Quantum information processing with trapped ions

Trapped ion experiments:

- Exploring quantum physics
- Elements of quantum computing
- Quantum simulations
- Precision spectroscopy with entangled states

Christian Roos
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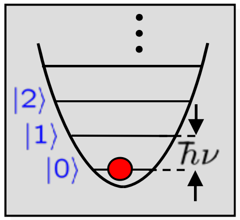
Quantum physics with trapped ions

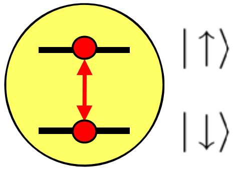
A single trapped ion: Realization of a quantum harmonic oscillator

Motional degrees of freedom



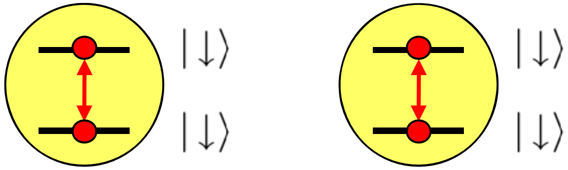
A single trapped ion: Realization of a quantum bit

Internal degrees of freedom



$$H \propto \sigma_x, \; H \propto \sigma_y$$

Strings of trapped ion: Entangled quantum bits



$$\Psi \propto |\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle$$

1952: Experiments with single atoms ?

In the first place it is fair to state that we are not *experimenting* with single particles, anymore than we can raise Ichtyosauria in the zoo.

..., this is the obvious way of registering the fact, that we *never* experiment with just *one* electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this envariably entails ridiculous consequences.

British Journal of the Philosophy of Science III (10), (1952)

E. Schrödinger

1953: Invention of the Paul trap

DEUTSCHES PATENTAMT

PATENTSCHRIFT

Nr. 944 900

KLASSE 42l GRUPPE 3 09

INTERNAT. KLASSE G 01 n ———

P 11054 IX/42l

Dr. Wolfgang Paul und Dr. Helmut Steinwedel, Bonn
sind als Erfinder genannt worden

Dr.-Ing. Wolfgang Paul, Bonn

Verfahren zur Trennung bzw. zum getrennten Nachweis von Ionen verschiedener spezifischer Ladung

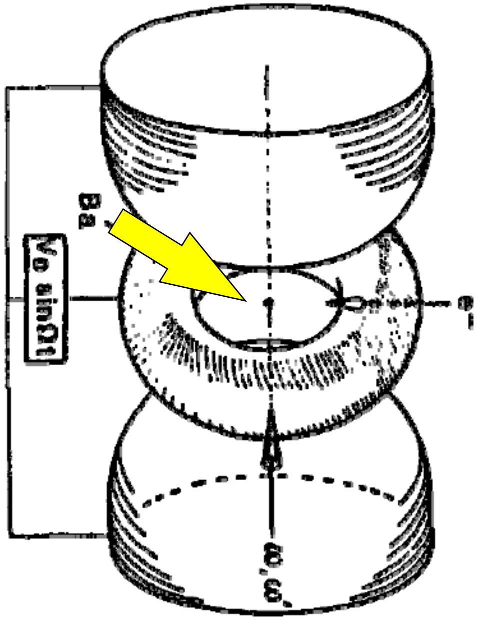
Patentiert im Gebiet der Bundesrepublik Deutschland vom 24. Dezember 1953 an

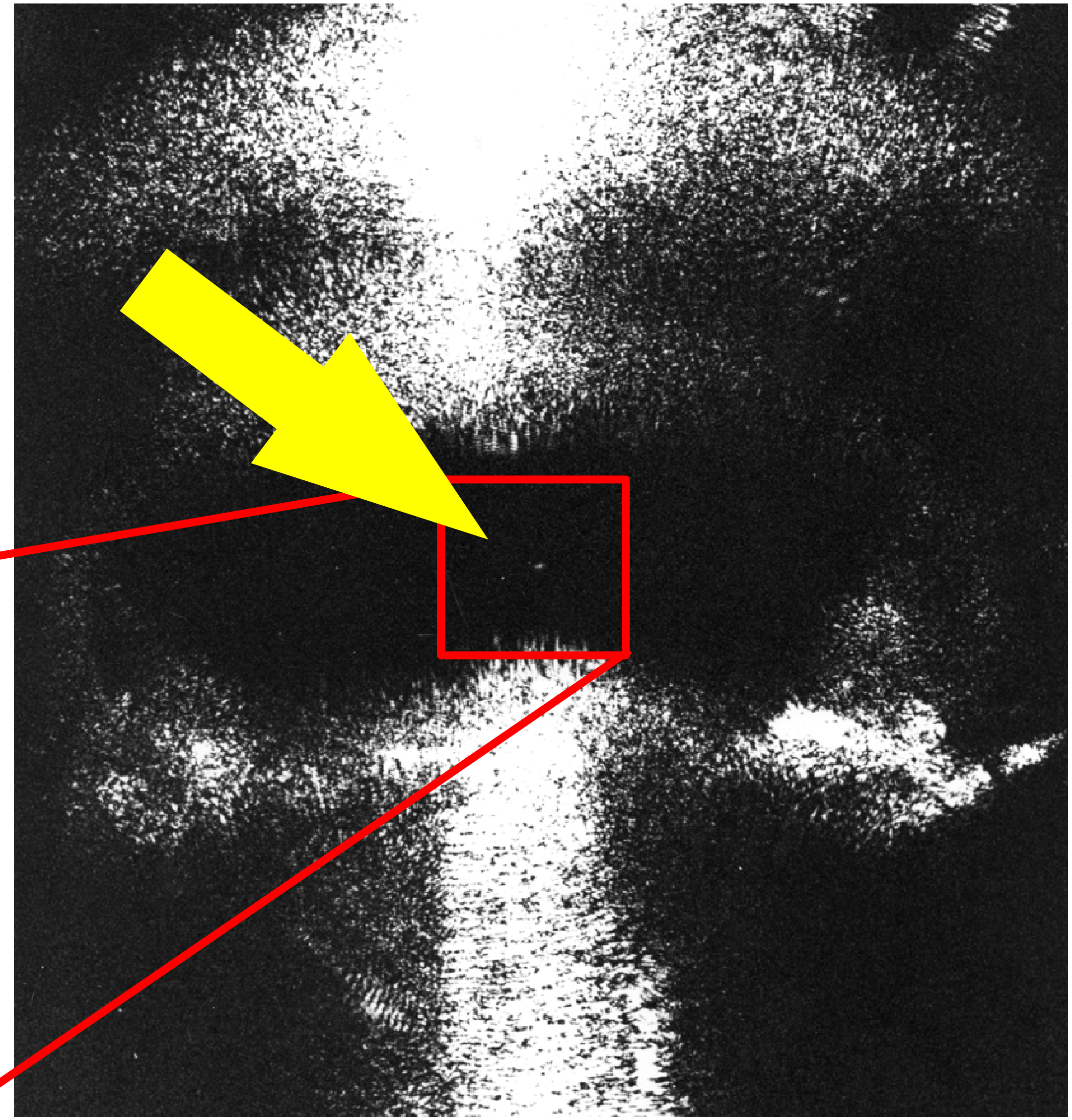
Patentanmeldung bekanntgemacht am 5. Januar 1956

Patenterteilung bekanntgemacht am 7. Juni 1956

W. Paul

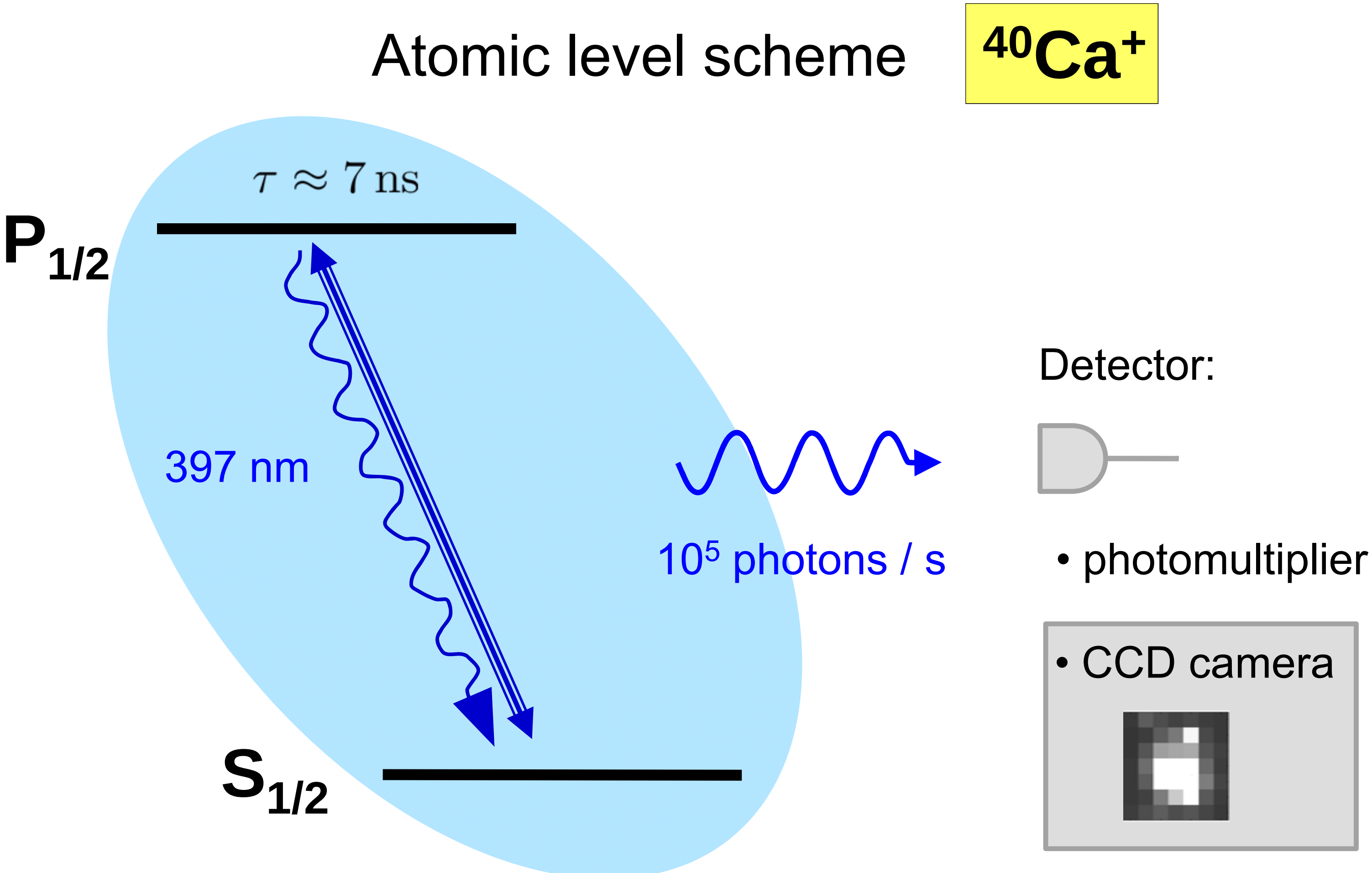
1978: Observation of single trapped ions



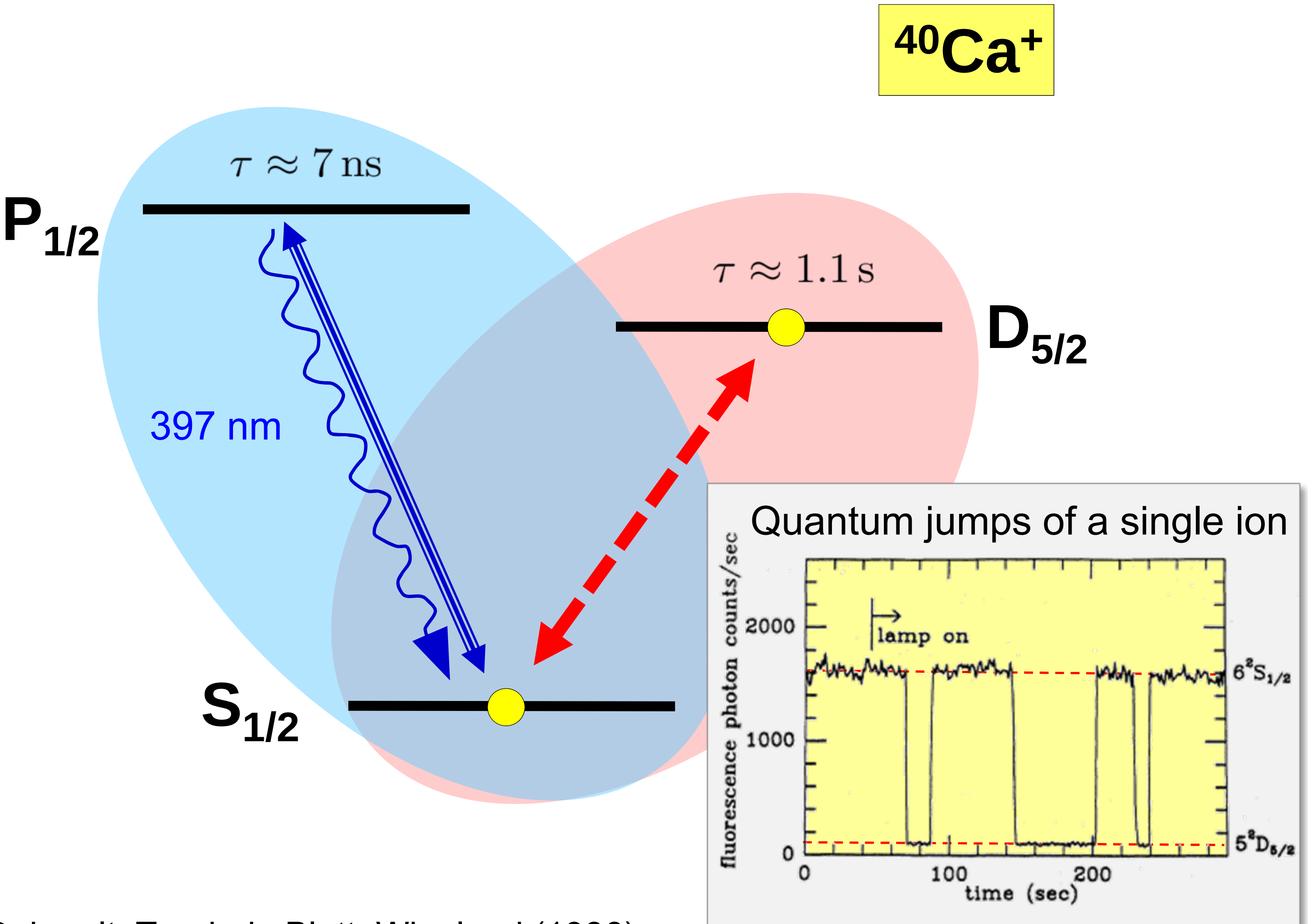


W. Neuhauser *et al.*, PRL **41**, 233 (1978), PRA (1980)

Fluorescence detection



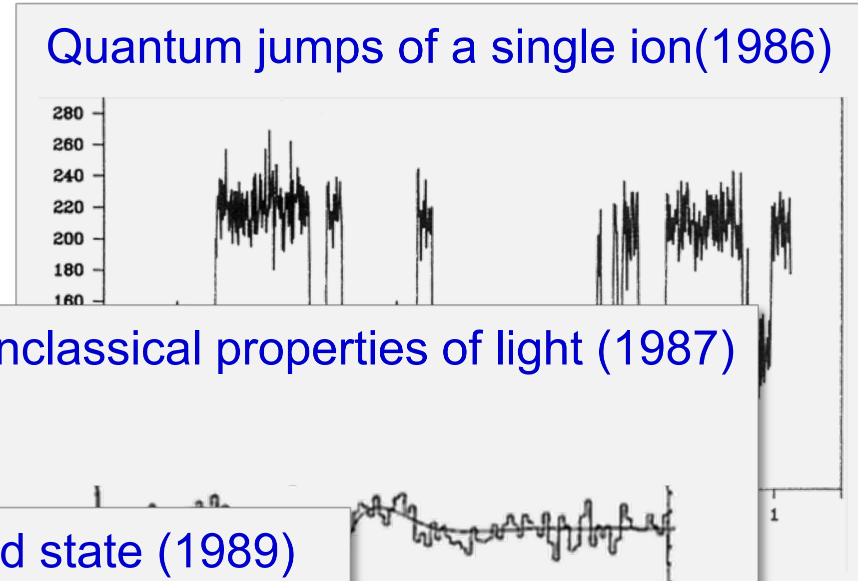
Detection of single absorption/emission events



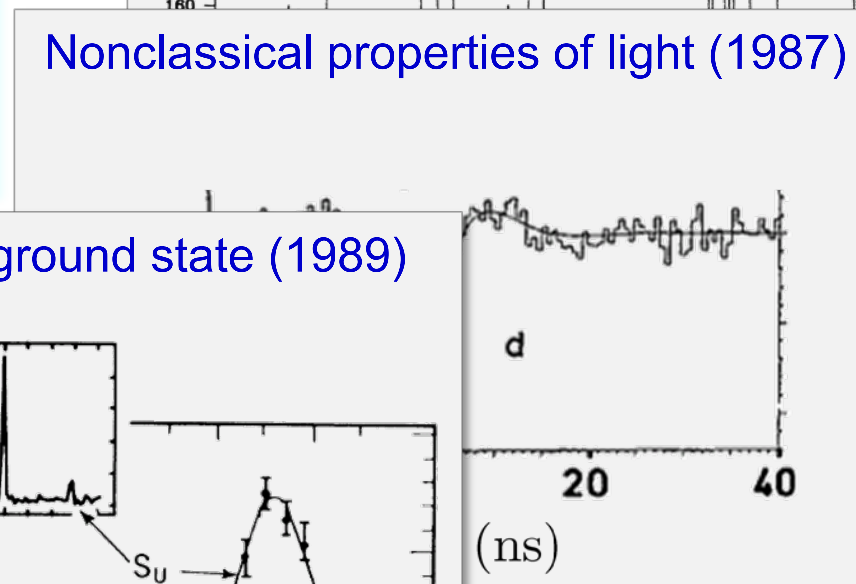
Experiments: Dehmelt, Toschek, Blatt, Wineland (1986)

Quantum physics with a single trapped ion

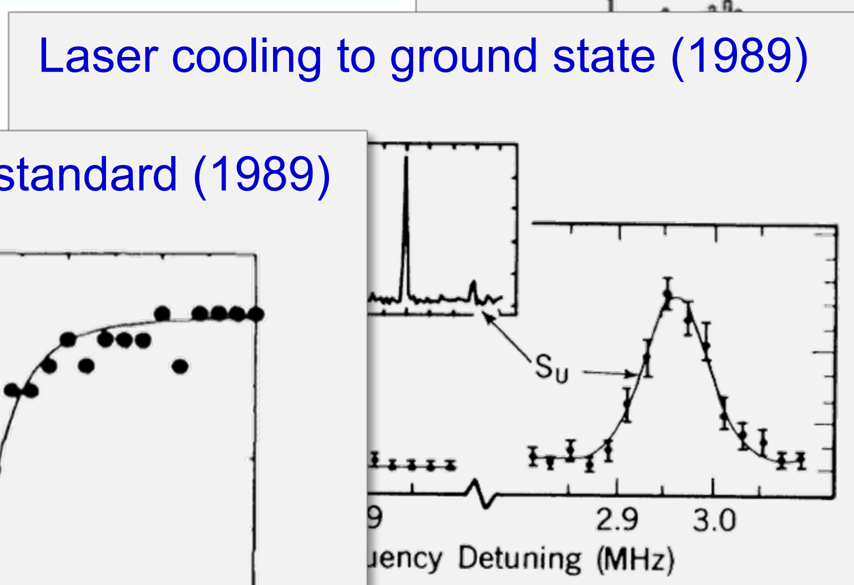
- Ion storage time: days
- Isolation from environment
- Near-unit quantum state detection
- Excellent quantum control



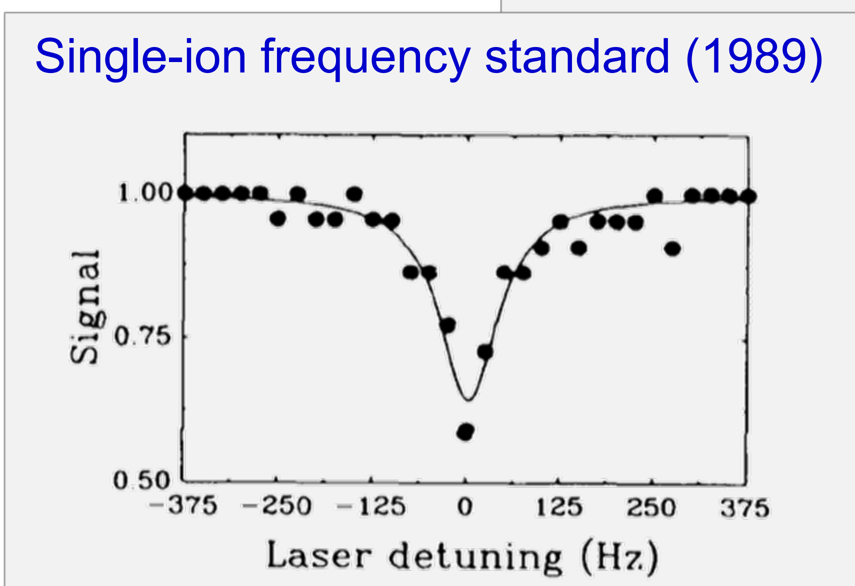
Quantum jumps of a single ion(1986)



Nonclassical properties of light (1987)



Laser cooling to ground state (1989)



Single-ion frequency standard (1989)

Quantum physics with a single trapped ion

Important tools:

Traps in UHV systems $\longrightarrow$ Isolation + long storage times

Narrow-band lasers $\longrightarrow$ Laser cooling

Photomultipliers $\longrightarrow$ Efficient quantum state detection

Areas of physics:

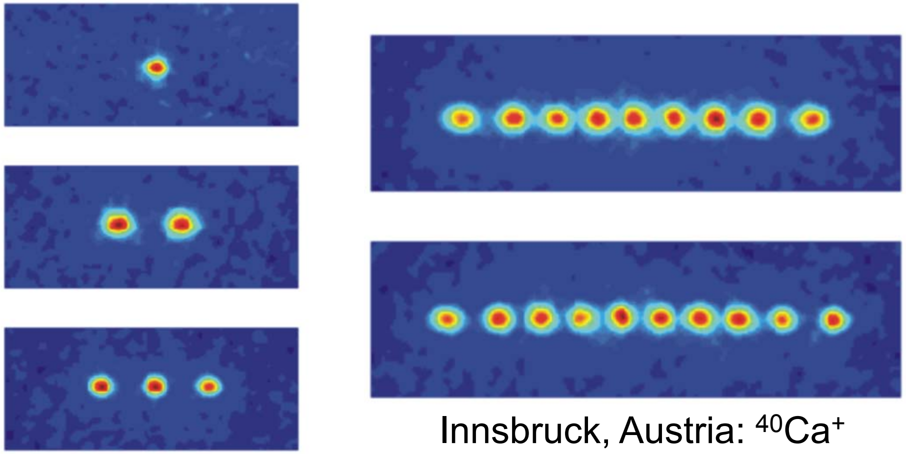
- Quantum optics: a single ion interacting with single photons
- Tests of quantum physics
- Single-ion optical clocks $\rightarrow$ Alastair Sinclair's lectures

