit gate

 $|\epsilon_1
angle|\epsilon_2
angle
ightarrow |\epsilon_1
angle|\epsilon_1\oplus\epsilon_2
angle$ 



### **Trapped ions for quantum information processing**

VOLUME 74, NUMBER 20

#### PHYSICAL REVIEW LETTERS

15 May 1995

#### **Quantum Computations with Cold Trapped Ions**

J. I. Cirac and P. Zoller\*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria (Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.



- State detection
- Single qubit gates

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Ion string

- Qubit register
- State detection
- Single qubit gates

Entangling gates



• individual addressing, spatially resolved fluorescence



- individual addressing, spatially resolved fluorescence
- coupling internal and motional states by laser takes on simple form



- individual addressing, spatially resolved fluorescence
- coupling internal and motional states by laser takes on simple form
- no direct state-dependent interactions between ions



#### Vibrational modes



## **Experimental setup**



## **Experimental setup**

XIXIX

### Linear ion trap

#### Linear ion trap



# Harmonic trapping potential in the centre



#### Anisotropic potential with

 $u_z \ll \nu_x, \nu_y \quad (\approx 1-5 \text{ MHz})$ 

linear chain of ions

## **Quantum physics with ion strings**

- based on Alastair's lectures: Theory of ion traps, laser cooling, two-level atoms, coupling of internal and vibrational states by laser light
- more emphasis on coherent atom-light interactions
- more ions: 2,3,4,...

#### Lecture plan:

- Encoding of quantum information in trapped ions
- Manipulation and measurement of quantum information
- Creation of entanglement
- Analysis of multi-qubit states
- Quantum gate operations
- Elements of quantum computing with trapped ions
- Entanglement for metrological applications
- Quantum simulation with trapped ions

