Quantum information processing with trapped ions

Part 2:

- Entangling quantum gates
- Elements of quantum computing
- Noise in trapped-ion experiments
- Precision spectroscopy with entangled states

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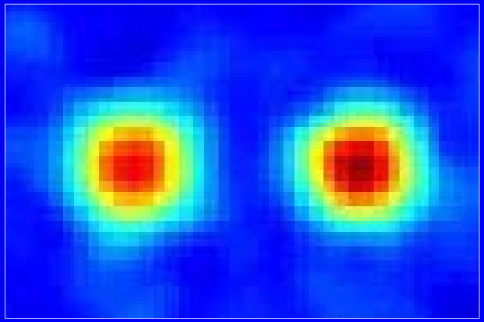
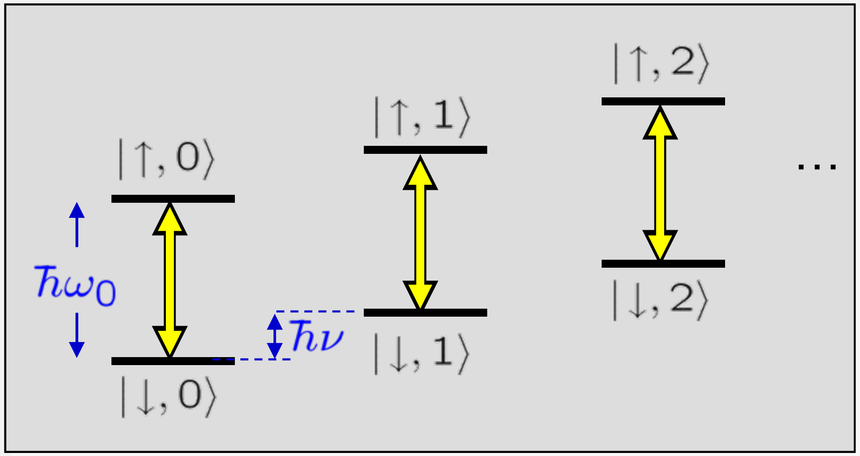
Entangling quantum gates



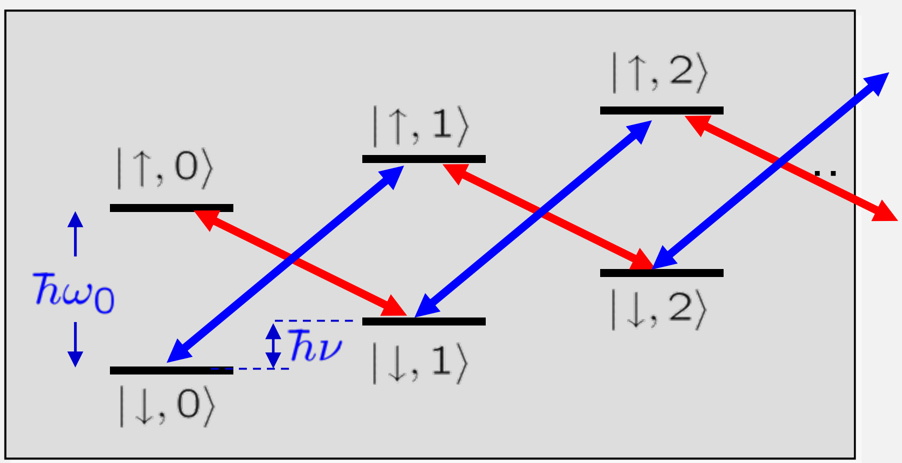
Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x,\ H \propto \sigma_y$$



qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$

$$H \propto \sigma_+ a + \sigma_- a^\dagger$$

$$\omega_{laser} = \omega_0 + \nu$$

$$H \propto \sigma_+ a^\dagger + \sigma_- a$$

Entangling quantum gates

Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	π	carrier
2	π	blue sideband

State transformation

$$|\downarrow\downarrow\rangle|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)|0\rangle$$

$$|\uparrow\downarrow\rangle|0\rangle \longrightarrow |\uparrow\uparrow\rangle|0\rangle$$