Quantum physics with ion strings

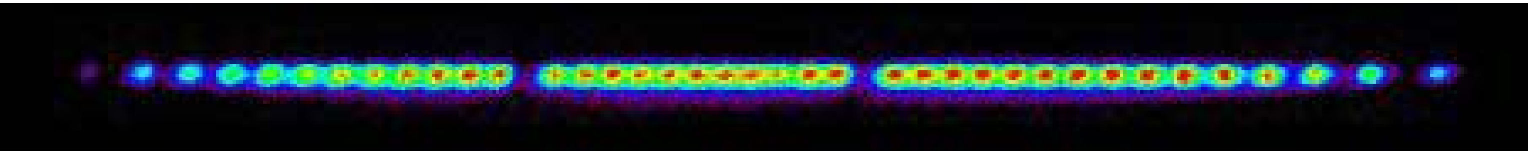
Ion crystals

At low temperatures:

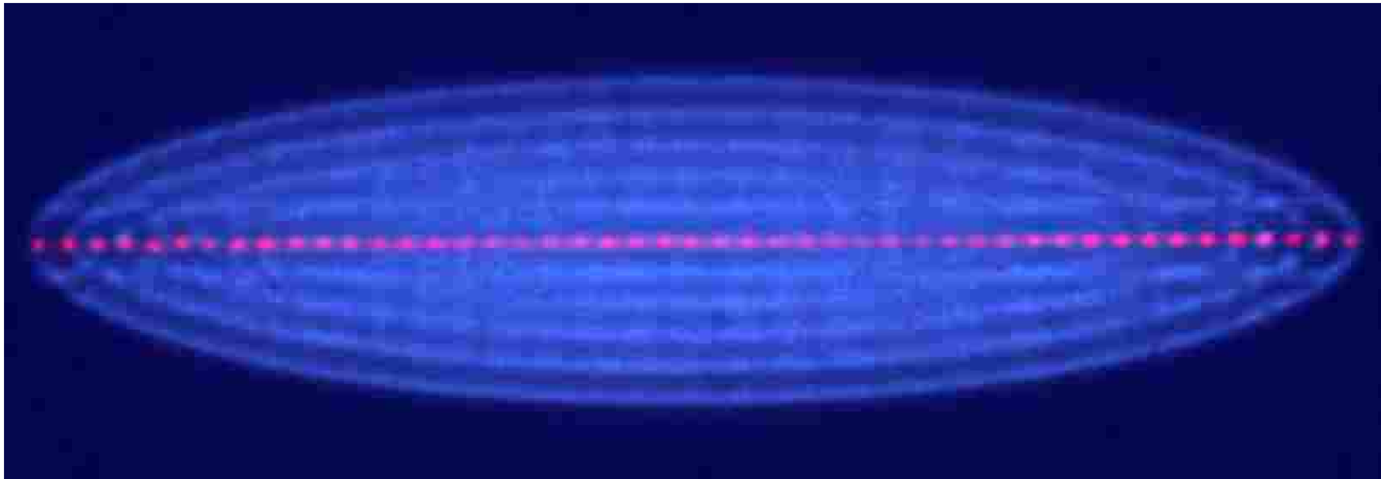
Equibrium positions determined by trapping forces and mutual Coulomb repulsion



Innsbruck, Austria: $^{40}Ca^+$



Boulder, USA: Hg^+



Aarhus, Denmark:
$^{40}Ca^+$ (red) and $^{24}Mg^+$ (blue)

Quantum physics with ion strings

Ion crystals

At low temperatures:

Equibrium positions determined by trapping forces and mutual Coulomb repulsion

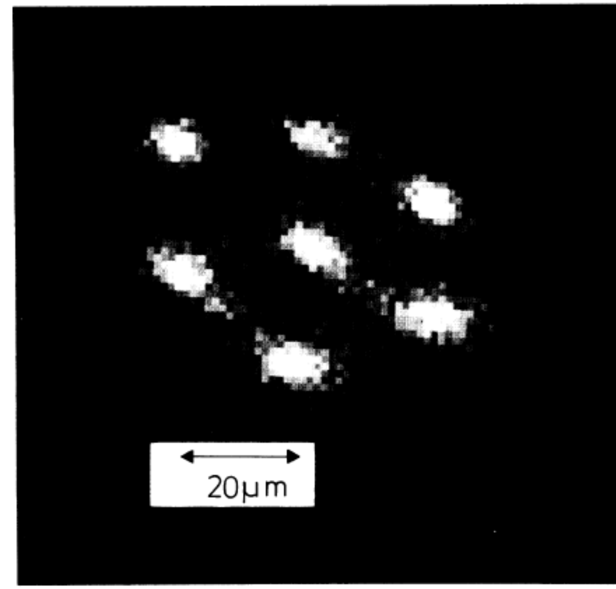


FIG. 3. Crystalline structure of seven $^{24}Mg^+$ ions observed

VOLUME 59, NUMBER 26 PHYSICAL REVIEW LETTERS 28 DECEMBER 1987

Observation of a Phase Transition of Stored Laser-Cooled Ions

F. Diedrich, E. Peik, J. M. Chen, W. Quint, and H. Walther
Max-Planck-Institut für Quantenoptik and Sektion Physik, Universität München, 8046 Garching, Federal Republic of Germany
(Received 8 July 1987)

Quantum physics and information processing

Journal of Statistical Physics, Vol. 22, No. 5, 1980

The Computer as a Physical System: A Microscopic Quantum Mechanical
as Represented by Tur

Paul Benioff[1,2]

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

Proc. R. Soc. Lond. A **425**, 73–90 (1989)

Quantum computational networks

By D. Deutsch

arXiv:quant-ph/9508027v2 25 Jan 1996

Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer*

Peter W. Shor†

Classical vs. quantum information processing

Bit:

Physical system with two distinct states 0 or 1

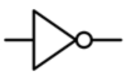
Quantum bit:

Two-level quantum system with state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Logic gates

Boolean logic operation



$0 \rightarrow 1$

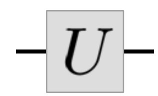
$1 \rightarrow 0$

$(\epsilon_1, \epsilon_2) \rightarrow \epsilon_1 \oplus \epsilon_2$

Quantum logic gate

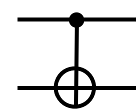
Unitary transformation

single qubit gate



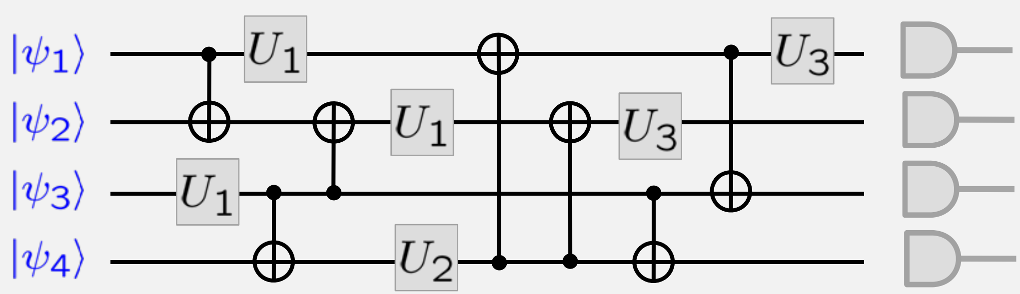
$|\psi\rangle \rightarrow U|\psi\rangle$

two-qubit gate



$|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$

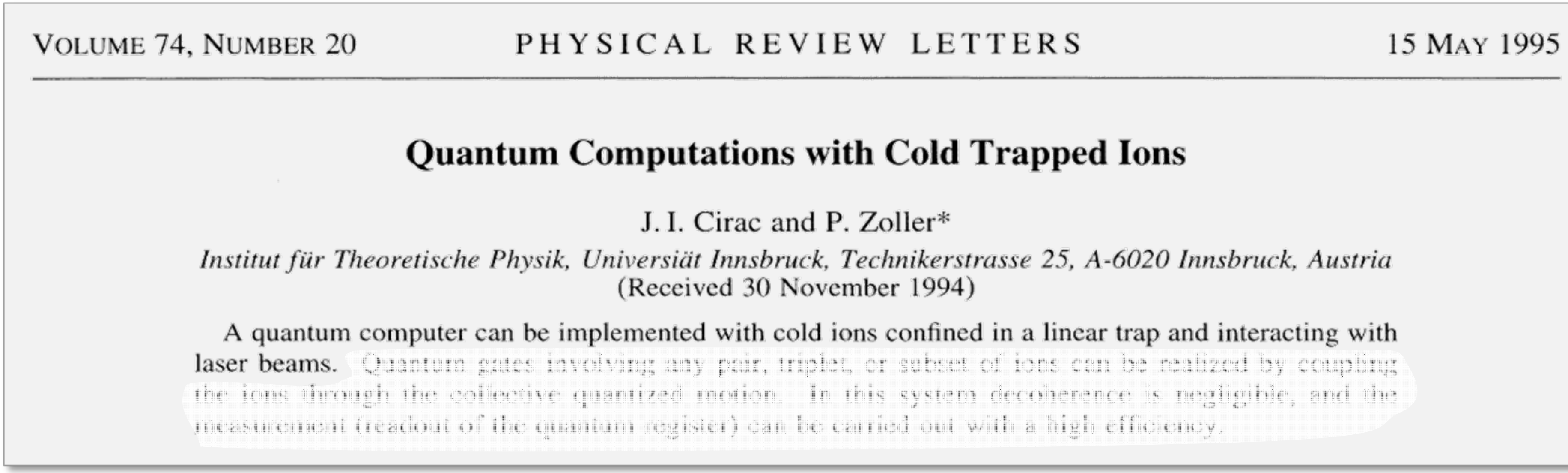
Quantum logic circuit



initialization

measurement

Trapped ions for quantum information processing

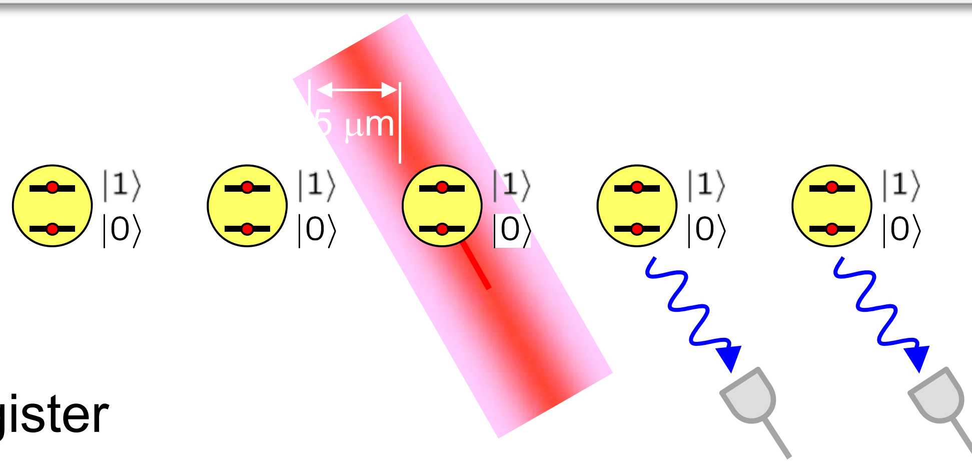

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*
Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

Ion string



- Qubit register
- State detection
- Single qubit gates

Trapped ions for quantum information processing

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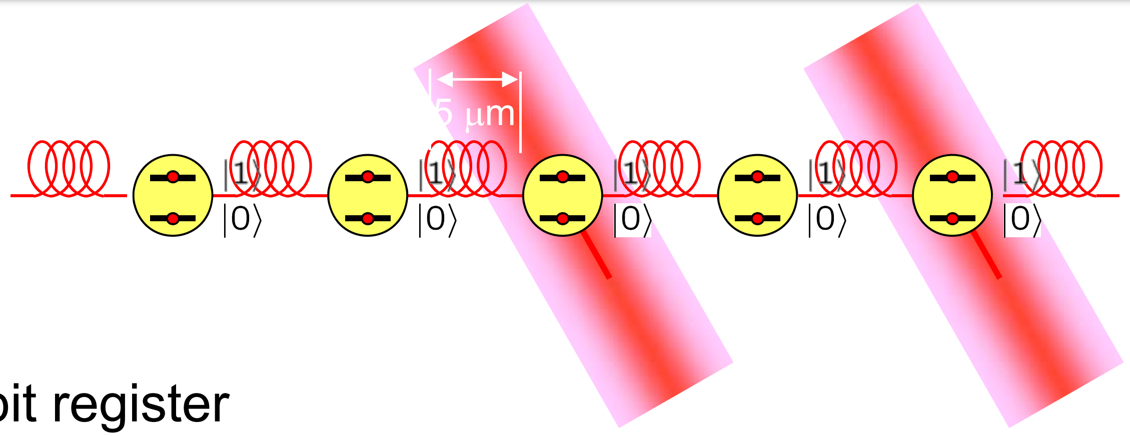
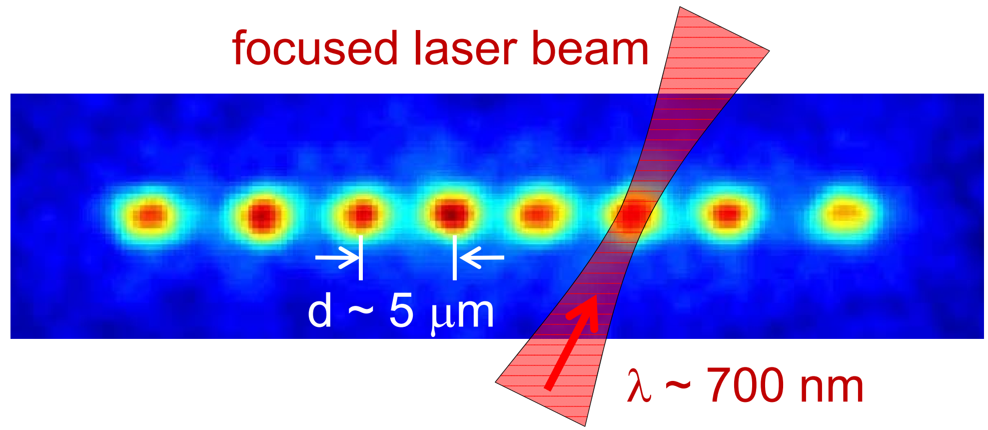


- Qubit register
- State detection
- Single qubit gates
- Entangling gates

Ion crystals: length scales

Trap frequencies:

$$\nu_z \quad \propto 1 \text{ MHz}$$
$$\nu_{x,y} \propto 5 \text{ MHz}$$



Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
d	$>$	λ	$>>$	z_0	$>>$	a_0
5 µm		700 nm		10 nm		50 pm

- individual addressing, spatially resolved fluorescence

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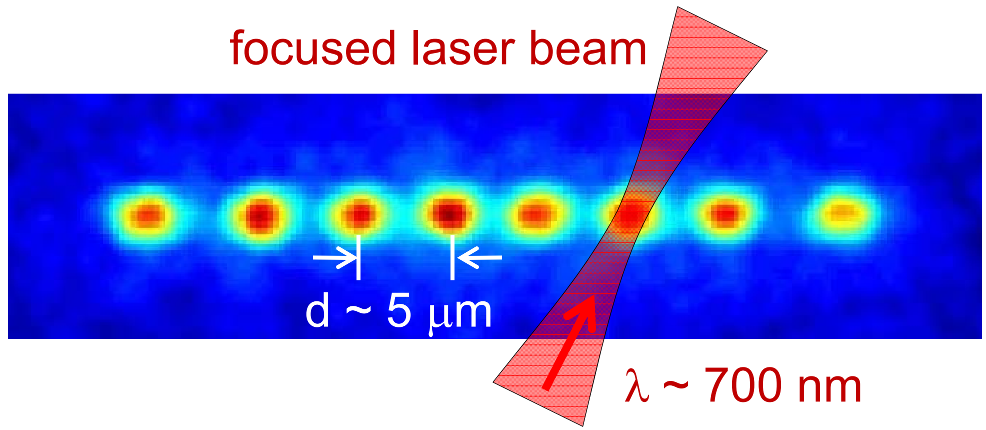

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- coupling internal and motional states by laser takes on simple form

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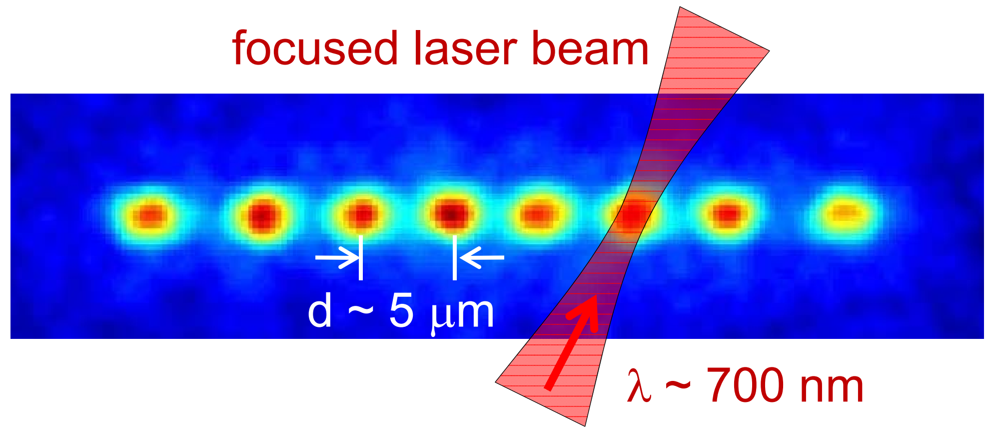

Length scales

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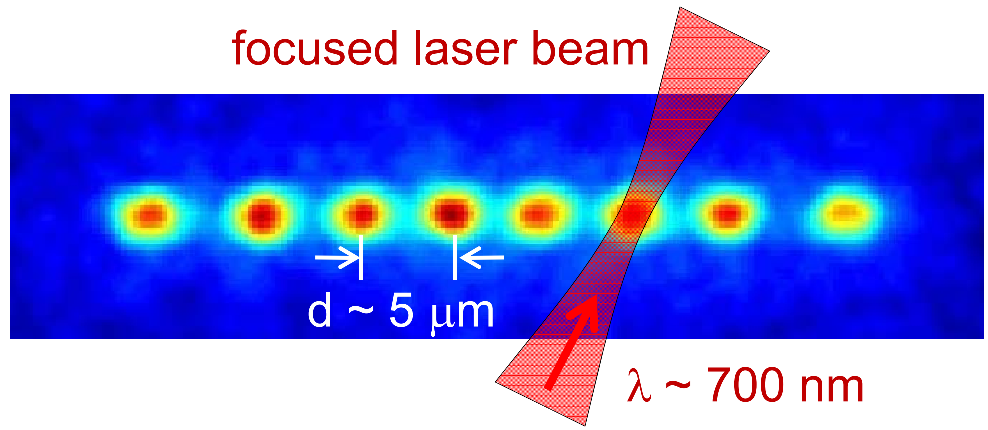
d	$>$	λ	$>>$	z_0	$>>$	a_0
5 μm		700 nm		10 nm		50 pm

- individual addressing, spatially resolved fluorescence
- coupling internal and motional states by laser takes on simple form
- no direct state-dependent interactions between ions

Ion crystals: length scales

Trap frequencies:

$$\nu_z \quad \propto 1\ \mathrm{MHz}$$
$$\nu_{x,y} \propto 5\ \mathrm{MHz}$$



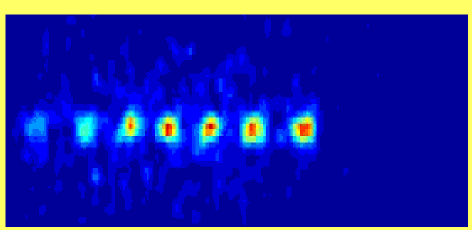
Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
d	>	λ	>>	z_0	>>	a_0
5 µm		700 nm		10 nm		50 pm