today

### **Trapped-ion quantum bits**

### Encoding, manipulation and measurement



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1	Hydrogen				For the most	accurate values	s of these and c	ther constants,	visit physics.nis	st.gov/constants	5		Natio	nal Institute	of Standards	tration and Technol	ogy	Helium
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Pirio 4	Potassium	Calcium	Scandium	Titanium	V Vanadium	Chromium	Manganese	Iron	Cobalt	Nickel	Copper		Gallium	Germanium	Asenic	Selenium	Bromine	Krypton
ď	39.0983 [Ar]4s	40.078 [Ar]4s <sup>2</sup>	44.95591 [Ar]3d4s <sup>2</sup>	47.867 [Ar]3d <sup>2</sup> 4s <sup>2</sup>	50.9415 [Ar]3d <sup>3</sup> 4s <sup>2</sup>	51.9961 [Ar]3d <sup>5</sup> 4s	54.93805 [Ar]3d <sup>5</sup> 4s <sup>2</sup>	55.845 [Ar]3d <sup>6</sup> 4s <sup>2</sup>	58.93320 [Ar]3d <sup>7</sup> 4s <sup>2</sup>	58.6934 [Ar]3d <sup>8</sup> 4s <sup>2</sup>	63.546 [Ar]3d <sup>10</sup> 4s	65.39 [Ar]3d <sup>10</sup> 4s <sup>2</sup>	69.723 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p	72.61 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>2</sup>	74.92160 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>3</sup>	78.96 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>4</sup>	79.904 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>5</sup>	83.80 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>6</sup>
	4.3407 37 <sup>2</sup> S <sub>1/2</sub>	6.1132 38 S.	6.5615 <b>39</b> <sup>2</sup> D <sub>312</sub>	6.8281 40 <sup>3</sup> Fa	6.7462 41 <sup>6</sup> D.o.	6.7665 42 <sup>7</sup> S <sub>2</sub>	7.4340 43 <sup>6</sup> San	7.9024 44 <sup>5</sup> Fe	7.8810 45 <sup>4</sup> Faa	7.6398 46 <sup>1</sup> S.	7.7264 47 <sup>2</sup> S.m	9 3942 48 <sup>1</sup> S.	5.9993 49 <sup>2</sup> P <sup>o</sup>	7.8994 50 <sup>3</sup> P-	9.7886 51 <sup>4</sup> S <sup>o</sup>	9.7524 52 <sup>3</sup> Pa	11.8138 53 <sup>2</sup> P <sup>o</sup>	13.9996 54 <sup>1</sup> S.
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te		Xe
5	Rubidium 85,4678	Strontium 87.62	Yttrium 88.90585	Zirconium 91.224	Niobium 92.90638	Molybdenum 95.94	Technetium (98)	Ruthenium 101.07	Rhodium 102.90550	Palladium 106.42	Silver 107.8682	Cadmium 112.411	Indium 114.818	Tin 118.710	Antimony 121.760	Tellurium 127.60	lodine 126.90447	Xenon 131.29
	[Kr]5s 4.1771	[Kr]5s <sup>+</sup> 5.6949	[Kr]4d5s* 6.2171	[Kr]4d*5s* 6.6339	[Kr]4d <sup>*</sup> 5s 6.7589	[Kr]4d°5s 7.0924	[Kr]4d <sup>*</sup> 5s <sup>*</sup> 7.28	[Kr]4d'5s 7.3605	[Kr]4d°5s 7.4589	[Kr]4d 8.3369	[Kr]4d <sup>16</sup> 5s 7.5762	[Kr]4d <sup>10</sup> 5s <sup>2</sup> 8.9938	[Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p 5.7864	[Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>2</sup> 7.3439	[Kr]4d <sup>19</sup> 5s <sup>2</sup> 5p <sup>3</sup> 8.6084	[Kr]4d <sup>10</sup> 5s <sup>4</sup> 5p <sup>4</sup> 9.0096	[Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>5</sup> 10.4513	[Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>6</sup> 12,1298
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6	Cesium	Barium	N.	Hafnium	Tantalum	Tungsten	Rhenium	Osmium	Iridium	Platinum	Gold	Mercury	Thallium	Lead	Bismuth	Polonium	Astatine	Radon
	[Xe]65	[Xe]6s <sup>2</sup> 5 2117	N.	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 6.8251	[Xe]4f <sup>14</sup> 5d <sup>3</sup> 6s <sup>2</sup>	[Xe]4f <sup>14</sup> 5d <sup>4</sup> 6s <sup>2</sup> 7 8640	[Xe]4f <sup>14</sup> 5d <sup>5</sup> 6s <sup>2</sup> 7 8335	[Xe]4f <sup>14</sup> 5d <sup>8</sup> 6s <sup>2</sup>	[Xe]4f <sup>14</sup> 5d <sup>7</sup> 6s <sup>2</sup>	[Xe]4f <sup>14</sup> 5d <sup>9</sup> 6s	[Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s	Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>1</sup>	[Hg]6p	[Hg]6p <sup>2</sup>	[Hg]6p <sup>3</sup>	(209) [Hg]6p <sup>4</sup>	(210) [Hg]6p <sup>5</sup>	(222) [Hg]6p <sup>6</sup>
	87_2S <sub>1/2</sub>	88 <sup>1</sup> S <sub>0</sub>		104 3F2?	105	106	107	108	109	110	111	112	0.1002		1,2030	0.417 1		10,7465
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	58	100	1	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Но	Er	Tm	Yb	Lu
Symbo		<b>6</b>	1. 1.	138,9055	140.116	140.90765	144.24	(145)	150.36	151.964	157.25	158,92534	162.50	Holmium 164.93032	167.26	168.93421	173.04	174.967
Nam	e Cer	ium	i, Y	5.5769	5.5387	5.473	5.5250	5.582	5.6436	5.6704	6.1501	5.8638	5.9389	6.0215	6.1077	6.1843	6.2542	5.4259
Atomi	c 140 t [Xe]4	.116 f5d6s <sup>2</sup>	1	AC	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr
	5.5	387	1	Actinium (227)	Thorium 232.0381	Protactinium 231.03588	Uranium 238.0289	Neptunium (237)	Plutonium (244)	Americium (243)	Curium (247)	Berkelium (247)	Californium (251)	Einsteinium (252)	Fermium (257)	Mendelevium (258)	Nobelium (259)	Lawrencium (262)
Gro	ound-state	Ionization Energy (eV)	``	(Rn]6d7s <sup>2</sup> 5.17	[Rn]6d <sup>2</sup> 7s <sup>2</sup> 6.3067	[Rn]5f <sup>2</sup> 6d7s <sup>2</sup> 5.89	[Rn]5f <sup>3</sup> 6d7s <sup>2</sup> 6.1941	[Rn]5f <sup>4</sup> 6d7s <sup>2</sup> 6.2657	[Rn]5f <sup>5</sup> 7s <sup>2</sup> 6.0262	[Rn]5f <sup>7</sup> 7s <sup>2</sup> 5,9738	[Rn]5f <sup>7</sup> 6d7s <sup>2</sup> 5.9915	[Rn]5f <sup>9</sup> 7s <sup>2</sup> 6.1979	[Rn]5f <sup>10</sup> 7s <sup>2</sup> 6.2817	[Rn]5f <sup>11</sup> 7s <sup>2</sup> 6.42	[Rn]5f <sup>12</sup> 7s <sup>2</sup> 6.50	[Rn]5f <sup>13</sup> 7s <sup>2</sup> 6.58	[Rn]5f <sup>14</sup> 7s <sup>2</sup> 6.65	[Rn]5f <sup>14</sup> 7s <sup>2</sup> 7p? 4.9 ?