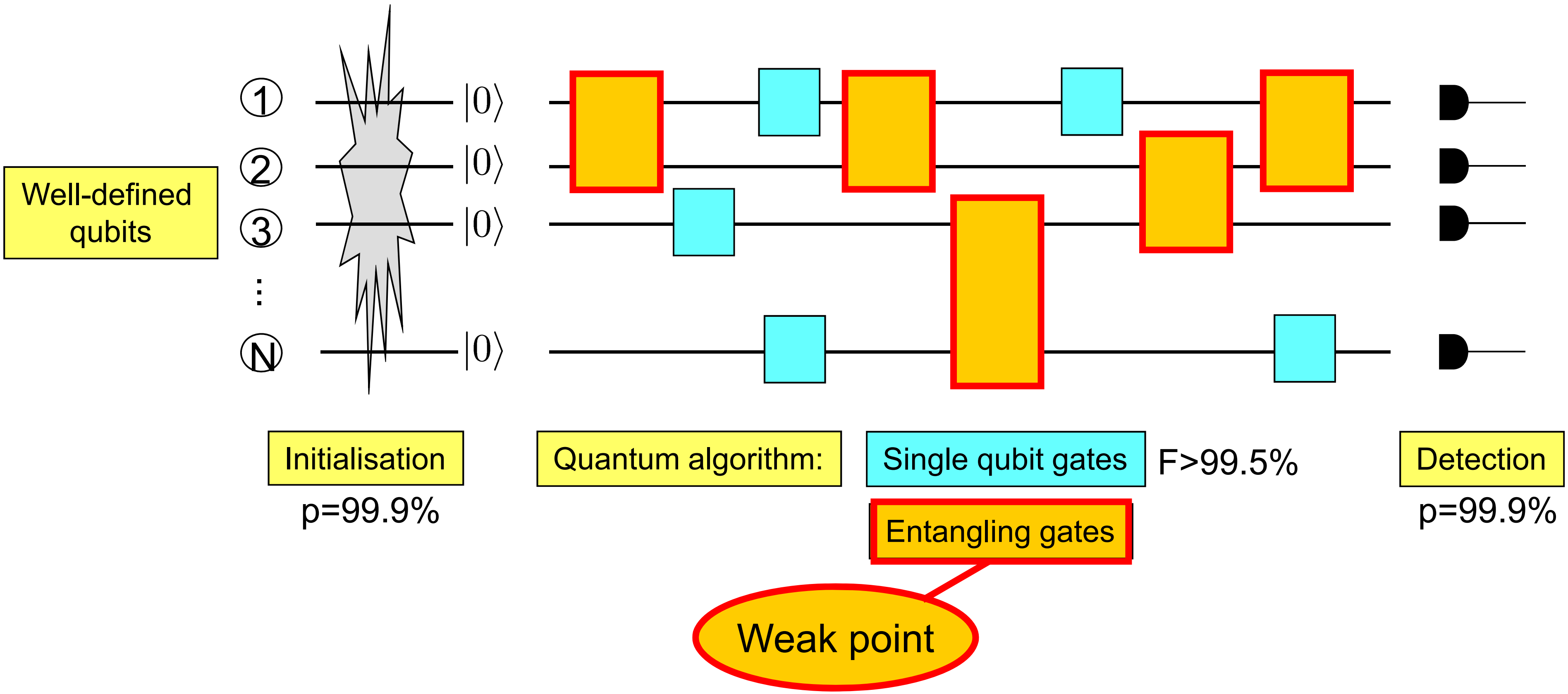
$$|\uparrow\uparrow\rangle|0\rangle \longrightarrow |\uparrow\uparrow\rangle|1\rangle$$

$$|\downarrow\uparrow\rangle|0\rangle \longrightarrow \alpha|\downarrow\downarrow\rangle|0\rangle + \beta|\uparrow\downarrow\rangle|1\rangle + |\downarrow\uparrow\rangle(\gamma|1\rangle + \delta|2\rangle)$$

- Can we devise a pulse sequence that maps the two-qubit state space onto itself?
- Can we devise a pulse sequence that product states onto Bell states?