Vibrational modes

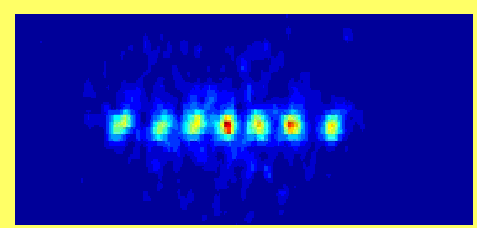
centre-of-mass mode

$$\nu = \nu_z$$

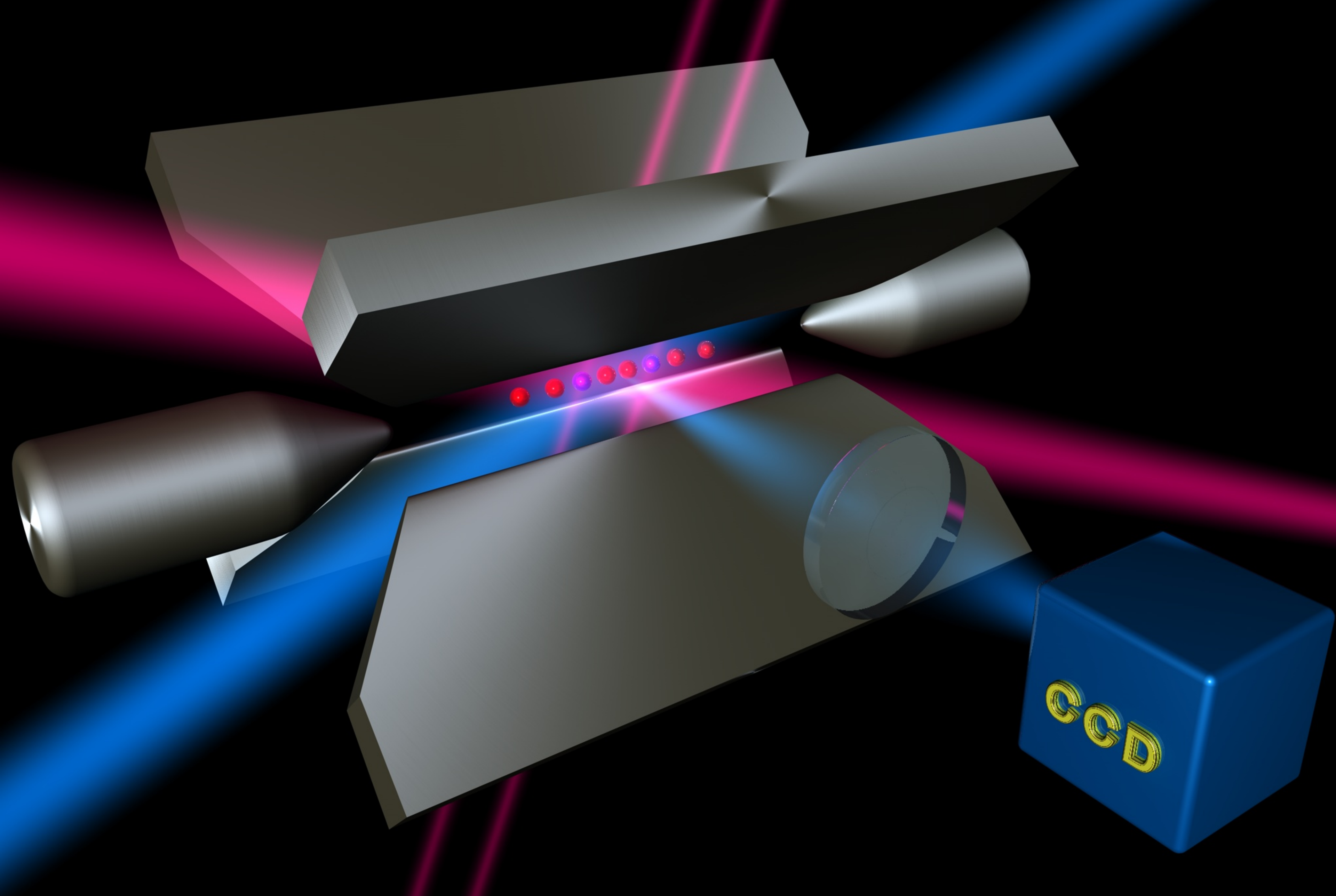


breathing mode

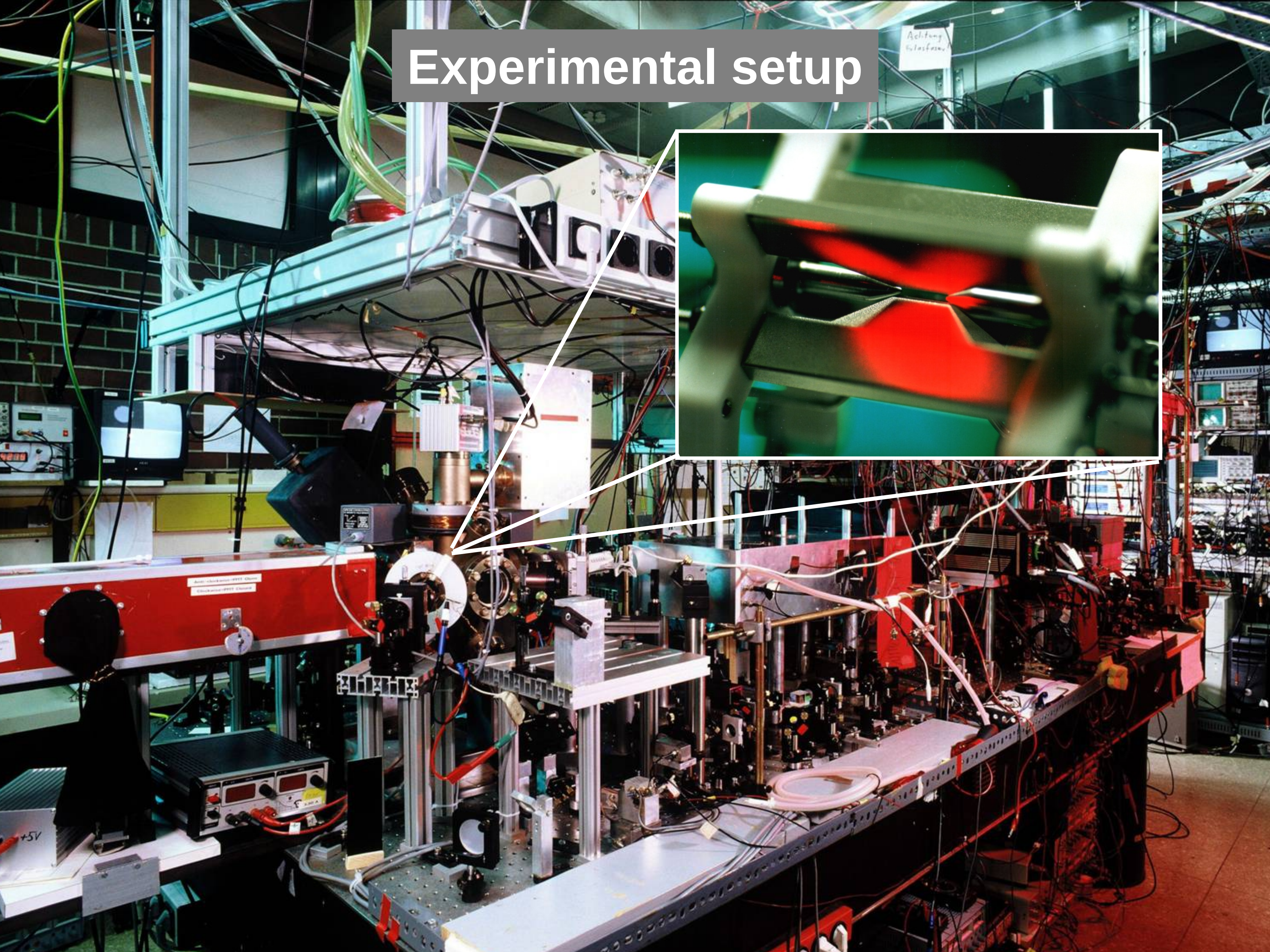
$$\nu = \sqrt{3}\,\nu_z$$



Experimental setup

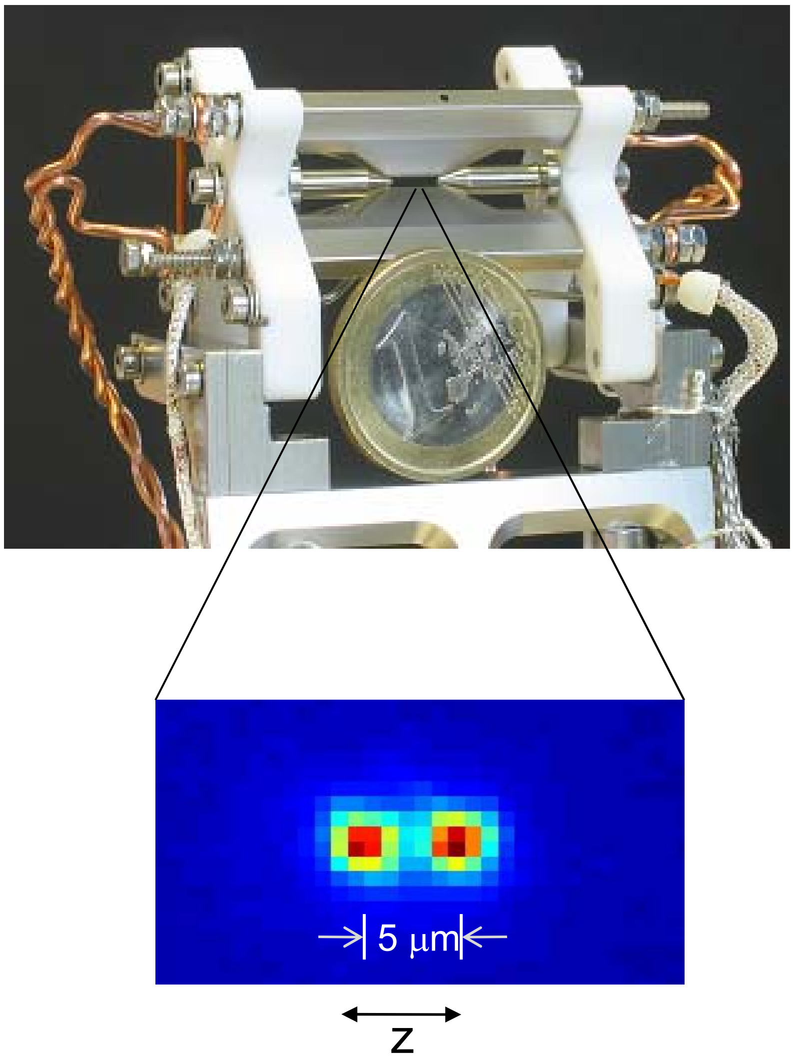


Experimental setup

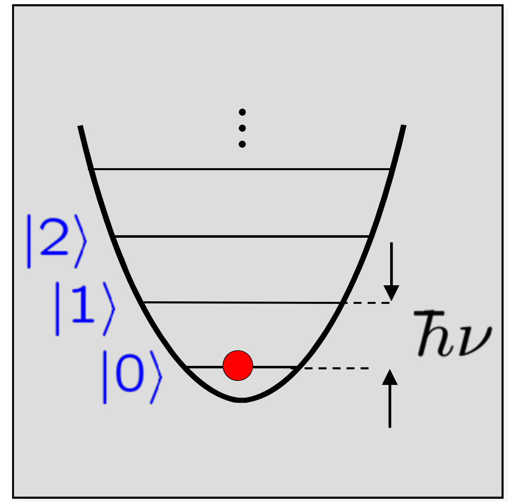


Linear ion trap

Linear ion trap



Harmonic trapping potential in the centre



Anisotropic potential with

$$\nu_z \ll \nu_x, \nu_y \qquad (\approx \text{1-5 MHz})$$

→ linear chain of ions

Quantum physics with ion strings

- based on Alastair's lectures: Theory of ion traps, laser cooling, two-level atoms, coupling of internal and vibrational states by laser light
- more emphasis on coherent atom-light interactions
- more ions: 2,3,4,…

Lecture plan:

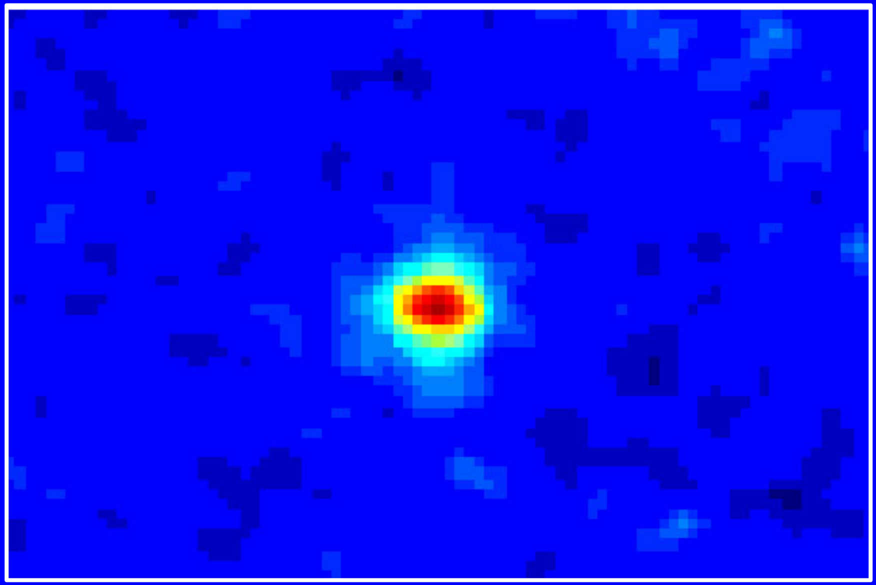
- Encoding of quantum information in trapped ions
- Manipulation and measurement of quantum information
- Creation of entanglement
- Analysis of multi-qubit states

today

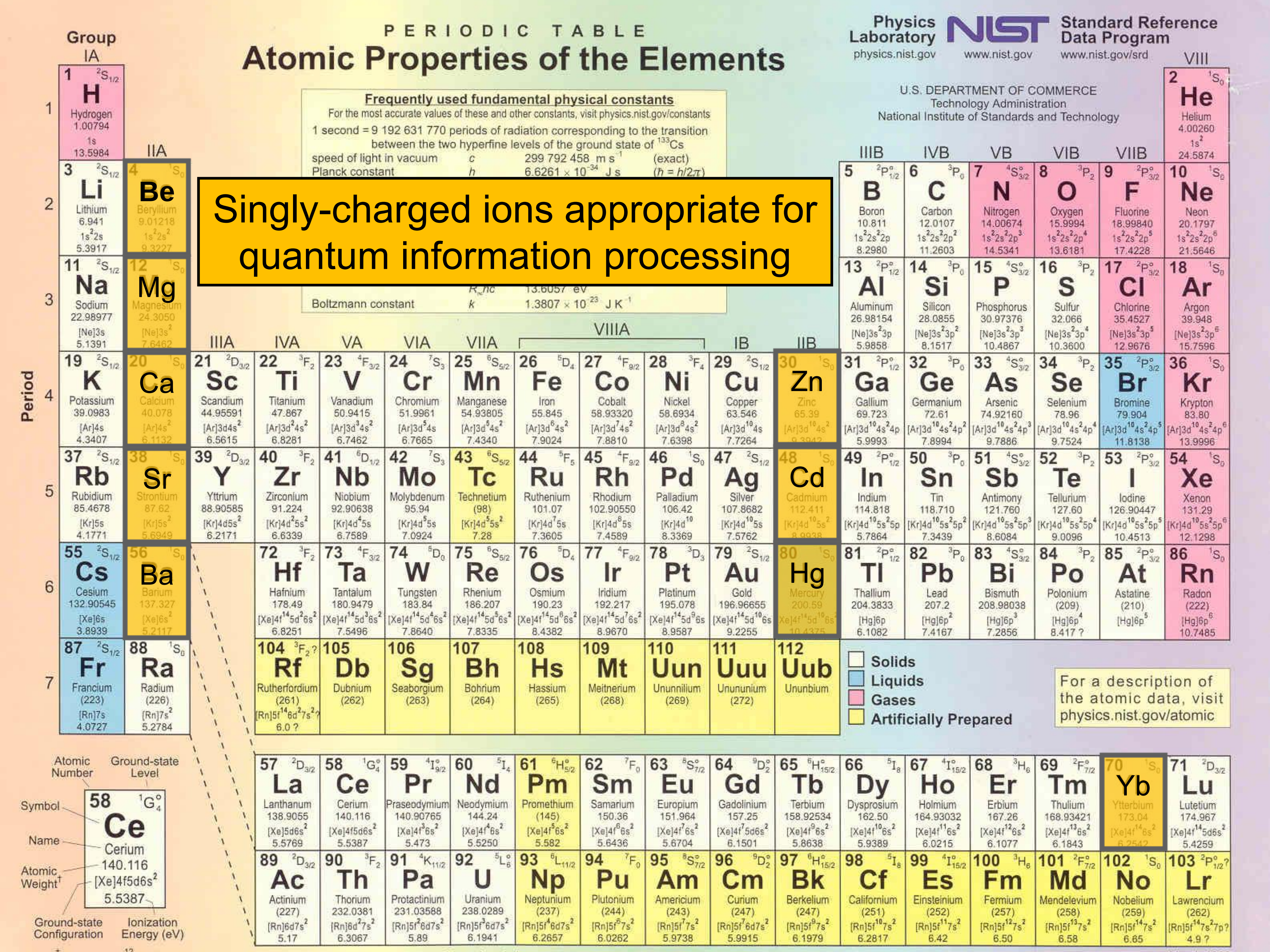
- Quantum gate operations
- Elements of quantum computing with trapped ions
- Entanglement for metrological applications
- Quantum simulation with trapped ions

Trapped-ion quantum bits

Encoding, manipulation and measurement

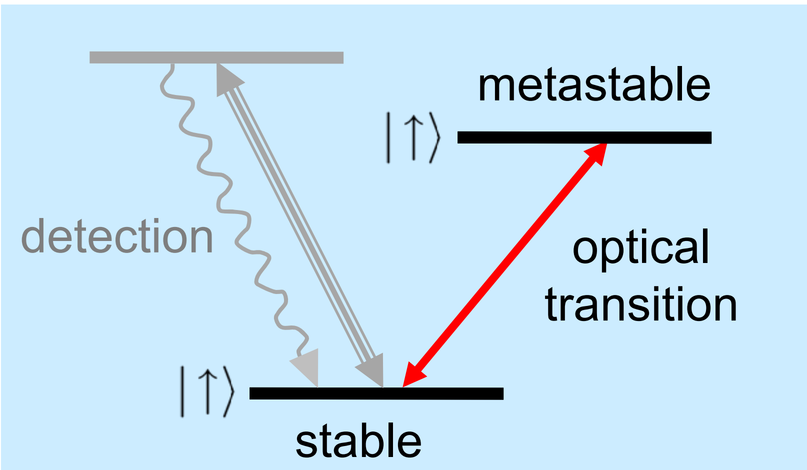


Singly-charged ions appropriate for quantum information processing



Trapped ion quantum bits

Ions with optical transition to metastable level: $^{40}Ca^+$, $^{88}Sr^+$, $^{172}Yb^+$

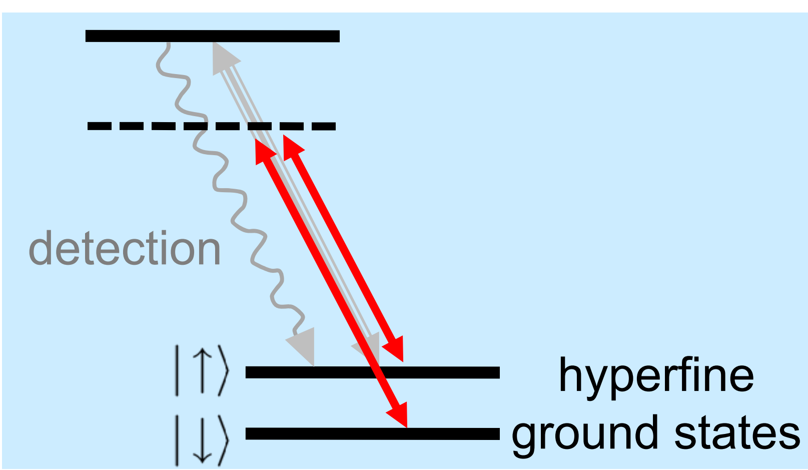


„optical qubit“

qubit manipulation requires ultrastable laser

$$\Psi = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$$

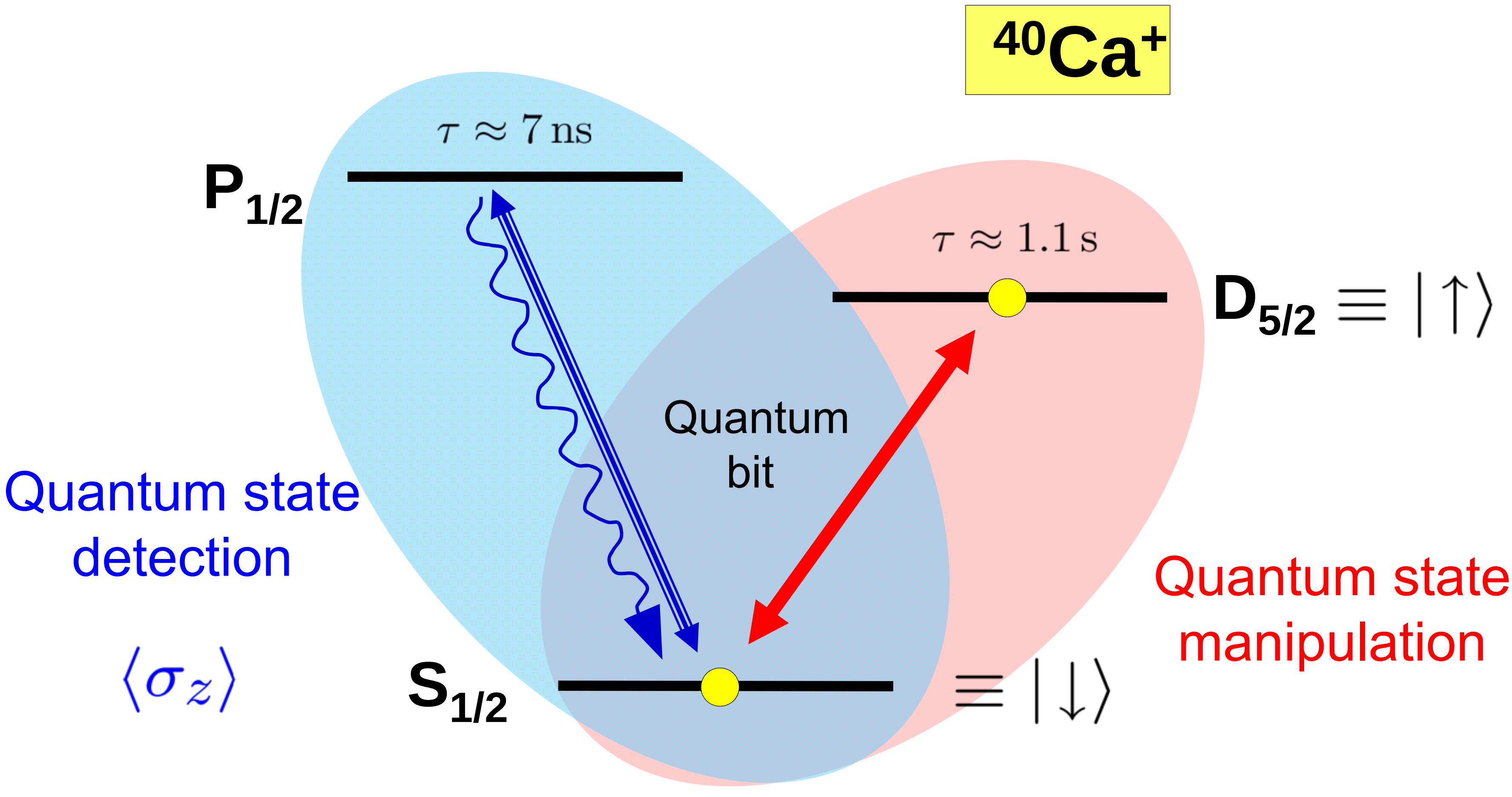
Ions with hyperfine structure: $^{9}Be^+$, $^{25}Mg^+$, $^{43}Ca^+$, $^{111}Cd^+$, $^{171}Yb^+$…



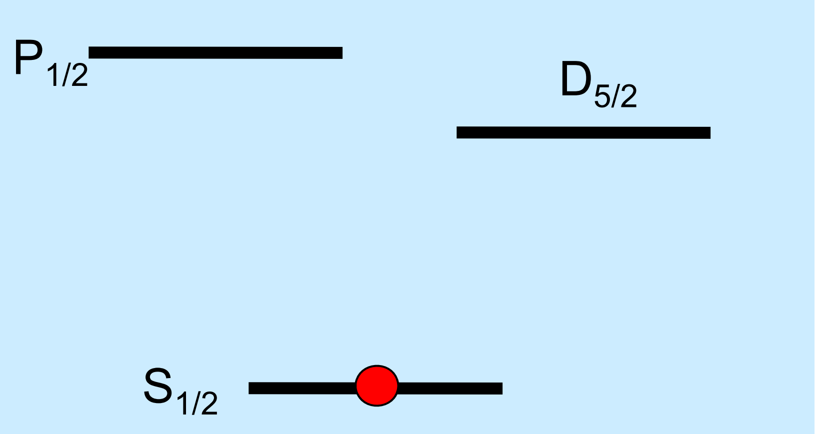
„hyperfine qubit“

qubit manipulation with microwaves or lasers (Raman transitions)