### **Trapped ion quantum bits**

lons with optical transition to metastable level: <sup>40</sup>Ca<sup>+</sup>,<sup>88</sup>Sr<sup>+</sup>,<sup>172</sup>Yb<sup>+</sup>



### "optical qubit"

qubit manipulation requires ultrastable laser

$$\Psi = \alpha |\!\downarrow\rangle + \beta |\!\uparrow\rangle$$

lons with hyperfine structure: <sup>9</sup>Be<sup>+</sup>, <sup>25</sup>Mg<sup>+</sup>, <sup>43</sup>Ca<sup>+</sup>, <sup>111</sup>Cd<sup>+</sup>, <sup>171</sup>Yb<sup>+</sup>...



"hyperfine qubit"

qubit manipulation with microwaves or lasers (Raman transitions)

### **Qubit manipulation and measurement**



### **Experimental sequence**



1. Initialization in a pure quantum state

### **Experimental sequence**



- 1. Initialization in a pure quantum state
- 2. Quantum state manipulation on  $S_{1/2} D_{5/2}$  transition

## **Experimental sequence**



- 1. Initialization in a pure quantum state
- 2. Quantum state manipulation on  $S_{1/2} D_{5/2}$  transition
- 3. Quantum state measurement by fluorescence detection

Two ions:

Spatially resolved detection with CCD camera:



50 experiments / s

Repeat experiments 100-200 times

## **Ion-laser interaction**



## **Qubit superposition states**

Schrödinger picture:

 $|\psi(t=0)\rangle \propto |\downarrow\rangle + |\uparrow\rangle \longrightarrow |\psi(t)\rangle \propto |\downarrow\rangle + e^{-i\omega_0 t}|\uparrow\rangle$ 

Phase evolution: for optical qubits  $\omega_0 \sim 10^{15} \text{ s}^{-1}$ 

Interaction picture:

 $|\psi(t)
angle \propto |\!\downarrow
angle \!+ |\!\uparrow
angle$  independent of time

 $|\psi(t)\rangle = \cos(\theta/2)|\downarrow\rangle + e^{i\phi}\sin(\theta/2)|\uparrow\rangle$ 