Quantum computing with trapped ions

Scheme:



- Conditional phase gate (Boulder, 2003): F ≈ 97 %
- Controlled-NOT gate (Innsbruck, 2003): F ≈ 93 %
- Mølmer-Sørensen gate (Innsbruck, 2008): F ≈ 99 %

Quantum Computations with Cold Trapped Ions

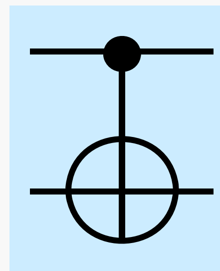
J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

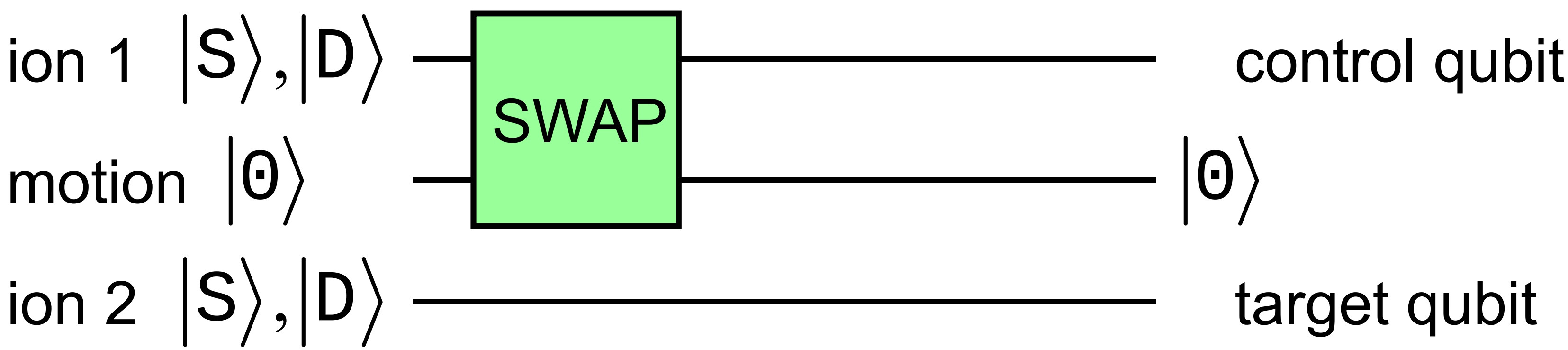
Cirac - Zoller two-ion controlled-NOT operation



$$
\begin{aligned}
|S\rangle|S\rangle &\rightarrow |S\rangle|S\rangle \\
|S\rangle|D\rangle &\rightarrow |S\rangle|D\rangle \\
|D\rangle|S\rangle &\rightarrow |D\rangle|D\rangle \\
|D\rangle|D\rangle &\rightarrow |D\rangle|S\rangle
\end{aligned}
$$

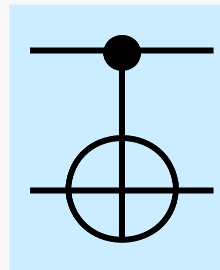
control target

Phys. Rev. Lett. **74**, 4091 (1995)



Cirac - Zoller two-ion controlled-NOT operation

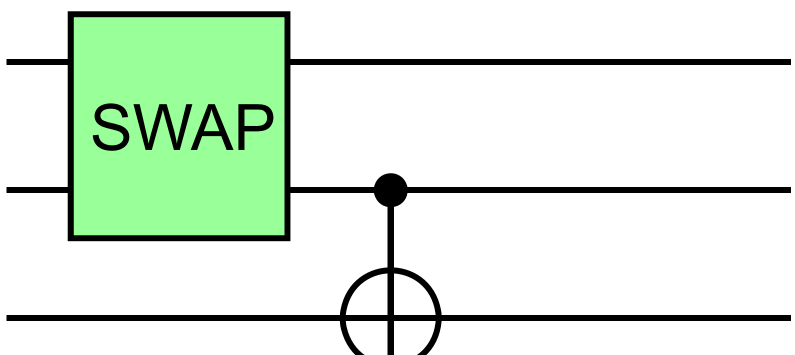
$|\varepsilon_1\rangle \quad |\varepsilon_2\rangle$