Qubit manipulation and measurement

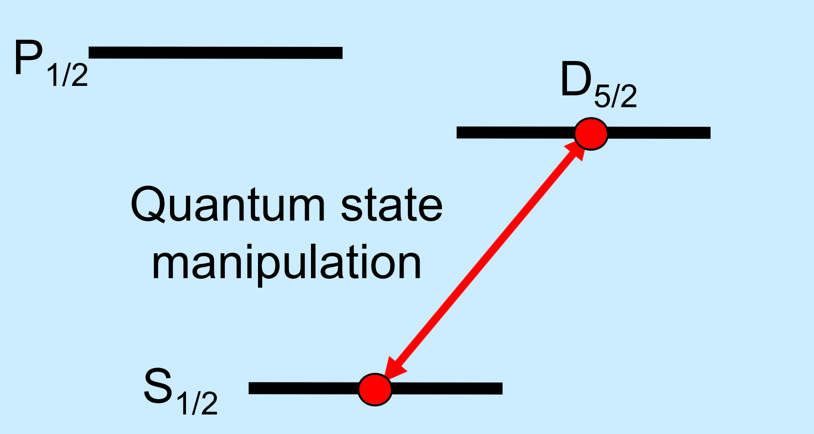


Experimental sequence



1. Initialization in a pure quantum state

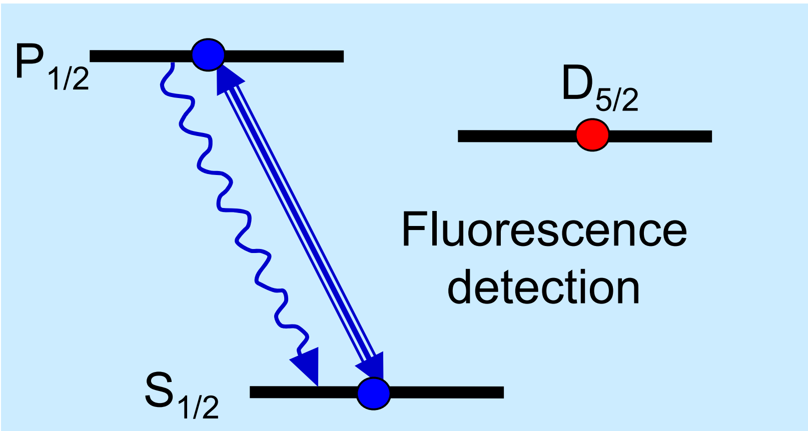
Experimental sequence



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition

Experimental sequence



1. Initialization in a pure quantum state
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Two ions:

Spatially resolved detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

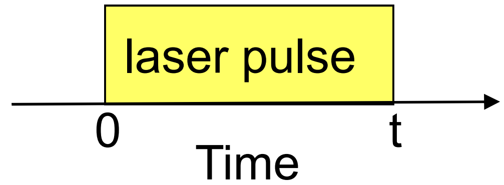
$|DD\rangle$

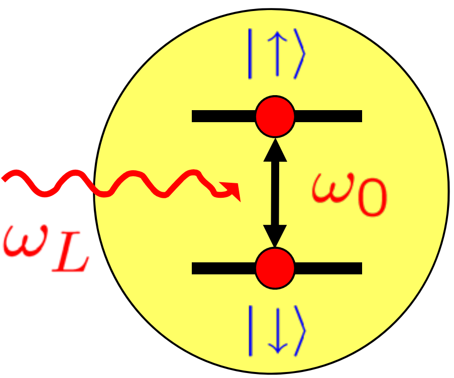
50 experiments / s

Repeat experiments 100-200 times

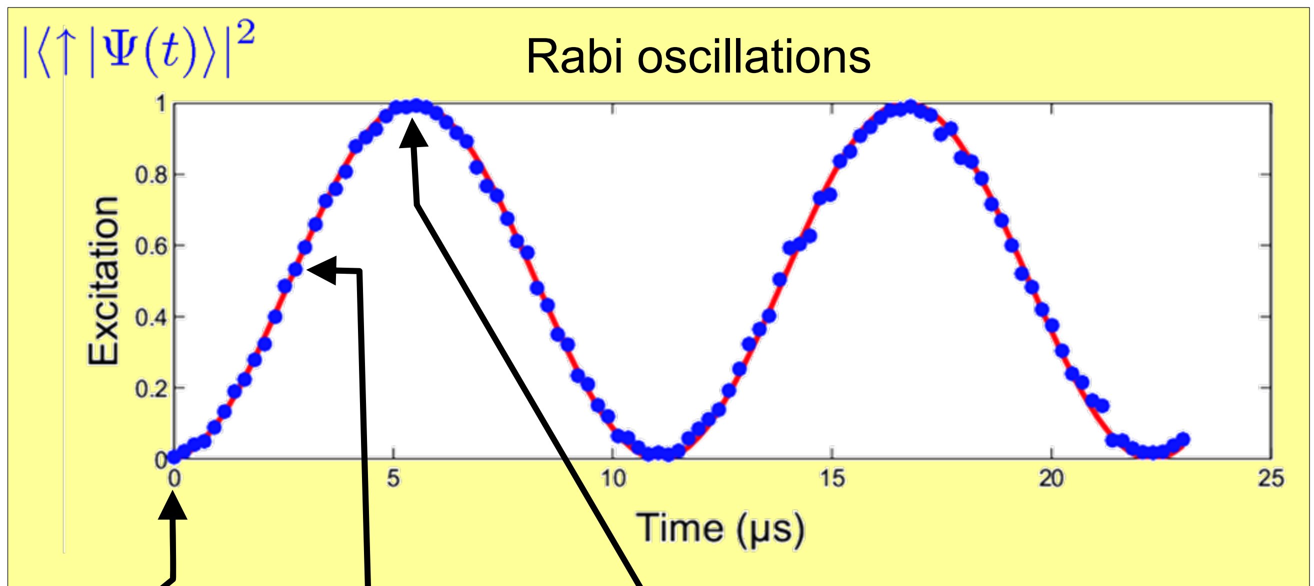
Ion-laser interaction

Resonant coherent excitation:





$$\omega_L = \omega_0$$



$$|\downarrow\rangle \xrightarrow{\pi/2} |\downarrow\rangle + |\uparrow\rangle \xrightarrow{\pi/2} |\uparrow\rangle$$

$$|\downarrow\rangle \xrightarrow{\pi} |\uparrow\rangle$$

Qubit superposition states

Schrödinger picture:

$$|\psi(t=0)\rangle \propto |\downarrow\rangle + |\uparrow\rangle \quad \longrightarrow \quad |\psi(t)\rangle \propto |\downarrow\rangle + e^{-i\omega_0 t}|\uparrow\rangle$$

Phase evolution: for optical qubits $\omega_0 \sim 10^{15}$ s^{-1}

Interaction picture:

$$|\psi(t)\rangle \propto |\downarrow\rangle + |\uparrow\rangle$$ independent of time

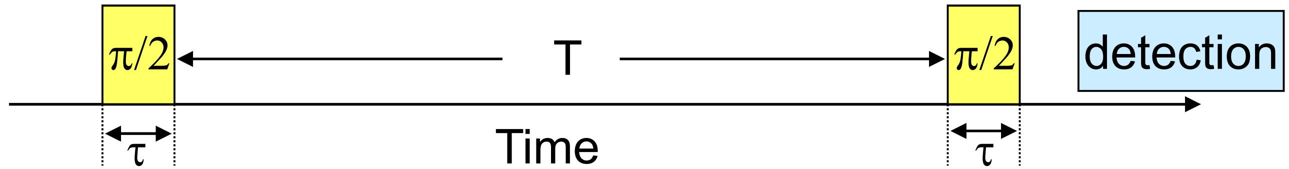
$$|\psi(t)\rangle = \cos(\theta/2)|\downarrow\rangle + e^{i\phi}\sin(\theta/2)|\uparrow\rangle$$

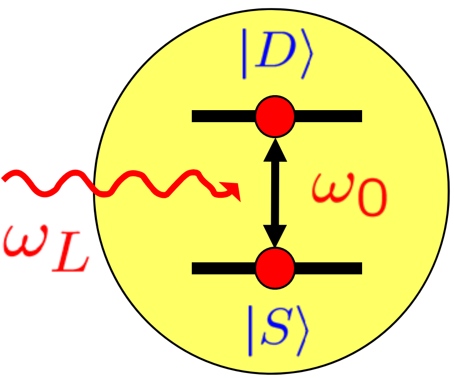
The phase ϕ of the superposition compares two oscillatory phenomena:

- Evolution of the Bloch vector in time
- Evolution of the electromagnetic field of the laser exciting the qubit

Ramsey spectroscopy for phase estimation

Two-pulse excitation:



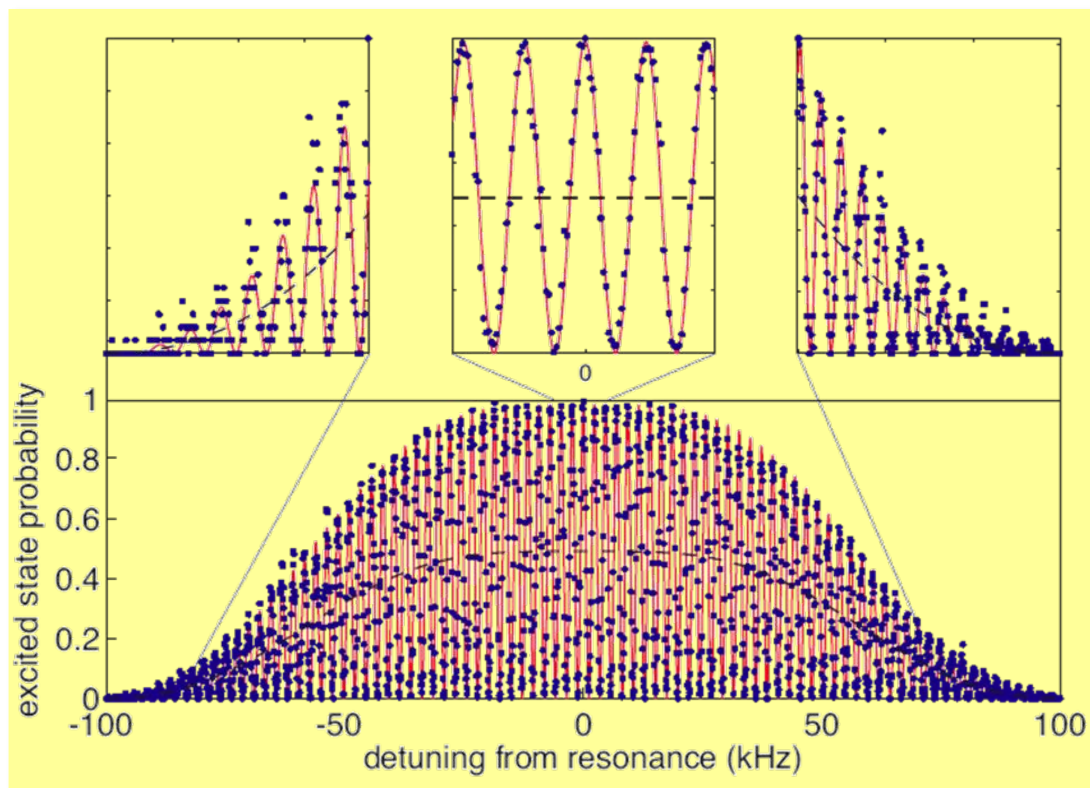


$$|S\rangle \xrightarrow{\pi/2} |S\rangle + |D\rangle \xrightarrow{\text{wait}} |S\rangle + e^{-i\Delta T}|D\rangle$$

$$\xrightarrow{\pi/2} |D\rangle$$

$$\xrightarrow{\pi/2} |S\rangle$$

$$\Delta = \omega_L - \omega_0 \approx 0$$

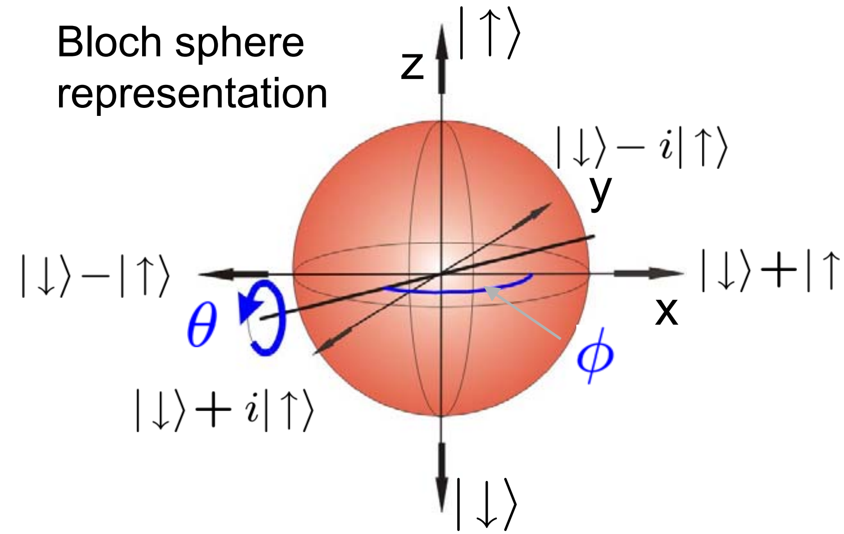


Resonant excitation in Bloch sphere picture

$$\boxed{H = \hbar\frac{\Omega}{2}(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})}$$

~ Laser intensity

Laser phase



Example: $\phi = 0 \longrightarrow H = \hbar\frac{\Omega}{2}\sigma_x$

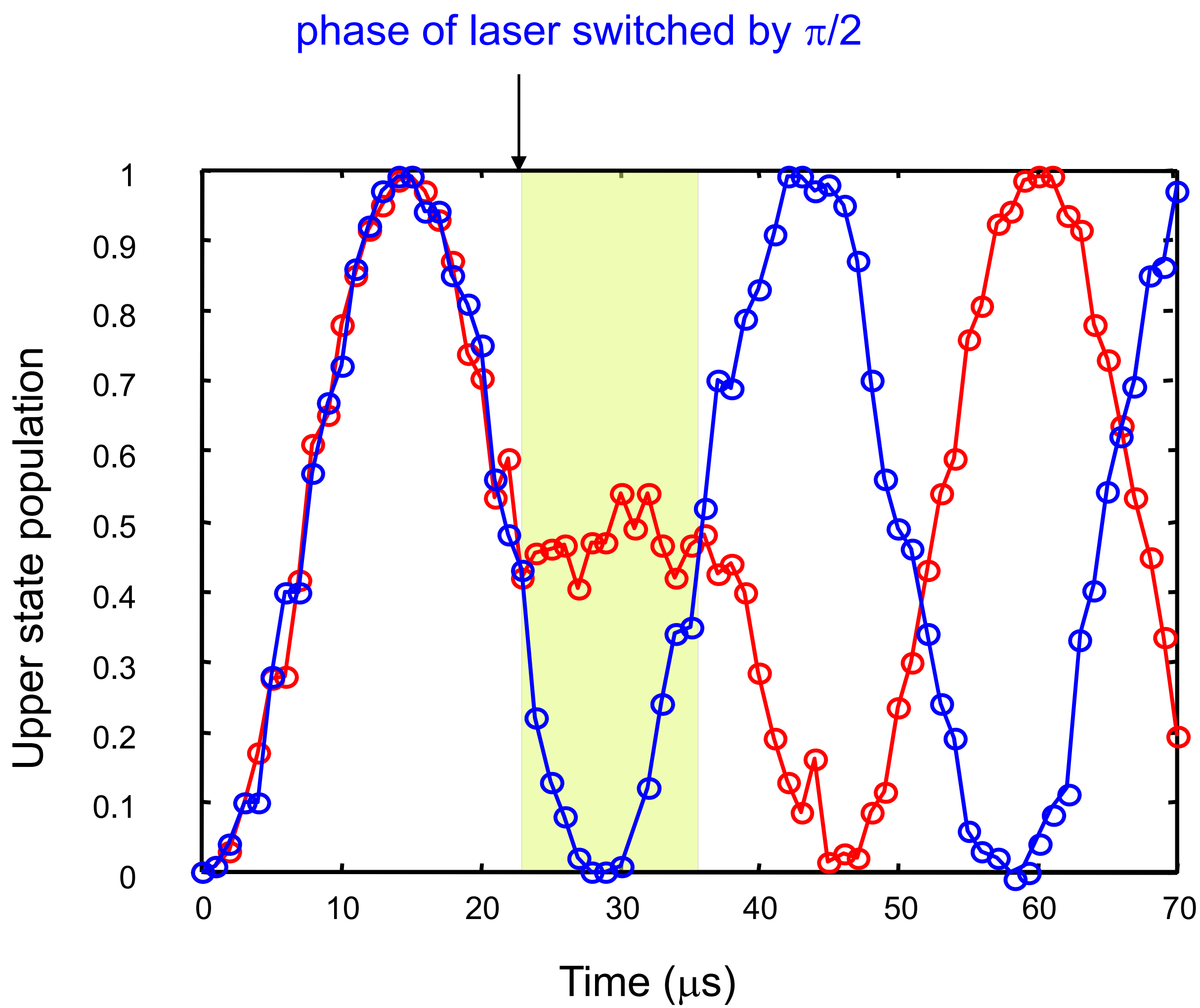
Time evolution operator:

$$U = \exp\left(-\frac{i}{\hbar}Ht\right) = \exp\left(-i\frac{\Omega t}{2}\sigma_x\right) = \cos\left(\frac{\Omega t}{2}\right) - i\sin\left(\frac{\Omega t}{2}\right)\sigma_x$$

For $\theta = \Omega t = \pi/2$

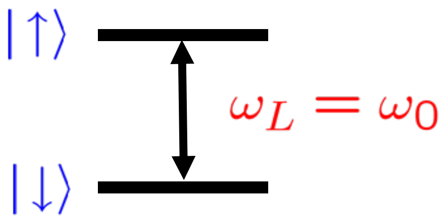
$$U|\downarrow\rangle = \frac{1}{\sqrt{2}}(I - i\sigma_x) = \frac{1}{\sqrt{2}}(|\downarrow\rangle - i|\uparrow\rangle)$$

Resonant qubit excitation



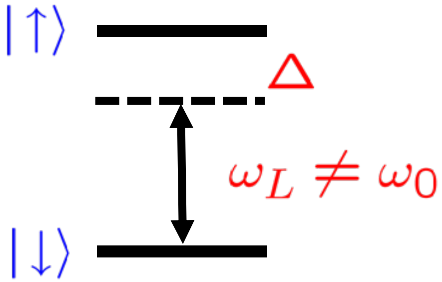
Qubit manipulation

Resonant excitation



$$H \propto \sigma_x \quad \text{or} \quad H \propto \sigma_y$$

Off-resonant excitation

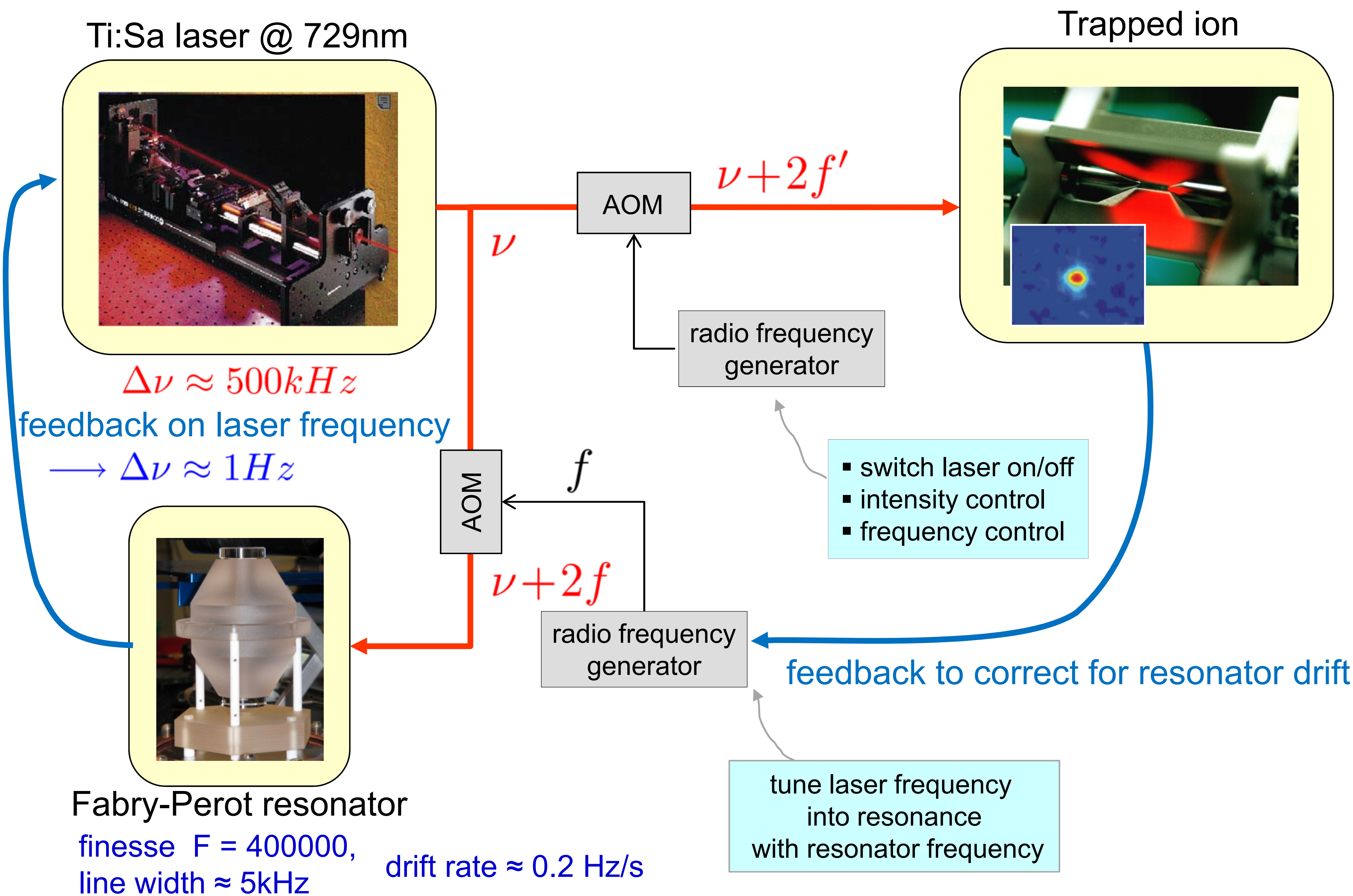


$$H \propto \sigma_z$$

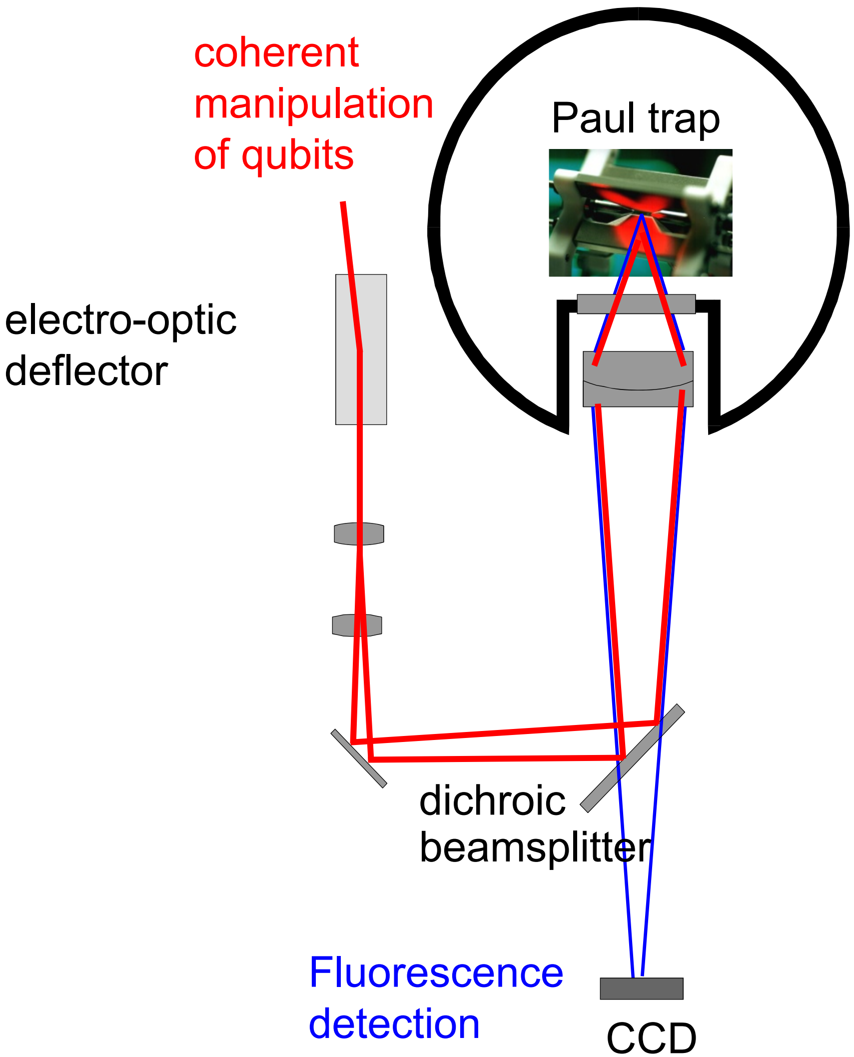
ac-Stark shifts shift qubit transition frequency

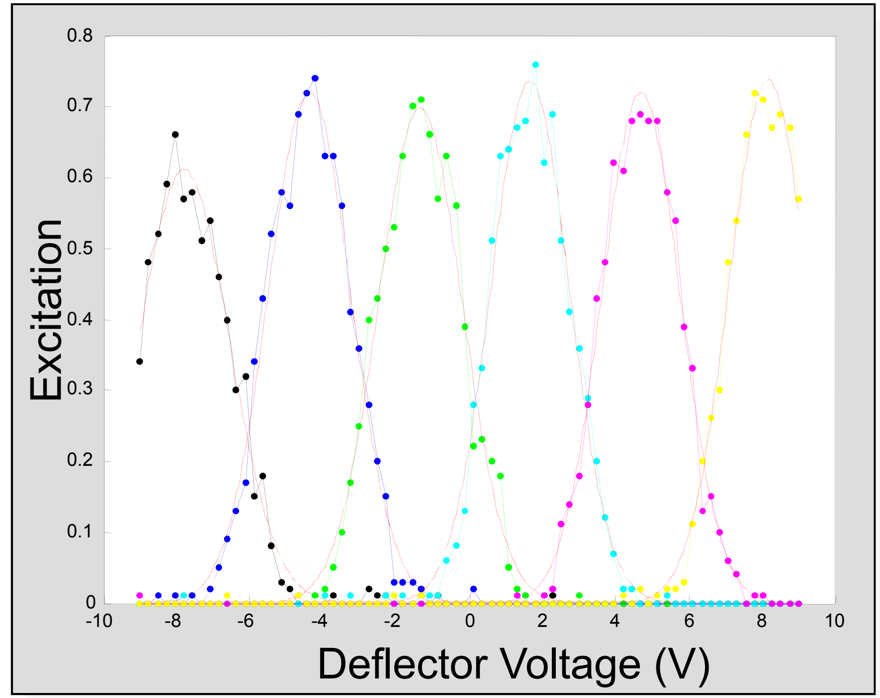
Arbitrary Bloch sphere rotations can be synthesized by a combination of laser pulses.