The phase  $\phi$  of the superposition compares two oscillatory phenomena:

- Evolution of the Bloch vector in time
- Evolution of the electromagnetic field of the laser exciting the qubit

### Ramsey spectroscopy for phase estimation



#### **Resonant excitation in Bloch sphere picture**



Example: 
$$\phi=0 \longrightarrow H=\hbarrac{\Omega}{2}\sigma_x$$

Time evolution operator:

$$U = \exp\left(-\frac{i}{\hbar}Ht\right) = \exp\left(-i\frac{\Omega t}{2}\sigma_x\right) = \cos\left(\frac{\Omega t}{2}\right) - i\sin\left(\frac{\Omega t}{2}\right)\sigma_x$$

For 
$$\theta = \Omega t = \pi/2$$
  
 $U|\downarrow\rangle = \frac{1}{\sqrt{2}}(I - i\sigma_x) = \frac{1}{\sqrt{2}}(|\downarrow\rangle - i|\uparrow\rangle)$ 

### **Resonant qubit excitation**



### **Qubit manipulation**

**Resonant excitation** 

$$\begin{array}{c|c} |\uparrow\rangle \\ \hline & \downarrow \\ |\downarrow\rangle \end{array} \begin{array}{c} \omega_L = \omega_0 \\ H \propto \sigma_x \\ \text{or} \\ H \propto \sigma_y \end{array}$$

#### **Off-resonant excitation**



$$H\propto\sigma_z$$

ac-Stark shifts shift qubit transition frequency

Arbitrary Bloch sphere rotations can be synthesized by a combination of laser pulses.

### Laser setup for manipulating the qubit



### Addressing of individual ions with a focussed laser beam





- inter ion distance: ~ 4 µm
- addressing waist: ~ 2 µm
- < 0.1% intensity on neighbouring ions

### **Measuring qubits**



detection errors  $\sim 0.1\%$ 

### **Further quantum measurements**





Coupling internal and vibrational degrees of freedom

Harmonic oscillator Quantum bit



motional states  $|0\rangle, |1\rangle, |2\rangle, |3\rangle, \dots$ 

internal states  $|\uparrow\rangle, |\downarrow\rangle$ 

### **Trapped-ion laser interactions**



qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, H \propto \sigma_y$$



### **Trapped-ion laser interactions**



qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, H \propto \sigma_y$$



#### qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$
  
 $H \propto \sigma_+ a + \sigma_- a^{\dagger}$ 

### **Trapped-ion laser interactions**



 $\hbar\omega_0$ 

qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, H \propto \sigma_y$$

 $\begin{array}{c|c} & |\uparrow, 2\rangle \\ \hline |\uparrow, 0\rangle & \hline & \sqrt{2}\eta\Omega \\ \hline & \eta\Omega \\ \hline & \eta\Omega \\ \hline & \eta\Omega \\ \hline & \downarrow, 1\rangle \\ \hline & \downarrow, 2\rangle \end{array}$ 

#### qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$
  
 $H \propto \sigma_+ a + \sigma_- a^{\dagger}$ 

$$\omega_{laser} = \omega_0 + \nu$$
$$H \propto \sigma_+ a^{\dagger} + \sigma_- a$$

### **Sideband excitation**

$$H^{(i)} = \frac{\hbar\Omega}{2}\sigma_{+}e^{-i\delta t + i\phi} \left(I + i\eta(a^{\dagger}e^{i\nu t} + ae^{-i\nu t}) + \mathcal{O}(\eta^{2})\right) + \text{h.c.}$$

TO

<u>Red sideband:</u>  $\delta = -\nu$ 

$$H_{int} = \frac{h\Omega}{2} i\eta \{\sigma_+ a e^{+i\phi} - \sigma_- a^{\dagger} e^{-i\phi}\}$$

$$|g,n
angle \longleftrightarrow |e,n-1
angle$$

'Jaynes-Cummings-Hamiltonian'