$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$
$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$
$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$
$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

control target

Phys. Rev. Lett. **74**, 4091 (1995)



Cirac - Zoller two-ion controlled-NOT operation

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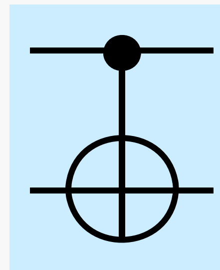
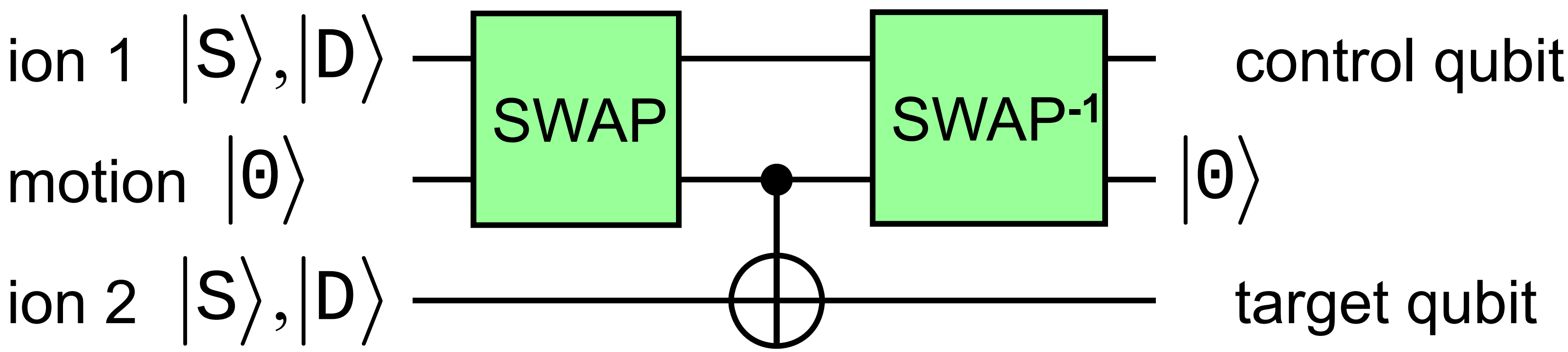


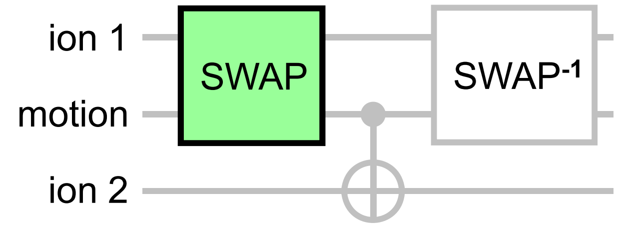
$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$
$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$
$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$
$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

control target

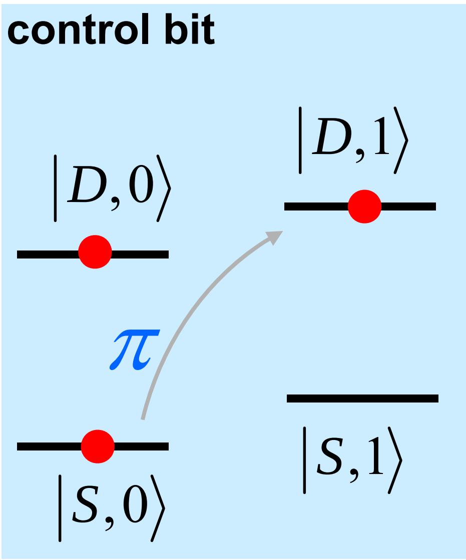
Phys. Rev. Lett. **74**, 4091 (1995)