Laser setup for manipulating the qubit



Addressing of individual ions with a focussed laser beam

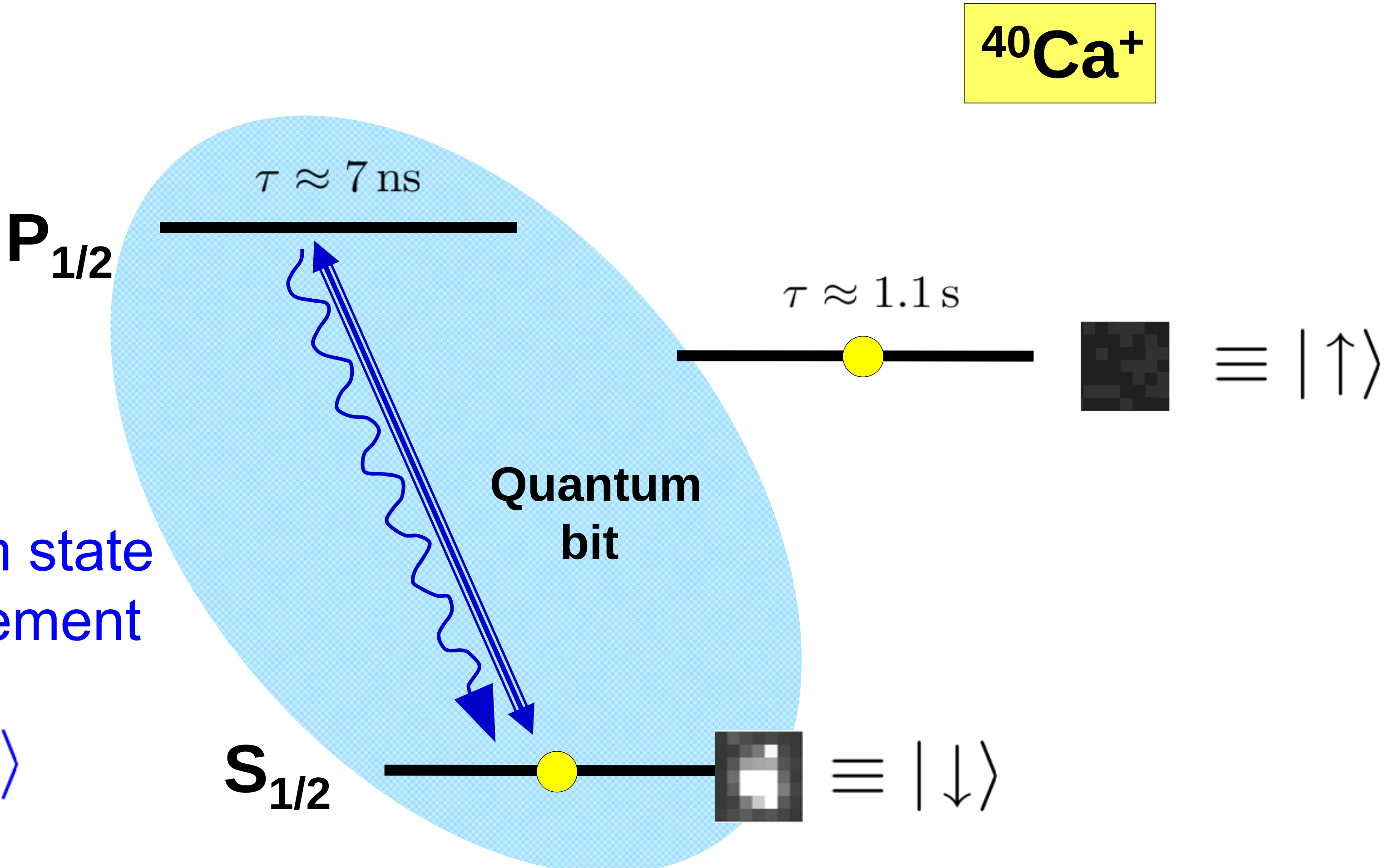




- inter ion distance: ~ 4 µm
- addressing waist: ~ 2 µm

< 0.1% intensity on neighbouring ions

Measuring qubits



Quantum state measurement

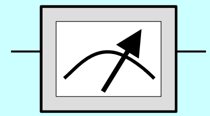
$\langle \sigma_z \rangle$

detection errors ~ 0.1%

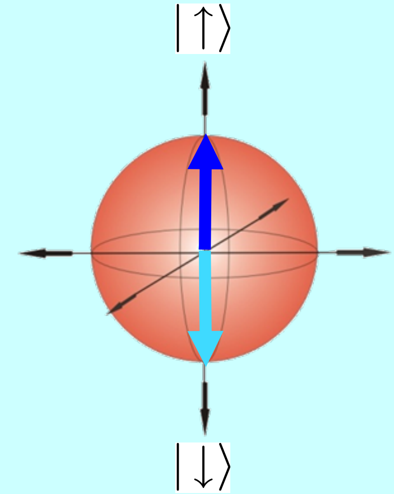
Further quantum measurements

Measurement of σ_z

Fluorescence measurements: $|\Psi\rangle$

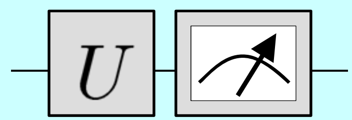


$$\langle\Psi|\sigma_z|\Psi\rangle$$

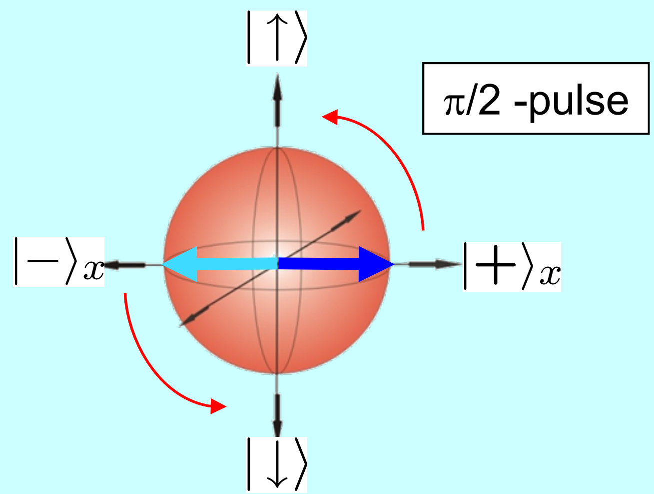


Measurement of σ_x

Unitary transformation +
fluorescence measurements $|\Psi\rangle$

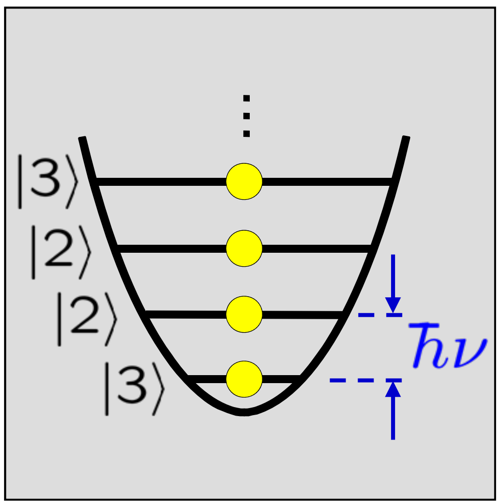


$$\langle U\Psi|\sigma_z|U\Psi\rangle = \langle\Psi|\underbrace{U^\dagger\sigma_z U}_{A = U^\dagger\sigma_z U}|\Psi\rangle$$

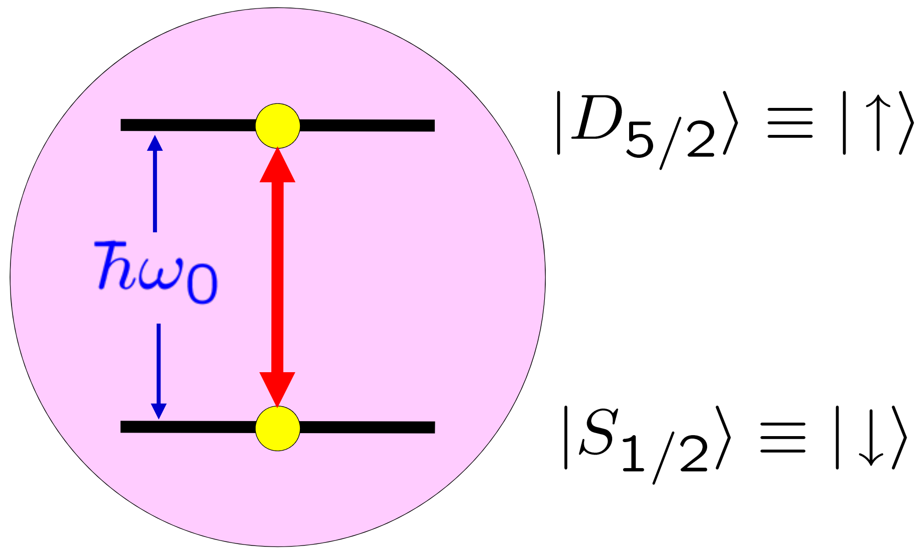


Coupling internal and vibrational degrees of freedom

Harmonic oscillator | Quantum bit



$\otimes$



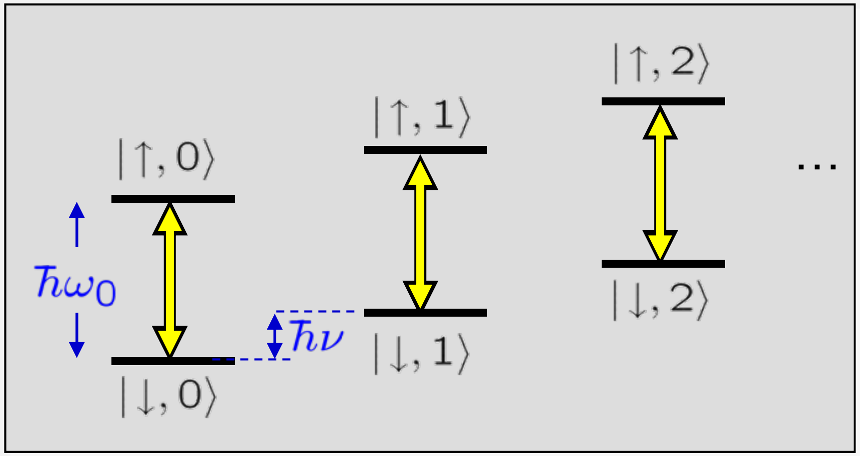
motional states

$|0\rangle, |1\rangle, |2\rangle, |3\rangle, \ldots$

internal states

$|\uparrow\rangle, |\downarrow\rangle$

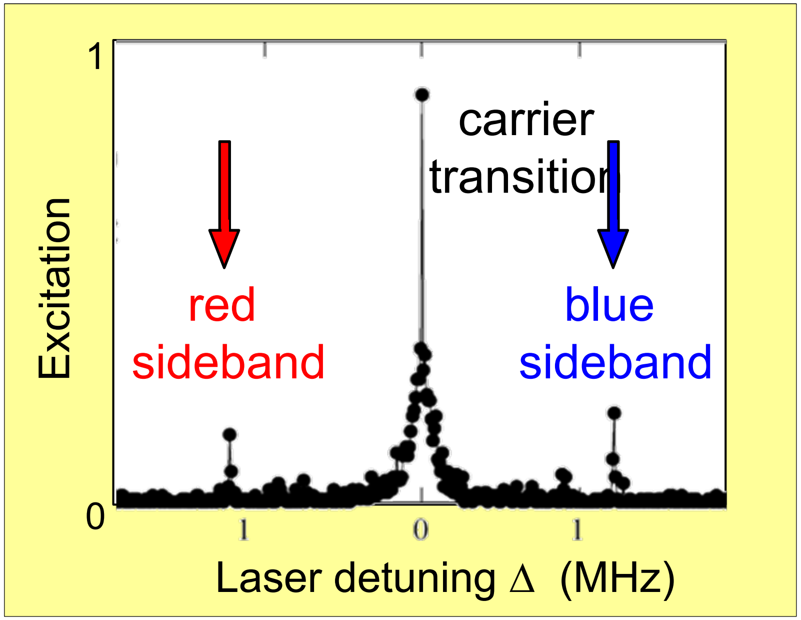
Trapped-ion laser interactions



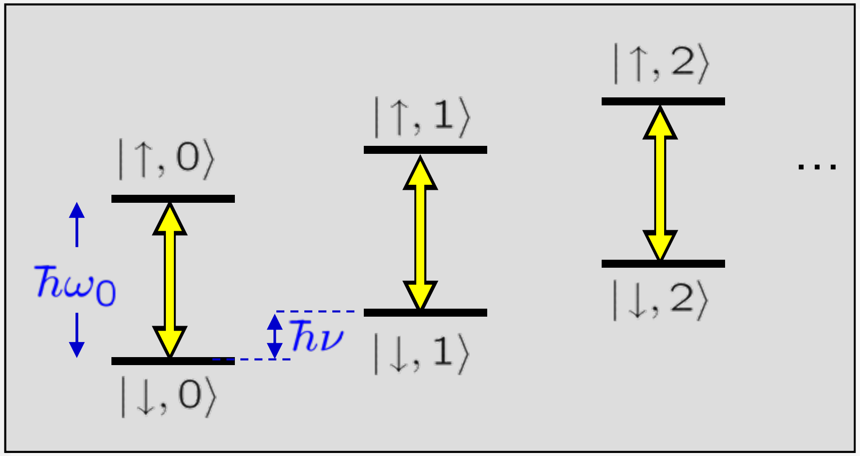
qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, \; H \propto \sigma_y$$



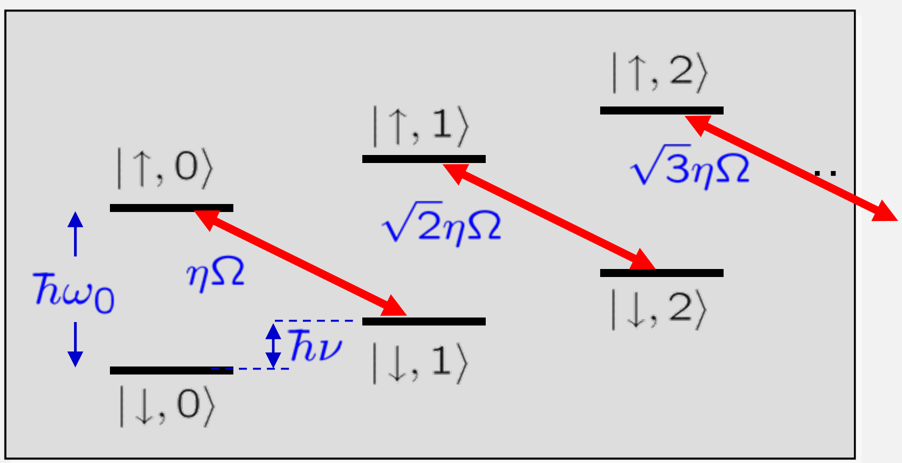
Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

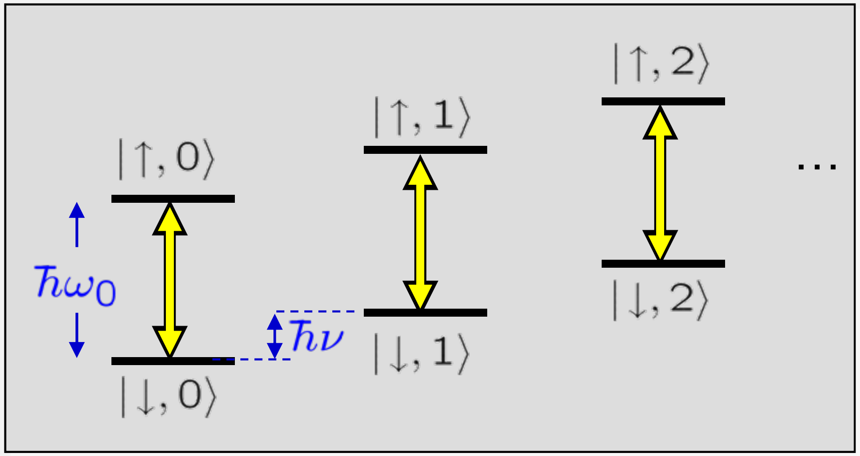
$$H \propto \sigma_x, \; H \propto \sigma_y$$



qubit-motion coupling

$$\boxed{\begin{gathered}\omega_{laser} = \omega_0 - \nu \\ H \propto \sigma_+ a + \sigma_- a^\dagger\end{gathered}}$$

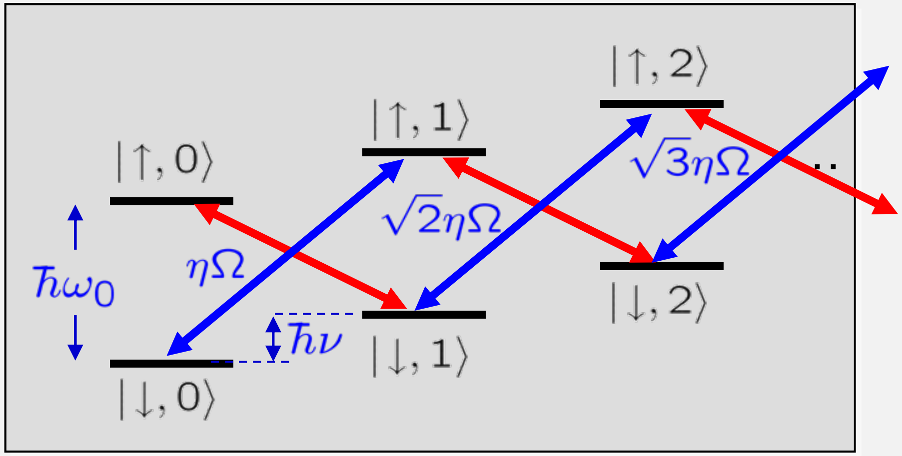
Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x,\ H \propto \sigma_y$$



qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$

$$H \propto \sigma_+ a + \sigma_- a^\dagger$$

$$\omega_{laser} = \omega_0 + \nu$$

$$H \propto \sigma_+ a^\dagger + \sigma_- a$$

Sideband excitation

$$H^{(i)} = \frac{\hbar\Omega}{2}\sigma_+ e^{-i\delta t + i\phi}\left(I + i\eta(a^\dagger e^{i\nu t} + a e^{-i\nu t}) + \mathcal{O}(\eta^2)\right) + \text{h.c.}$$

Red sideband: $\delta = -\nu$

$$H_{int} = \frac{\hbar\Omega}{2} i\eta\{\sigma_+ a\, e^{+i\phi} - \sigma_- a^\dagger e^{-i\phi}\}$$

$$|g, n\rangle \longleftrightarrow |e, n-1\rangle$$

'Jaynes-Cummings-Hamiltonian'

- Coupling strength dependent on n

Blue sideband: $\delta = +\nu$

$$H_{int} = \frac{\hbar\Omega}{2} i\eta\{\sigma_+ a^\dagger e^{+i\phi} - \sigma_- a\, e^{-i\phi}\}$$

$$|g, n\rangle \longleftrightarrow |e, n+1\rangle$$

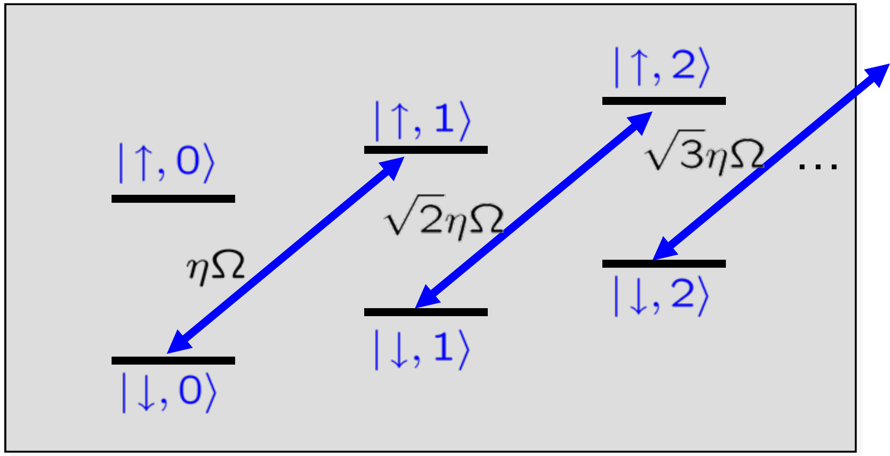
'anti-Jaynes-Cummings-Hamiltonian'