Coupling strength dependent on n

<u>Blue sideband:</u>  $\delta = +\nu$ 

$$|g,n
angle \longleftrightarrow |e,n+1
angle$$

$$H_{int} = \frac{\hbar\Omega}{2} i\eta \{\sigma_+ a^{\dagger} e^{+i\phi} - \sigma_- a e^{-i\phi}\}$$

'anti-Jaynes-Cummings-Hamiltonian'

Coupling strength dependent on n

#### Coherent excitation on the sideband

"Blue sideband" pulses:

 $|\uparrow,2\rangle$ 

 $|\uparrow,1
angle$ 



## Entangling a pair of trapped ions + detecting the entanglement



lon	Pulse length	Transition				







lon	Pulse length	Transition
1	π/2	blue sideband





lon	Pulse length	Transition
1	π/2	blue sideband
2	π	carrier





lon	Pulse length	Transition			
1	π/2	blue sideband			
2	π	carrier			
2	π	blue sideband			



### Measuring the entangled state

We hope to create the state

$$\psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$$

There are no pure states in experimental physics!

The state created in the experiment has to be described by a density matrix  $ho_{exp}$  .

How can we analyze the state  $\rho_{exp}$  we created?



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How can we analyze the state  $\rho_{exp}$  we created?

Option 1:

Measure how close  $\rho_{exp}$  is to  $|\psi\rangle$  by measuring the fidelity  $F = \langle \psi | \rho_{exp} | \psi \rangle$ 

Option 2:

Carry out measurements that allow to completely determine  $\rho_{exp}$ 

 $\rightarrow$  Quantum state tomography

#### **Reconstruction of the density matrix**

Representation of  $\rho$  as a sum of orthogonal observables  $A_i$ :

$$\rho = \sum_{i} \lambda_i A_i \text{ with } Tr(A_i A_j) = \delta_{ij}$$

 $\rho$  is completely detemined by the expectation values <A\_i> :

$$\langle A_j \rangle = Tr(\rho A_j) = \sum_i \lambda_i Tr(A_i A_j) = \lambda_j$$

For a two-ion system :  $A_i \in \{\sigma_i^{(1)} \otimes \sigma_j^{(2)}, \sigma_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}\}$ 

Joint measurements of all spin components

$$\sigma_i^{(1)}\otimes\sigma_j^{(2)}$$

$$\rho_R = \sum_{i=1}^{16} \langle A_i \rangle A_i$$

### **Measurement of spin expectation values**

Measurement of  $\langle \sigma_z \rangle$  : Fluorescence measurement

$$\langle \sigma_z \rangle = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}$$
  
=  $\rho_{DD} - \rho_{SS}$ 



Rotation of the Bloch sphere prior to state measurement:

$$\langle \sigma_z \rangle_{U\rho U^{-1}} = Tr(\sigma_z U\rho U^{-1})$$
$$= Tr(\underbrace{U^{-1}\sigma_z U\rho}_{\stackrel{!}{=}\sigma_x})$$





### **Bell state analysis**

Measurement of  $\langle \sigma_z \rangle$ :

$$\langle \sigma_z \rangle \qquad = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}$$

Measurement of  $\langle \sigma_x \rangle$  ,  $\langle \sigma_y \rangle$  :

Rotation of the Bloch sphere prior to state measurement:

prepare Bell state 200 repetitions no rotation  $\langle \sigma_z^{(1)} \rangle, \langle \sigma_z^{(2)} \rangle, \langle \sigma_z^{(1)} \sigma_z^{(2)} \rangle$ measure prepare Bell state 200 repetitions ion #1, y - rotation ion #2, identity  $\langle \sigma_x^{(1)} \rangle, \langle \sigma_z^{(2)} \rangle, \langle \sigma_x^{(1)} \sigma_z^{(2)} \rangle$ measure **9 different** settings prepare Bell state 200 repetitions ion #1, x - rotation ion #2, x - rotation  $\langle \sigma_{y}^{(1)} \rangle, \langle \sigma_{y}^{(2)} \rangle, \langle \sigma_{y}^{(1)} \sigma_{y}^{(2)} \rangle$ measure