CNOT gate (1)



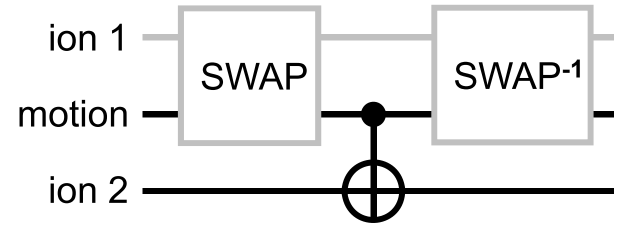
SWAP operation : π-pulse on blue sideband



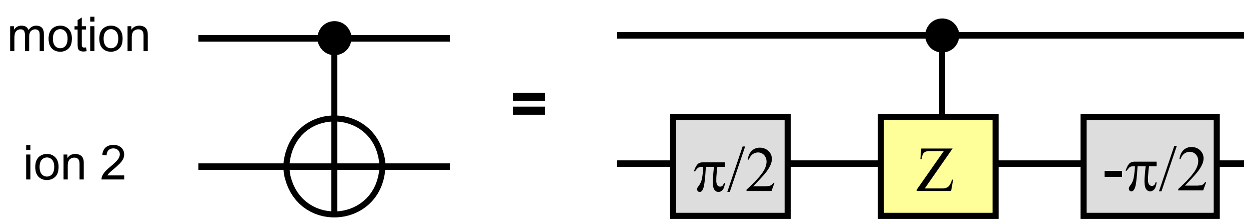
$$(\alpha|S\rangle + \beta|D\rangle)|0\rangle$$

$$\longrightarrow \quad |D\rangle(\alpha|0\rangle + \beta|1\rangle)$$

CNOT gate (2)



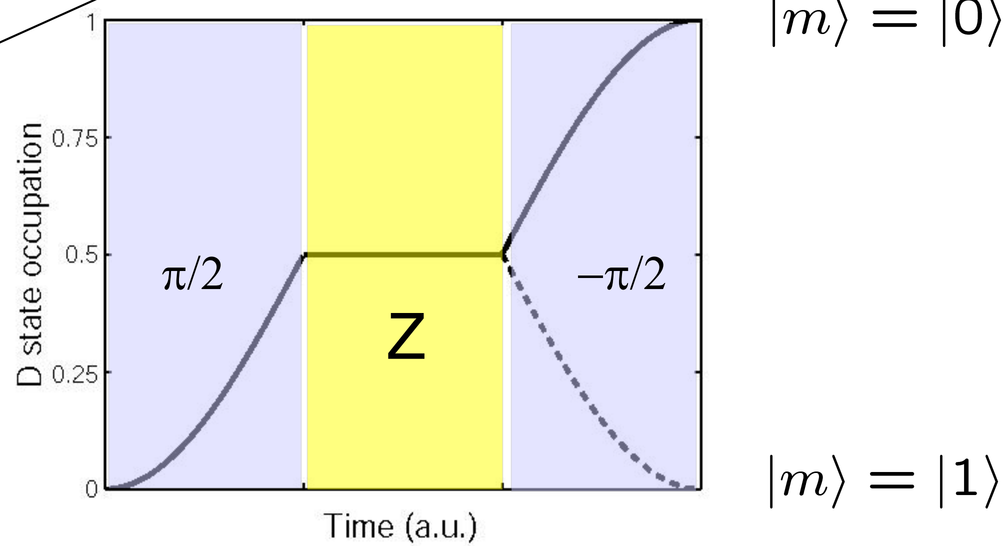
CNOT between motion and ion 2 :