- Coupling strength dependent on n

Coherent excitation on the sideband

„Blue sideband" pulses:

$$|\downarrow\rangle|n\rangle \longleftrightarrow |\uparrow\rangle|n+1\rangle$$

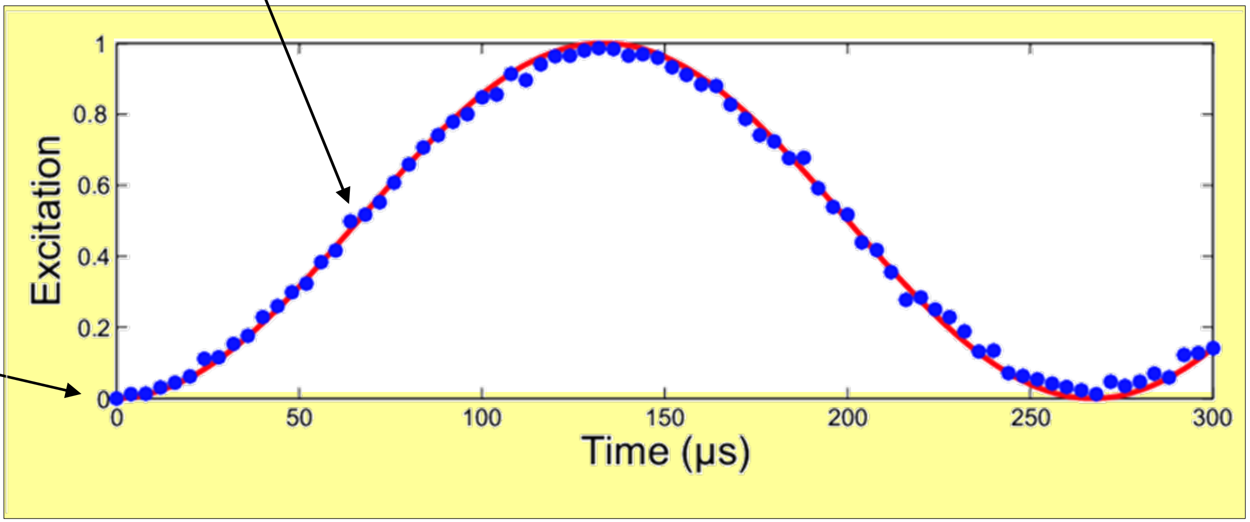


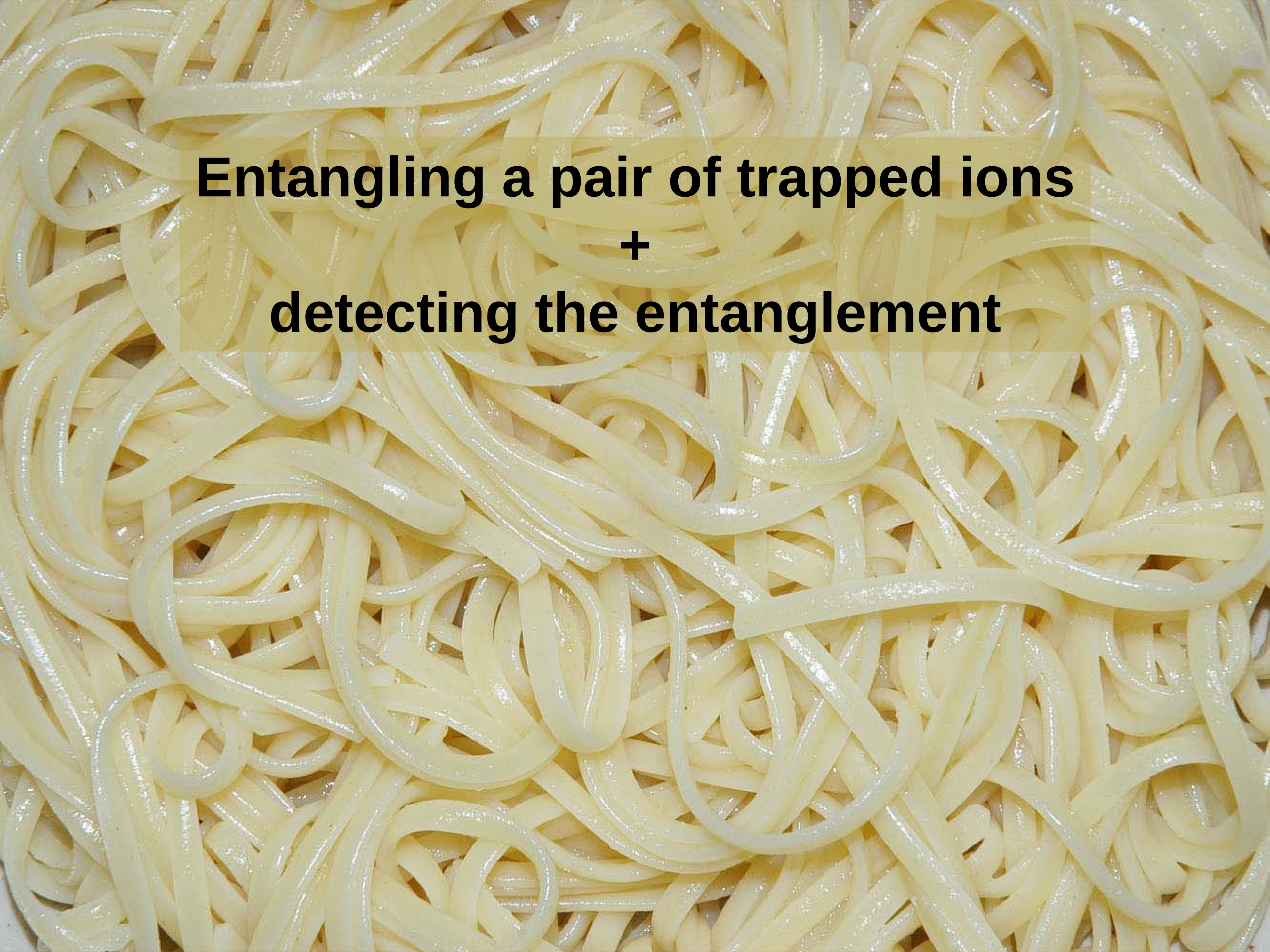
$\theta = \pi/2$: Entanglement between internal and motional state !

$$\frac{1}{\sqrt{2}}(|\downarrow, n=0\rangle + |\uparrow, n=1\rangle)$$

upper state population

$$|\downarrow, n=0\rangle$$

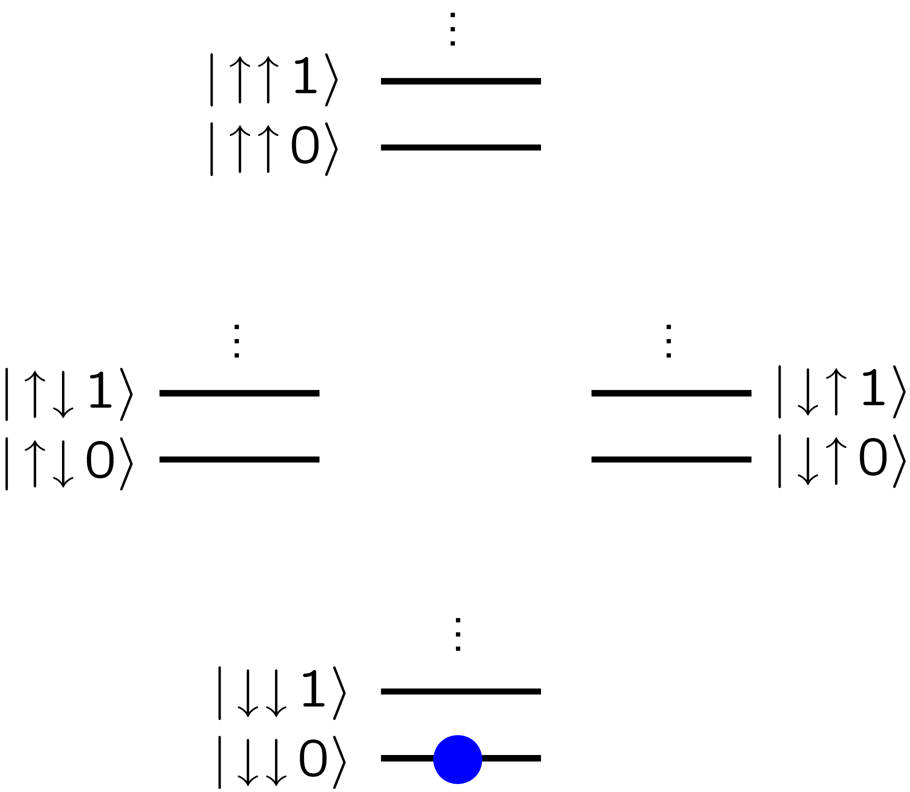




Entangling a pair of trapped ions
+
detecting the entanglement

Generation of Bell states

Two-ion energy levels

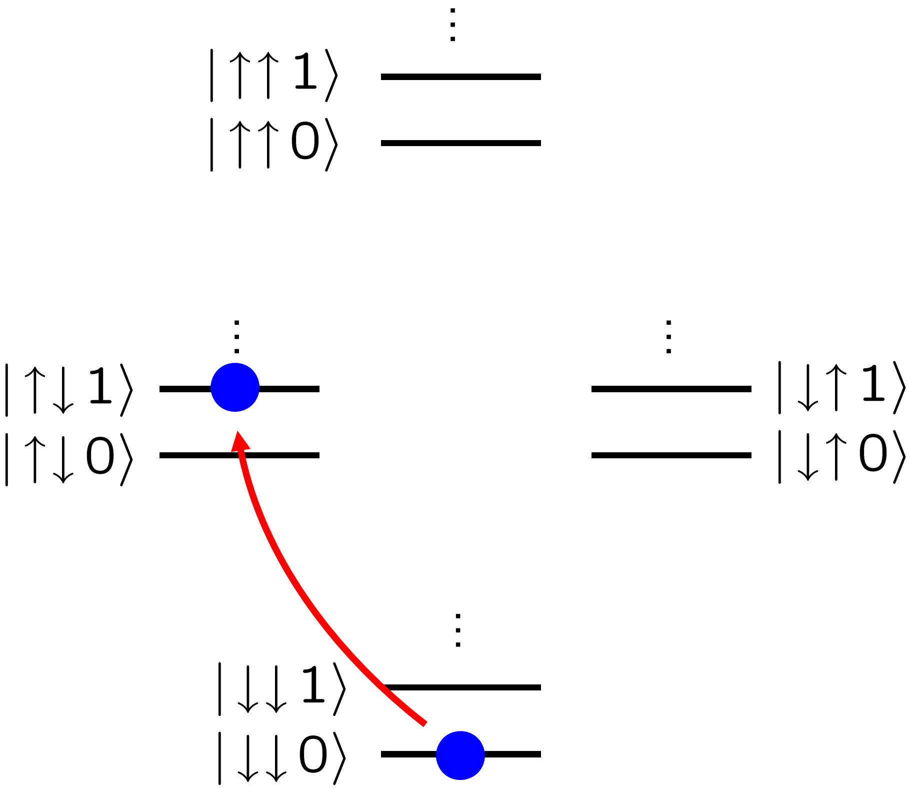


Pulse sequence:

Ion	Pulse length	Transition

$|\downarrow\downarrow 0\rangle$

Generation of Bell states

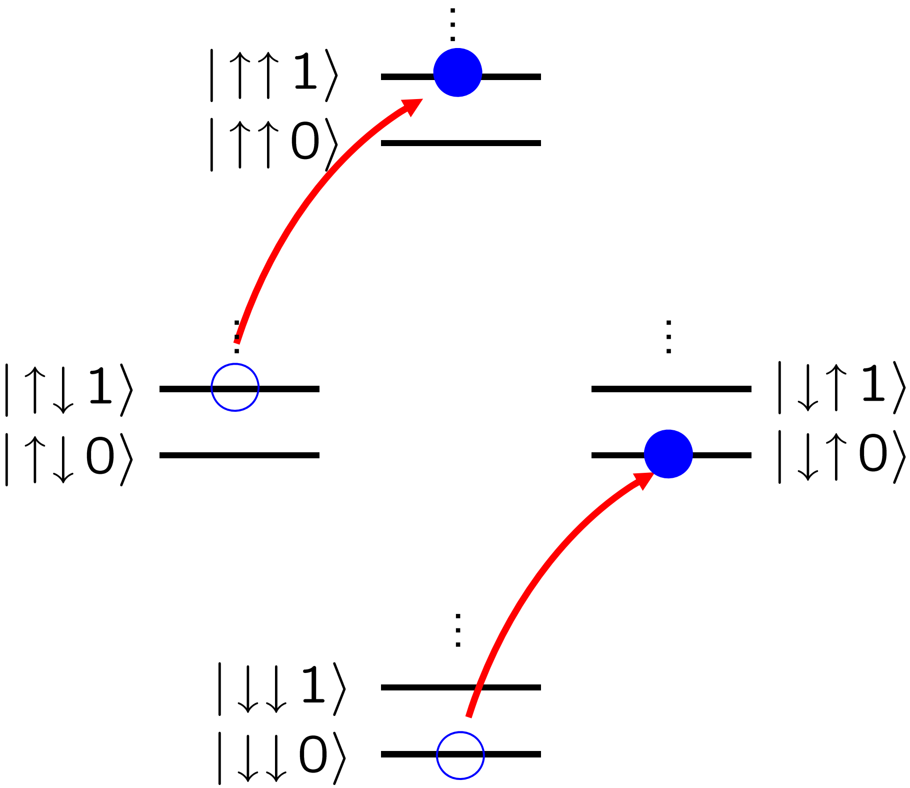


Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband

$$|\downarrow\downarrow 0\rangle + |\uparrow\downarrow 1\rangle$$

Generation of Bell states

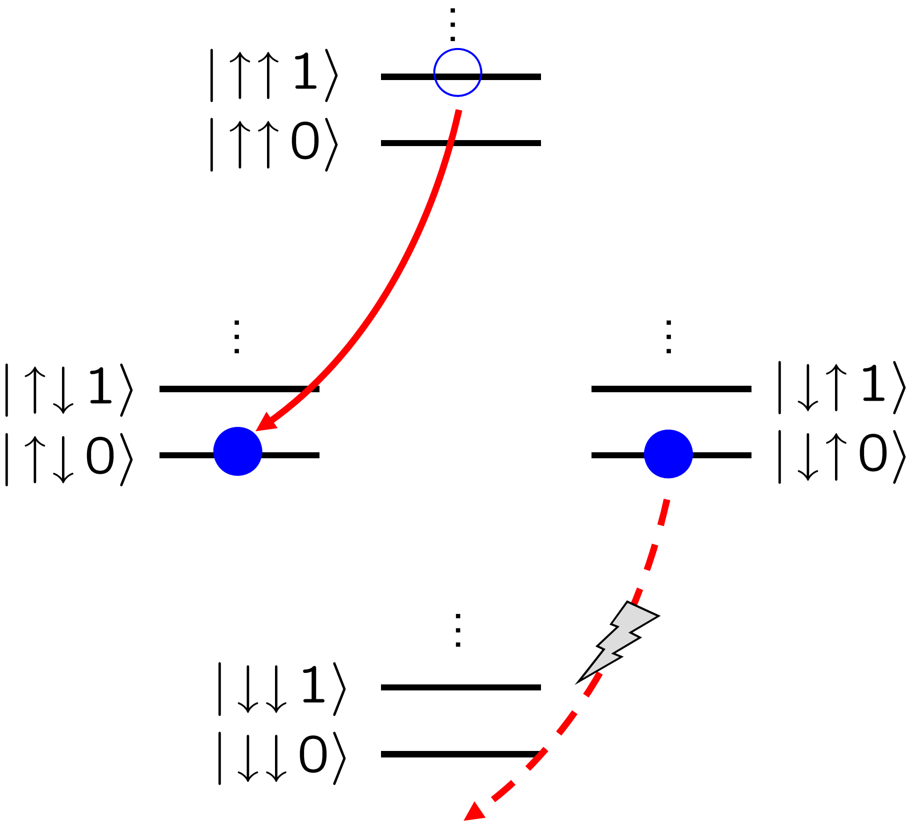


Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	π	carrier

$$|\downarrow\uparrow 0\rangle + |\uparrow\uparrow 1\rangle$$

Generation of Bell states



Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	π	carrier
2	π	blue sideband

$$(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)|0\rangle$$

Measuring the entangled state

We hope to create the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$

There are no pure states in experimental physics!

The state created in the experiment has to be described by a density matrix ρ_{exp} .

How can we analyze the state ρ_{exp} we created?

Fluorescence detection with CCD camera:

$|\downarrow\downarrow\rangle$

$|\downarrow\uparrow\rangle$

$|\uparrow\downarrow\rangle$

$|\uparrow\uparrow\rangle$

Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

Measuring the entangled state

We hope to create the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$

There are no pure states in experimental physics!

The state created in the experiment has to be described by a density matrix ρ_{exp} .

How can we analyze the state ρ_{exp} we created?

Option 1:

Measure how close ρ_{exp} is to $|\psi\rangle$ by measuring the fidelity $F = \langle\psi|\rho_{exp}|\psi\rangle$

Option 2:

Carry out measurements that allow to completely determine ρ_{exp}

$\longrightarrow$ Quantum state tomography

Reconstruction of the density matrix

Representation of ρ as a sum of orthogonal observables A_i :

$$\rho = \sum_i \lambda_i A_i \text{ with } Tr(A_i A_j) = \delta_{ij}$$

ρ is completely detemined by the expectation values $<A_i>$:

$$\langle A_j \rangle = Tr(\rho A_j) = \sum_i \lambda_i Tr(A_i A_j) = \lambda_j$$

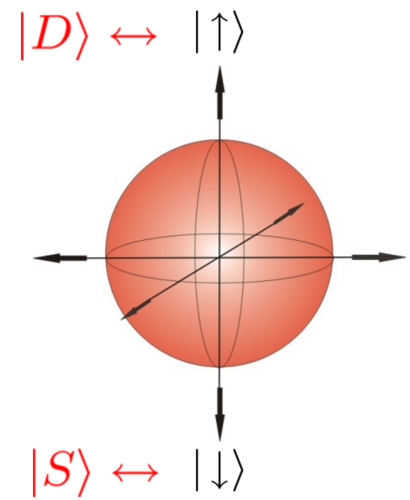
For a two-ion system : $A_i \in \{\sigma_i^{(1)} \otimes \sigma_j^{(2)}, \sigma_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}\}$

$\longrightarrow$ Joint measurements of all spin components $\sigma_i^{(1)} \otimes \sigma_j^{(2)}$

$$\rho_R = \sum_{i=1}^{16} \langle A_i \rangle A_i$$

Measurement of spin expectation values

Measurement of $\langle \sigma_z \rangle$: Fluorescence measurement



$$\langle \sigma_z \rangle \quad = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}$$

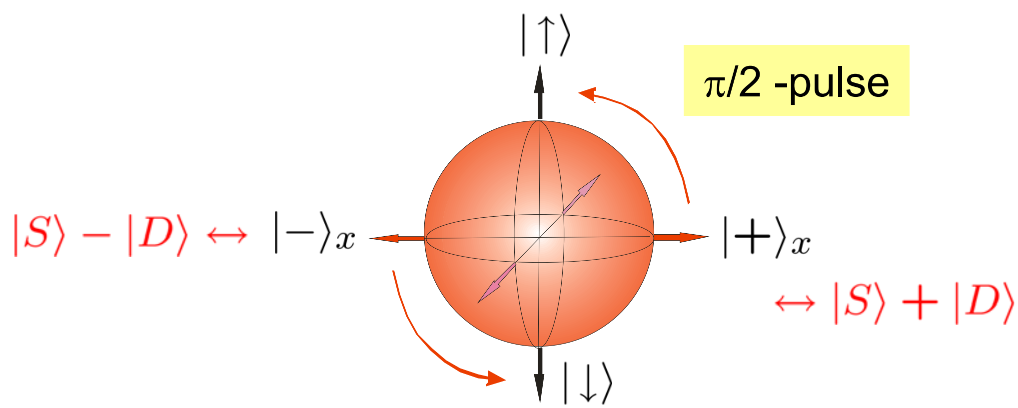
$$= \rho_{DD} - \rho_{SS}$$

Measurement of $\langle \sigma_x \rangle$, $\langle \sigma_y \rangle$:

Rotation of the Bloch sphere prior to state measurement:

$$\langle \sigma_z \rangle_{U\rho U^{-1}} = Tr(\sigma_z U \rho U^{-1})$$

$$= Tr(\underbrace{U^{-1}\sigma_z U}_{\overset{!}{=}\,\sigma_x} \rho)$$



Bell state analysis

Measurement of $\langle \sigma_z \rangle$:

$$\langle \sigma_z \rangle \qquad = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}$$

Measurement of $\langle \sigma_x \rangle$, $\langle \sigma_y \rangle$:

Rotation of the Bloch sphere prior to state measurement:

$$\langle \sigma_z \rangle_{U\rho U^{-1}} = Tr(\sigma_z U \rho U^{-1})$$

$$= Tr(\underbrace{U^{-1}\sigma_z U}_{\stackrel{!}{=}\sigma_x}\rho)$$

Measurement time: 40 s

prepare Bell state
no rotation
measure

200 repetitions

$\langle \sigma_z^{(1)} \rangle, \langle \sigma_z^{(2)} \rangle, \langle \sigma_z^{(1)} \sigma_z^{(2)} \rangle$

↓

prepare Bell state
ion #1, y - rotation
ion #2, identity
measure

200 repetitions

$\langle \sigma_x^{(1)} \rangle, \langle \sigma_z^{(2)} \rangle, \langle \sigma_x^{(1)} \sigma_z^{(2)} \rangle$

↓

⋮

9 different settings

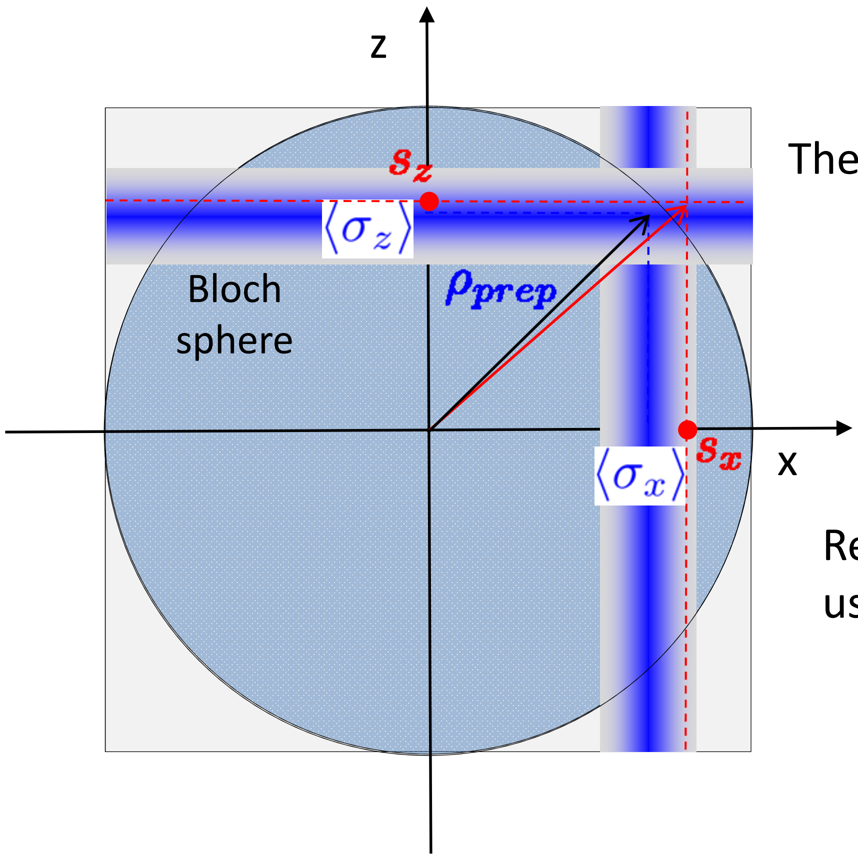
↓

prepare Bell state
ion #1, x - rotation
ion #2, x - rotation
measure

200 repetitions

$\langle \sigma_y^{(1)} \rangle, \langle \sigma_y^{(2)} \rangle, \langle \sigma_y^{(1)} \sigma_y^{(2)} \rangle$

Example: Tomography of a qubit



The experimental procedure prepares the state ρ_{prep}

$$\rho_{prep} = \frac{1}{2}(I + \langle \sigma_x \rangle \sigma_x + \langle \sigma_y \rangle \sigma_y + \langle \sigma_z \rangle \sigma_z)$$

Reconstruction by estimation of $\langle \sigma_x \rangle, \langle \sigma_y \rangle, \langle \sigma_z \rangle$ using a finite number of copies of the state:

$$s_z = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \;, \; s_x = \ldots, \; s_y = \ldots$$

$$\rho_{tomo} = \frac{1}{2}(I + s_x \sigma_x + s_y \sigma_y + s_z \sigma_z) \neq \rho_{prep}$$

ρ_{tomo} might not be within the Bloch sphere !