#### **Example: Tomography of a qubit**



The experimental procedure prepares the state  $\rho_{prep}$ 

$$ho_{prep} = rac{1}{2} (I + \langle \sigma_x 
angle \sigma_x + \langle \sigma_y 
angle \sigma_y + \langle \sigma_z 
angle \sigma_z)$$

Reconstruction by estimation of  $\langle \sigma_x \rangle, \langle \sigma_y \rangle, \langle \sigma_z \rangle$ using a finite number of copies of the state:

$$egin{aligned} s_{m{z}} &= rac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \;, \; s_{m{x}} = \dots, \; s_{m{y}} = \dots \ &
ho_{tomo} &= rac{1}{2}(I + s_{m{x}}\sigma_{m{x}} + s_{m{y}}\sigma_{m{y}} + s_{m{z}}\sigma_{m{z}}) 
eq 
ho_{prep} \end{aligned}$$



*Ptomo* might not be within the Bloch sphere !

### **Maximum likelihood estimation**

Is  $\rho_R = \sum_i \langle A_i \rangle A_i$  positive semidefinite ? ... not necessarily:

with a finite number of measurements, we can only estimate expectation values

#### Maximum likelihood estimation:

(Hradil '97, Banaszek '99)

In N experiments, the quantum state is projected onto the outcomes  $|y_j\rangle$ .

 $f_j$  : relative frequency of the outcome  $|y_j
angle$ 

On the set of density matrices  $\rho$ , look for the one that maximizes

$$\mathcal{L}(\rho) = \prod_{j} \langle y_j | \rho | y_j \rangle^{N f_j}$$

Maximize 
$$L(\rho) = \sum_j f_j \log \langle y_j | \rho | y_j \rangle$$

### Bell state reconstruction with maximum likelihood estimation



- State fidelity:  $\langle \psi | \rho_{tomo} | \psi 
  angle = 0.91$
- Violation of a Bell inequality:  $\langle \rho_x^{(1)} \rho_{x-z}^{(2)} \rangle + \langle \rho_x^{(1)} \rho_{x+z}^{(2)} \rangle + \langle \rho_z^{(1)} \rho_{x-z}^{(2)} \rangle \langle \rho_z^{(1)} \rho_{x+z}^{(2)} \rangle = 2.52(6) > 2$
- Entanglement of formation:  $E(\rho_{tomo}) = 0.79$

Pulse sequence:







$$\begin{array}{ccc} |DSS1\rangle & ---- |SDS1\rangle & ---- |SSD1\rangle \\ |DSS0\rangle & ---- |SDS0\rangle & ---- |SSD0\rangle \end{array}$$

 $\ket{DDS,\mathsf{0}}$ 







### W – state: |SDD> + |DSD> + |DDS>



#### **Reconstructed W – state: experiment and theory**

$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$$



experimental result

theoretical expectation

C. F. Roos et al., Science **304**, 1478 (2004)

#### **More ions: Four-ion W-states**





#### **Five-ion W-states**

 $\Psi_5 = \frac{1}{\sqrt{5}} (|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + |SDDDD\rangle + |SDDDD\rangle)$ 



### **Six-ion W-states**

 $\Psi_{6} = \frac{1}{\sqrt{6}} (|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$ 



729 settings, measurement time: 40 minutes

#### **Seven-ion W-states**



2187 settings, measurement time: 2 hours

### **Eight-ion W-states**



H. Häffner et al., Nature 438, 643 (2005)

6561 settings, measurement time: 10 hours