DISASTER !!!

Maximum likelihood estimation

Is $\rho_R = \sum_i \langle A_i \rangle \, A_i$ positive semidefinite ? … not necessarily:

with a finite number of measurements, we can only estimate expectation values

Maximum likelihood estimation: (Hradil '97, Banaszek '99)

In N experiments,the quantum state is projected onto the outcomes $|y_j\rangle$.

f_j : relative frequency of the outcome $|y_j\rangle$

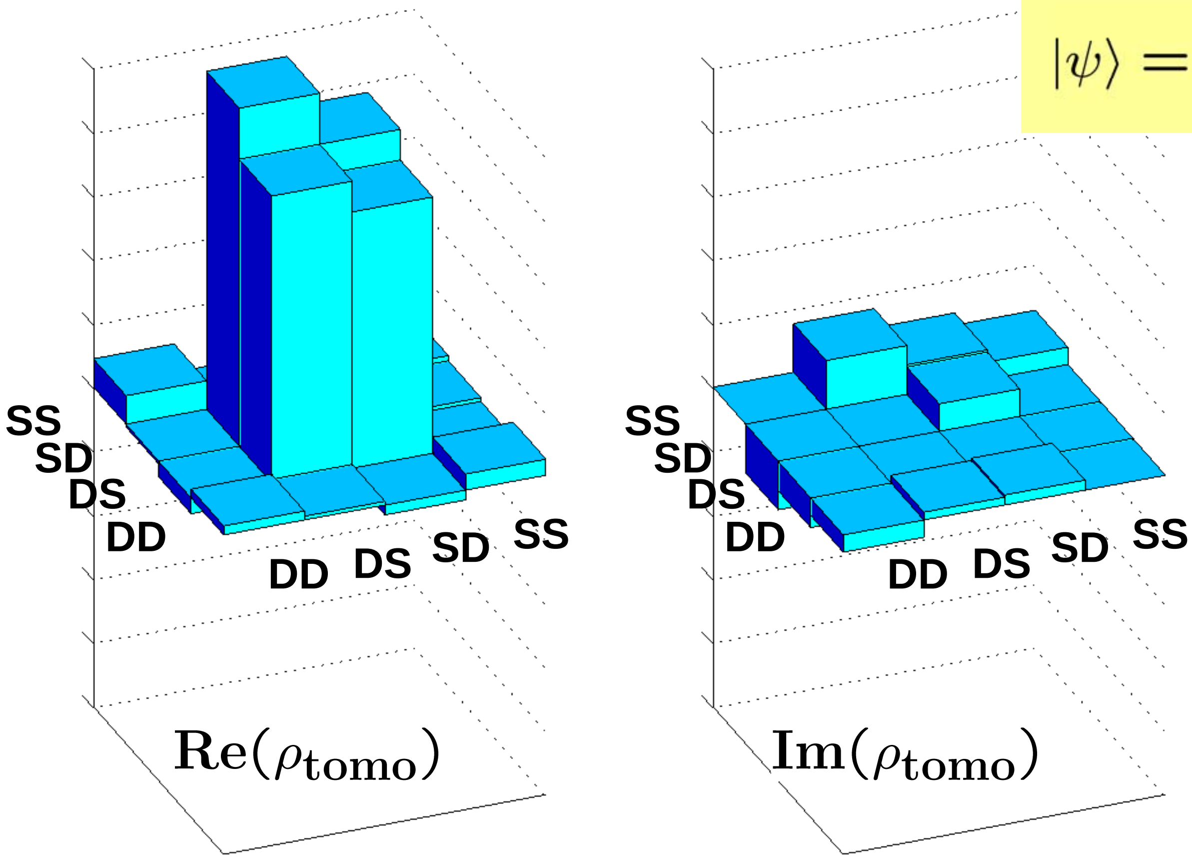
On the set of density matrices ρ, look for the one that maximizes

$$\mathcal{L}(\rho) = \prod_j \langle y_j|\rho|y_j\rangle^{N f_j}$$

$$\boxed{\text{Maximize } L(\rho) = \textstyle\sum_j f_j \log\langle y_j|\rho|y_j\rangle}$$

Bell state reconstruction with maximum likelihood estimation

$$|\psi\rangle = \frac{1}{2}(|SD\rangle + |DS\rangle)$$

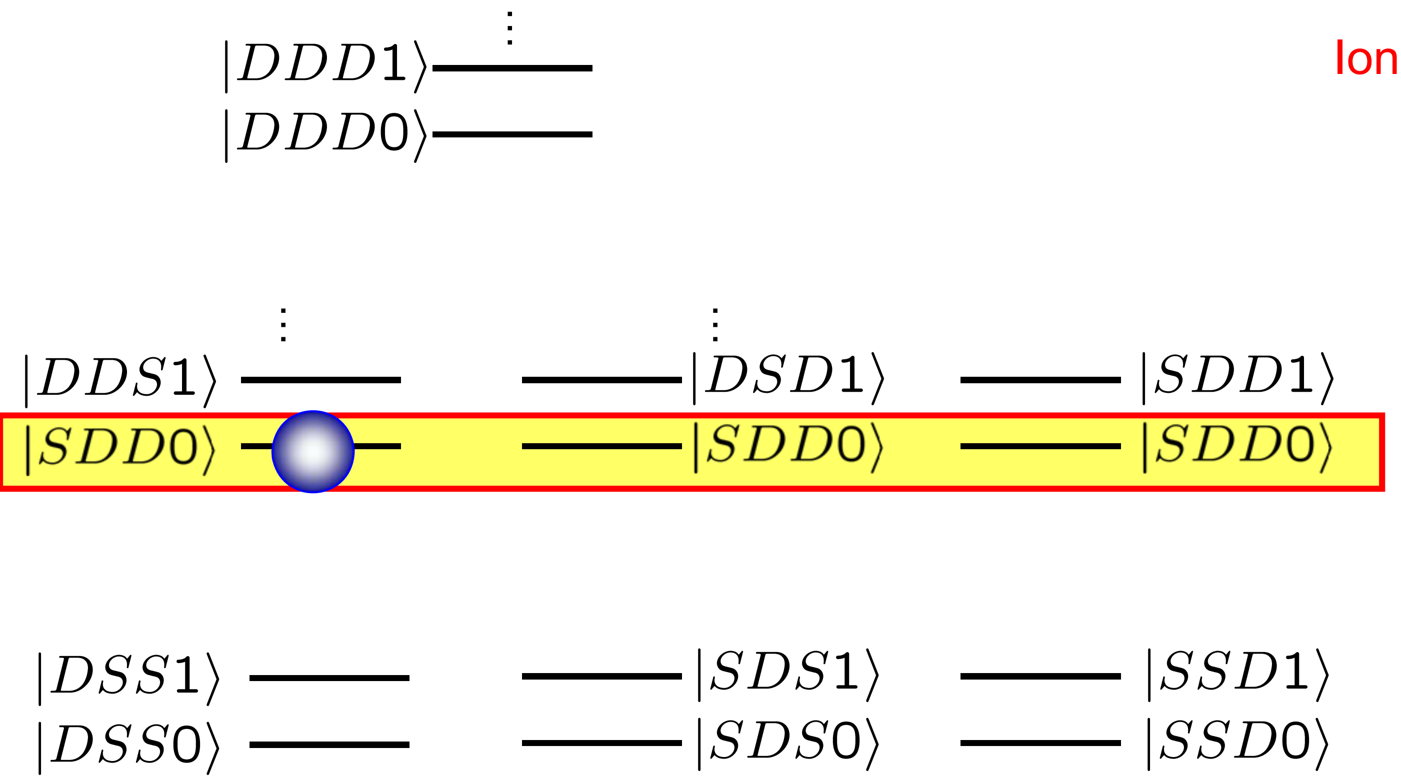


- State fidelity: $\langle\psi|\rho_{tomo}|\psi\rangle = 0.91$
- Violation of a Bell inequality: $\langle\rho_x^{(1)}\rho_{x-z}^{(2)}\rangle + \langle\rho_x^{(1)}\rho_{x+z}^{(2)}\rangle + \langle\rho_z^{(1)}\rho_{x-z}^{(2)}\rangle - \langle\rho_z^{(1)}\rho_{x+z}^{(2)}\rangle = 2.52(6) > 2$
- Entanglement of formation: $E(\rho_{tomo}) = 0.79$

Generation of multi-qubit entangled states: W-states

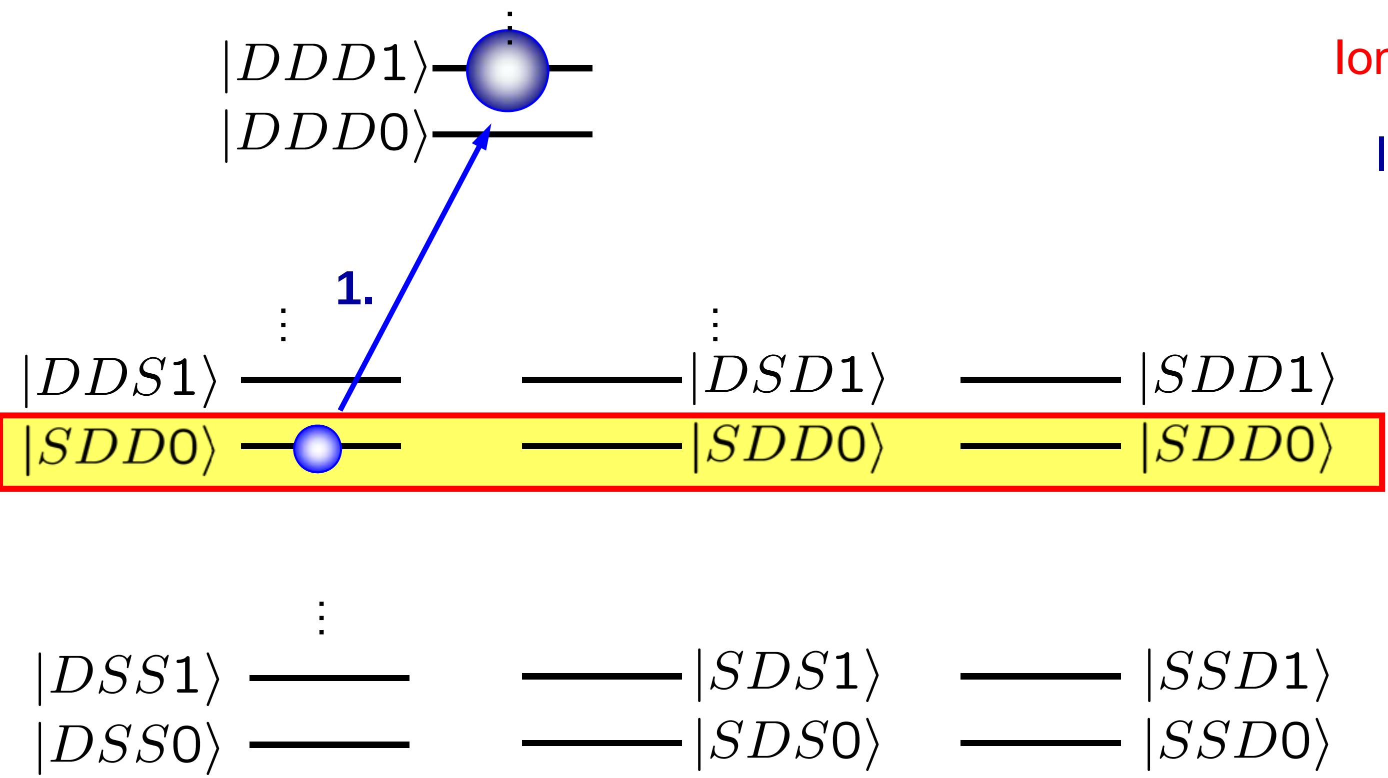
Pulse sequence:

Ion 2,3: π , carrier



$$|DDS, 0\rangle$$

Generation of multi-qubit entangled states: W-states



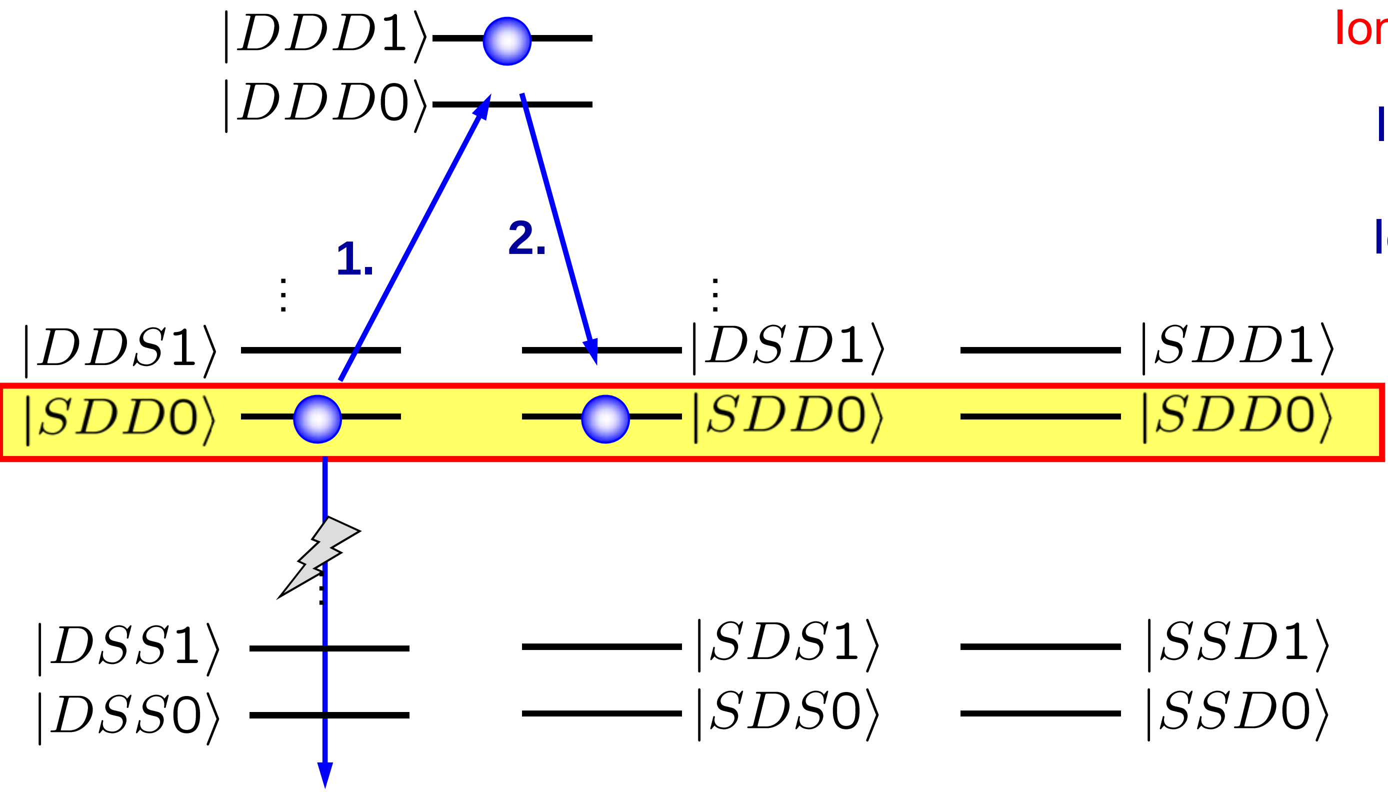
Pulse sequence:

Ion 2,3: π , carrier

Ion 1: θ_1 , blue sideband

$$\sqrt{\frac{1}{3}}|DDS,0\rangle + \sqrt{\frac{2}{3}}|DDD,1\rangle$$

Generation of multi-qubit entangled states: W-states



Pulse sequence:

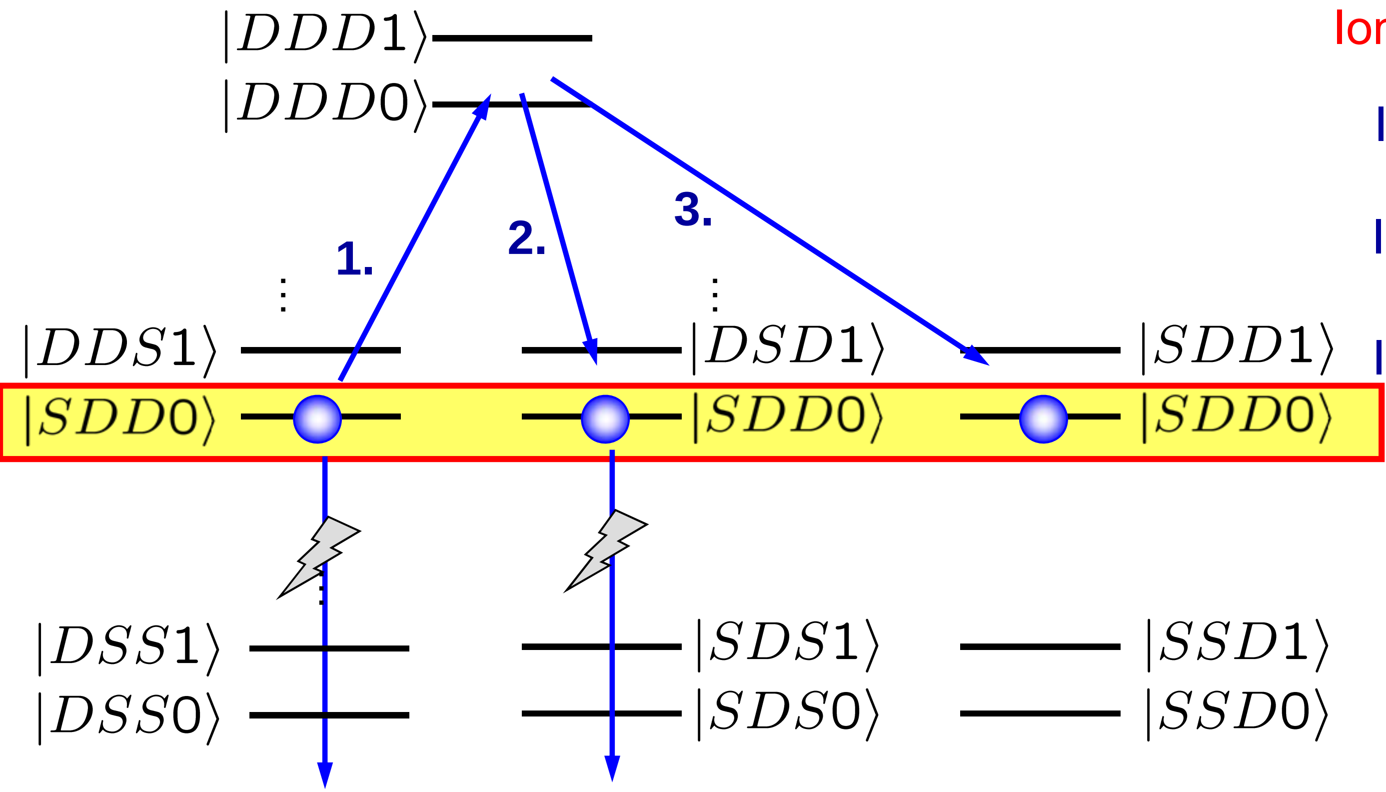
Ion 2,3: π , carrier

Ion 1: θ_1 , blue sideband

Ion 2: θ_2 , blue sideband

$$\sqrt{\frac{1}{3}}|DDS,0\rangle + \sqrt{\frac{1}{3}}|DSD,0\rangle + \sqrt{\frac{1}{3}}|DDD,1\rangle$$

Generation of multi-qubit entangled states: W-states



Pulse sequence:

Ion 2,3: π , carrier

Ion 1: θ_1 , blue sideband

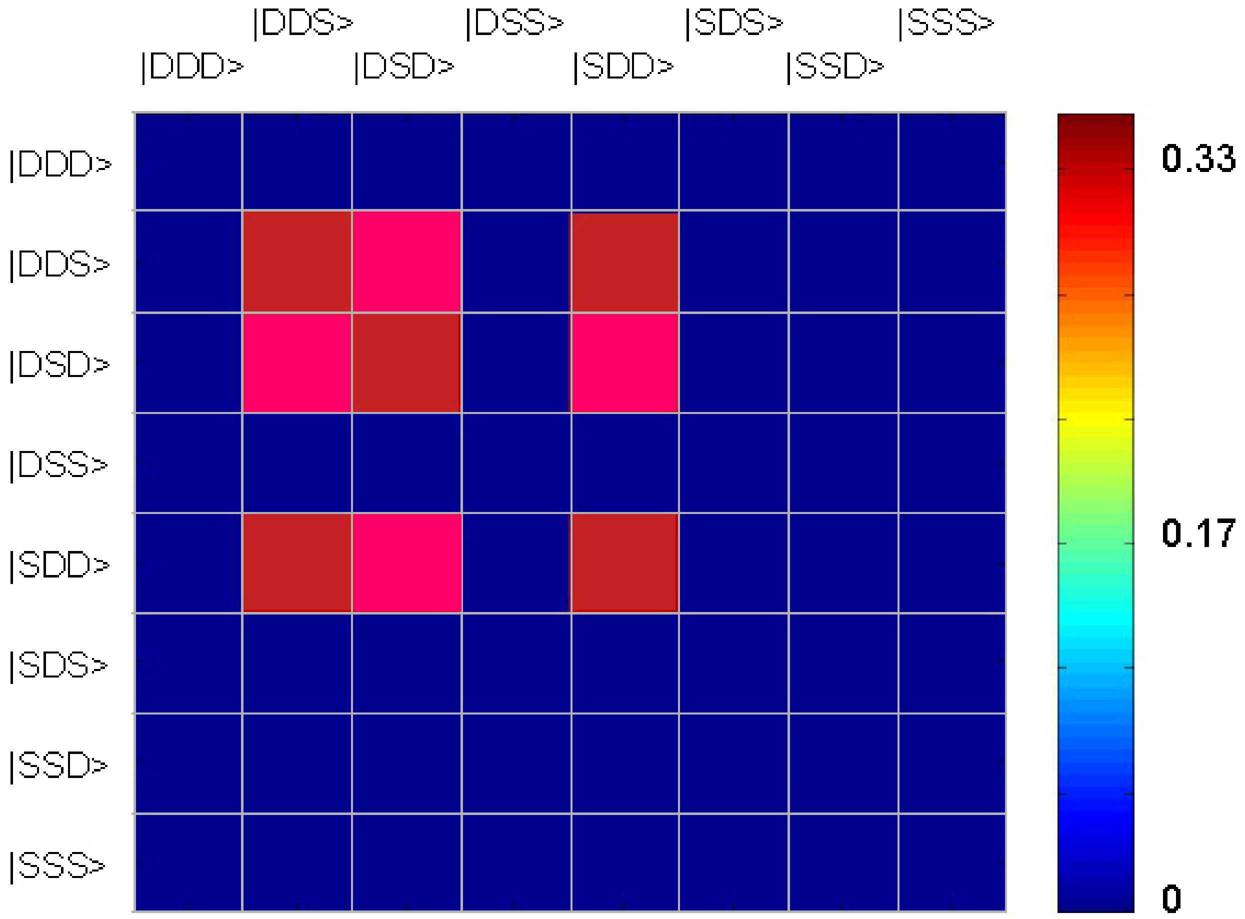
Ion 2: θ_2 , blue sideband

Ion 3: θ_3 , blue sideband

$$\sqrt{\frac{1}{3}}|DDS,0\rangle + \sqrt{\frac{1}{3}}|DSD,0\rangle + \sqrt{\frac{1}{3}}|SDD,0\rangle$$

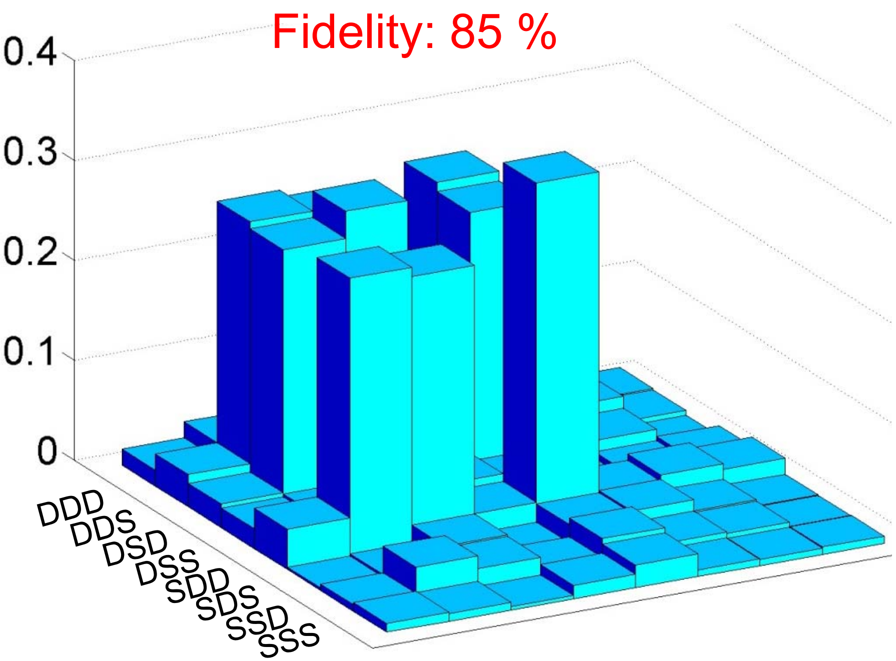
W – state: |SDD> + |DSD> + |DDS>

Ideal 3-ion density matrix

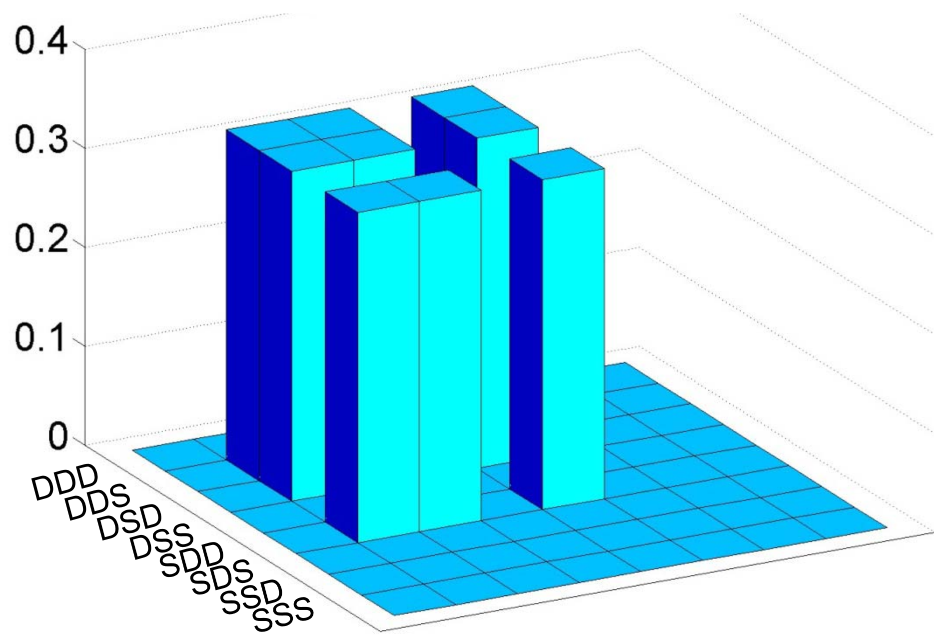


Reconstructed W – state: experiment and theory

$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$$



experimental result

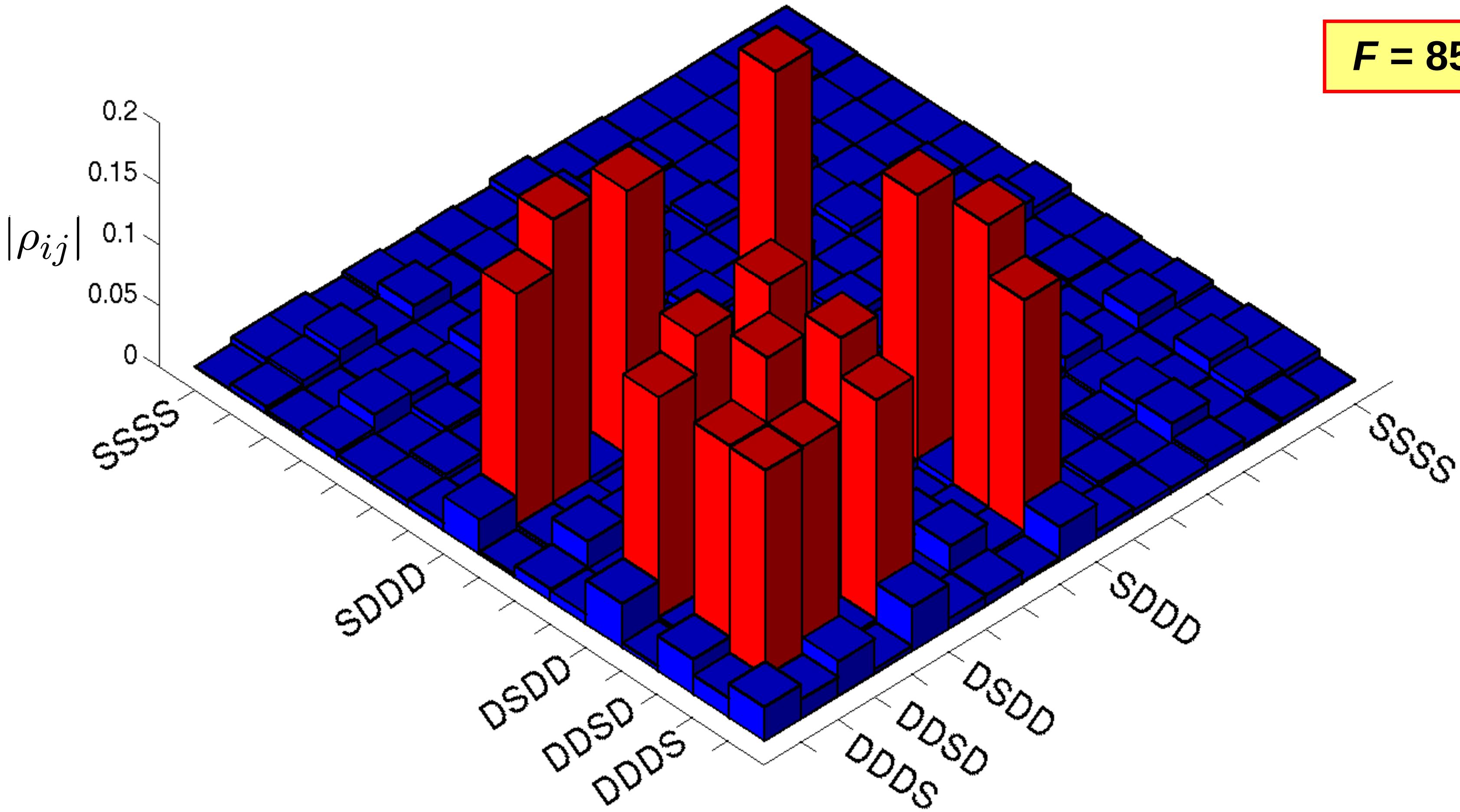


theoretical expectation

C. F. Roos *et al*., Science **304**, 1478 (2004)

More ions: Four-ion W-states

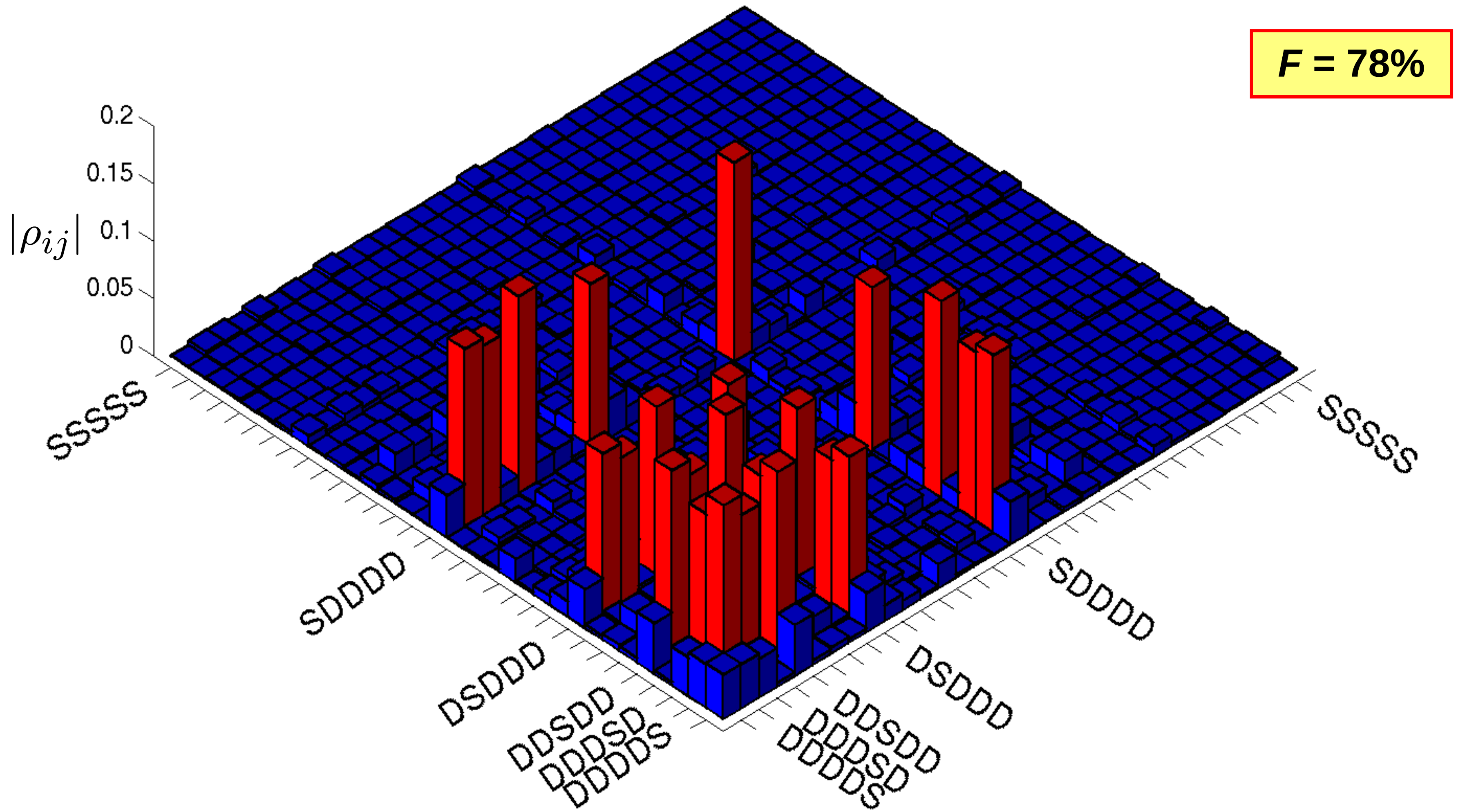
$$\Psi_4 = \frac{1}{\sqrt{4}}(|DDDS\rangle + |DDSD\rangle + |DSDD\rangle + |SDDD\rangle)$$



F = 85%

Five-ion W-states

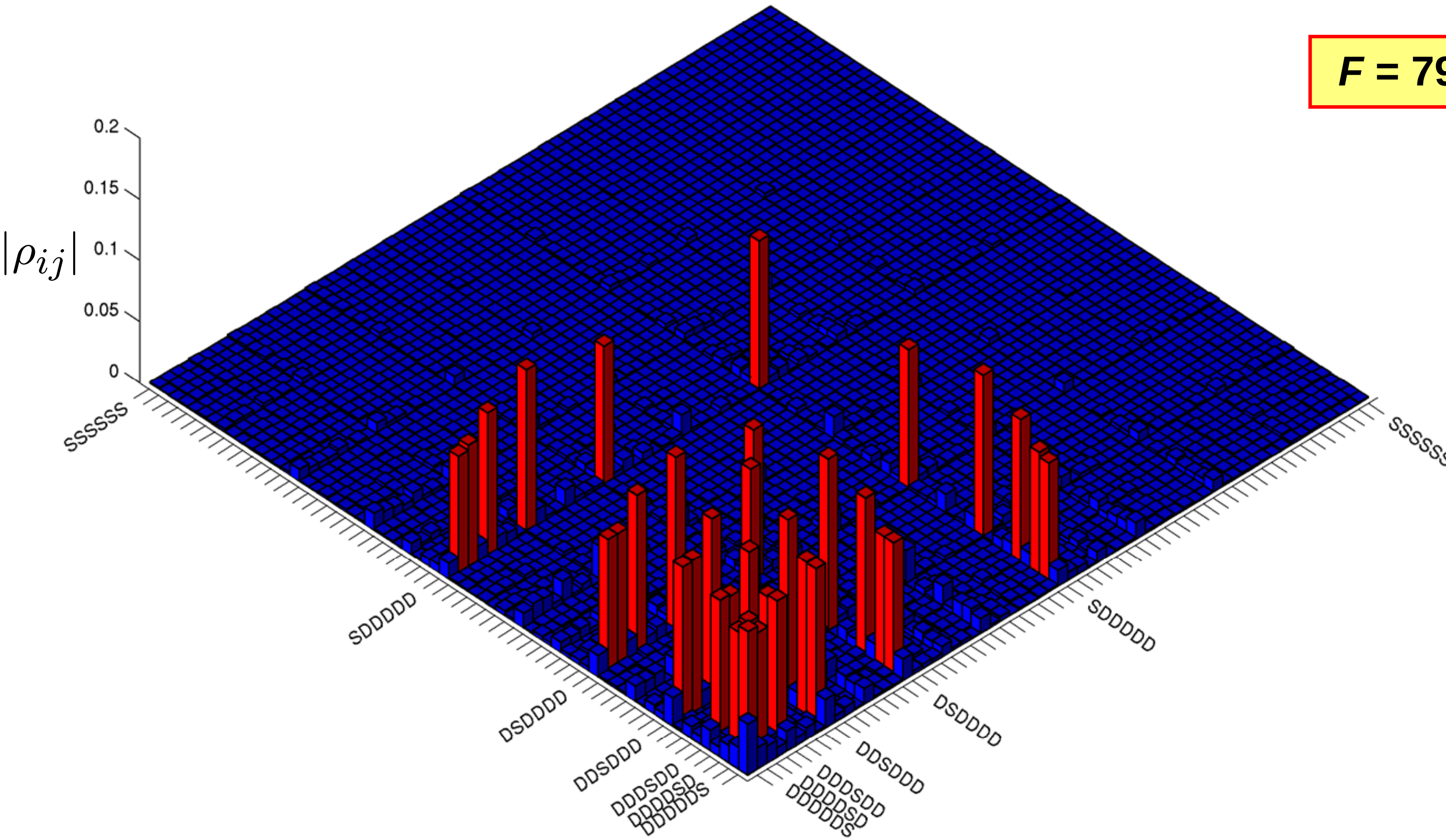
$$\Psi_5 = \frac{1}{\sqrt{5}}(|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + |DSDDD\rangle + |SDDDD\rangle)$$



Six-ion W-states

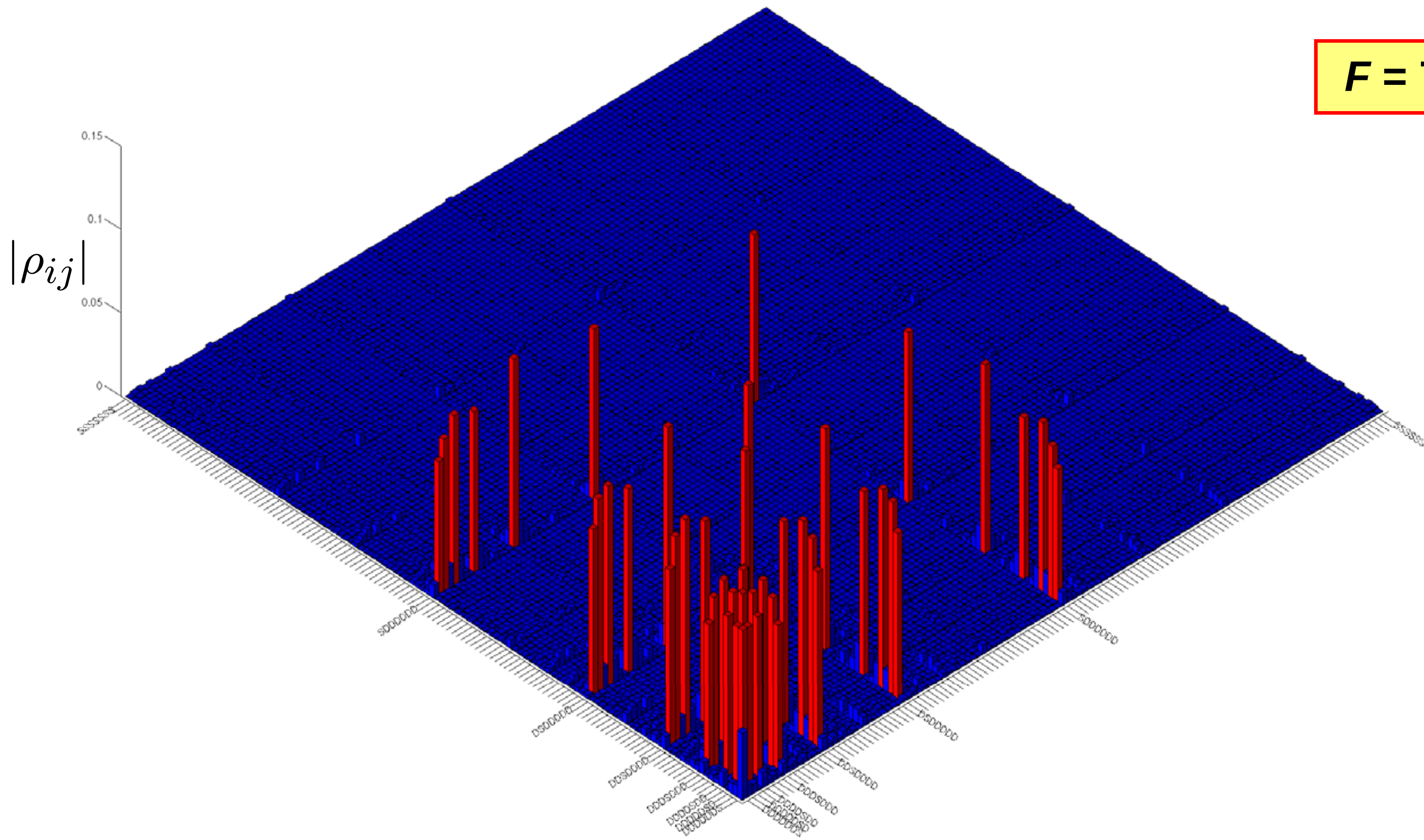
$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle +$$
$$|DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$$

F = 79%



729 settings, measurement time: 40 minutes

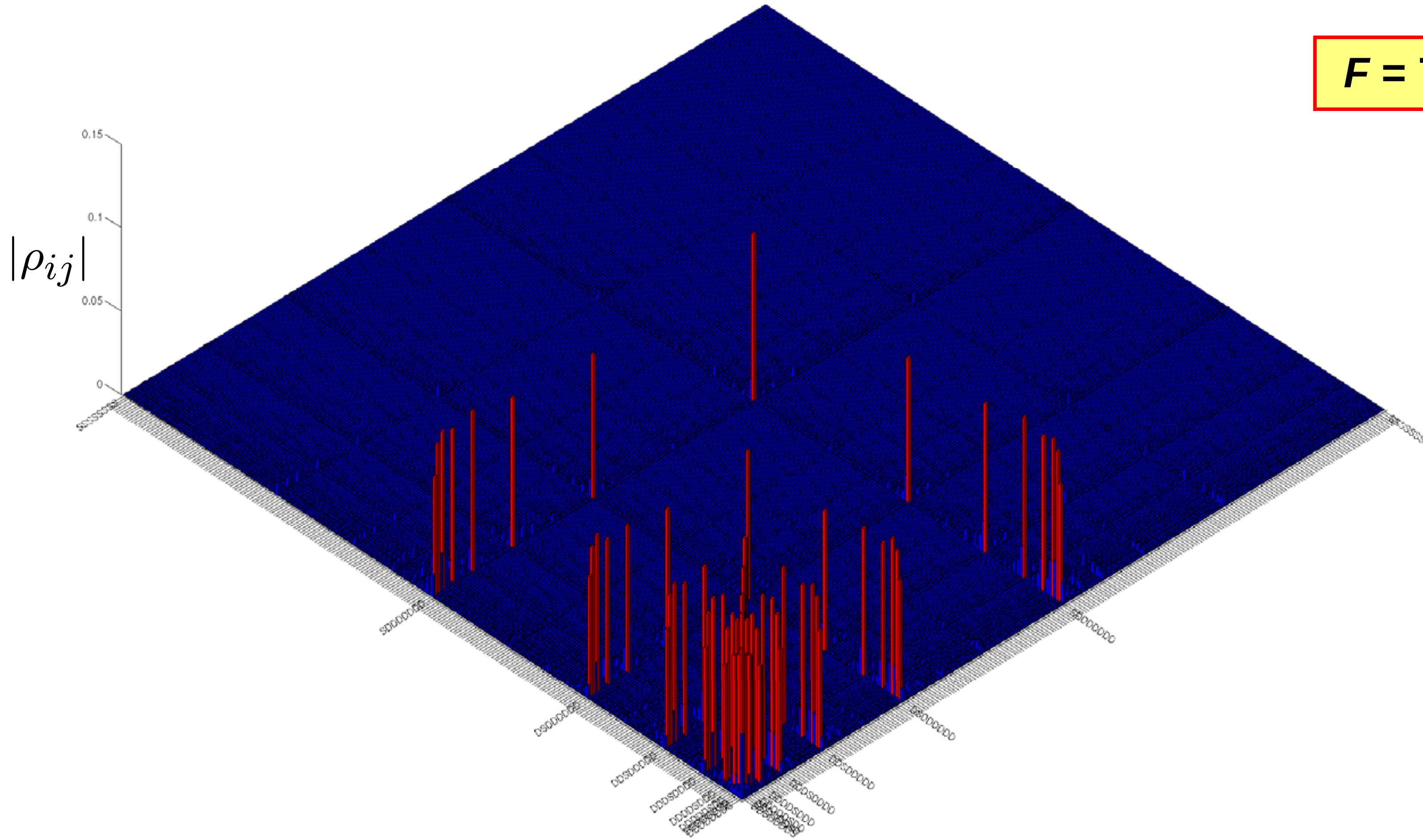
Seven-ion W-states



***F* = 76%**

2187 settings, measurement time: 2 hours

Eight-ion W-states



F = 72%

H. Häffner *et al*., Nature **438**, 643 (2005)

6561 settings, measurement time: 10 hours