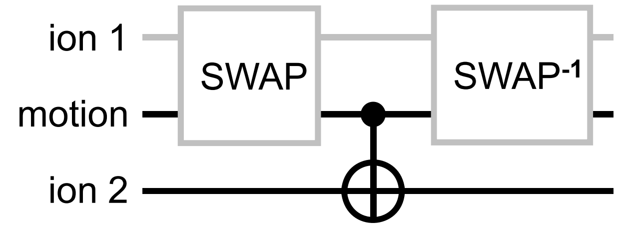
Conditional phase shift:

changes relative phase between |S> and |D> by 0 or π depending on motional state

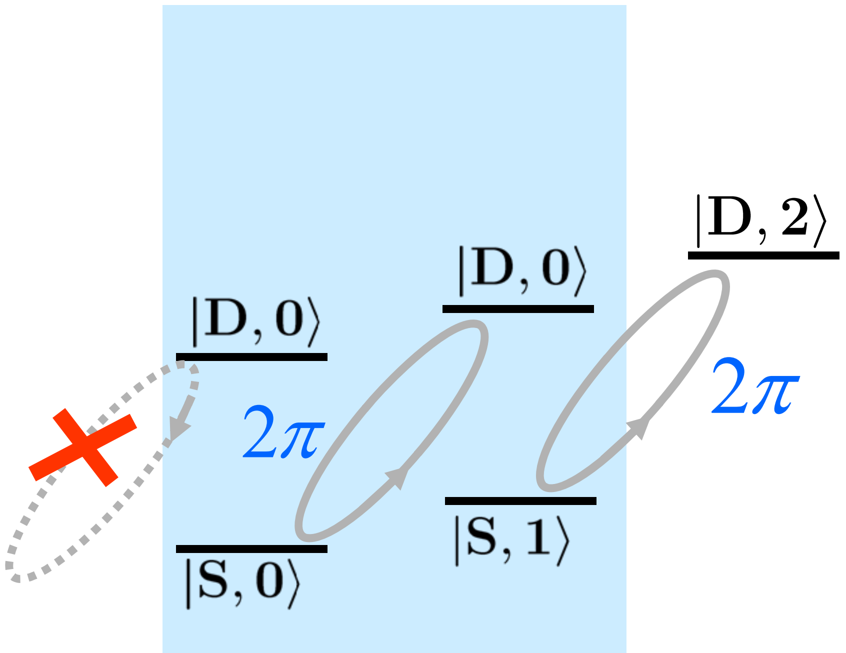
$$
\begin{array}{cc}
 & \begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\
U_\Phi = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\end{array}
$$



CNOT gate (3)



Conditional phase shift operation:



Rabi frequency:

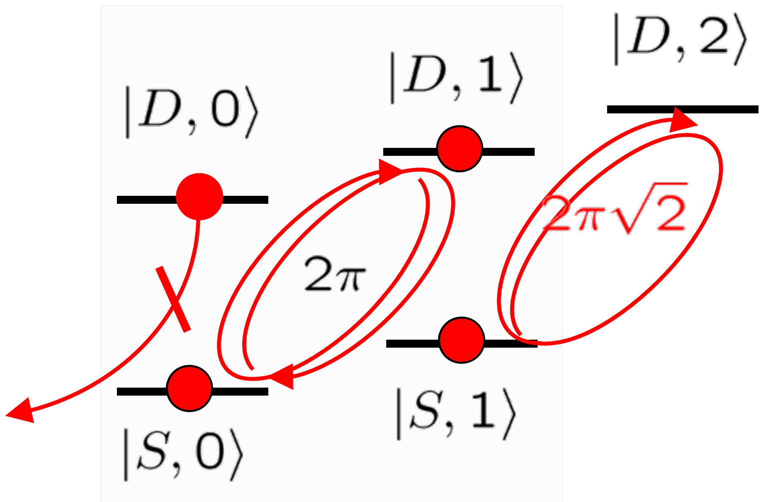
Blue SB: $\eta\Omega\sqrt{n+1}$

Composite pulse phase gate

Effect:
phase factor of **-1**
for all, except |D,0 >

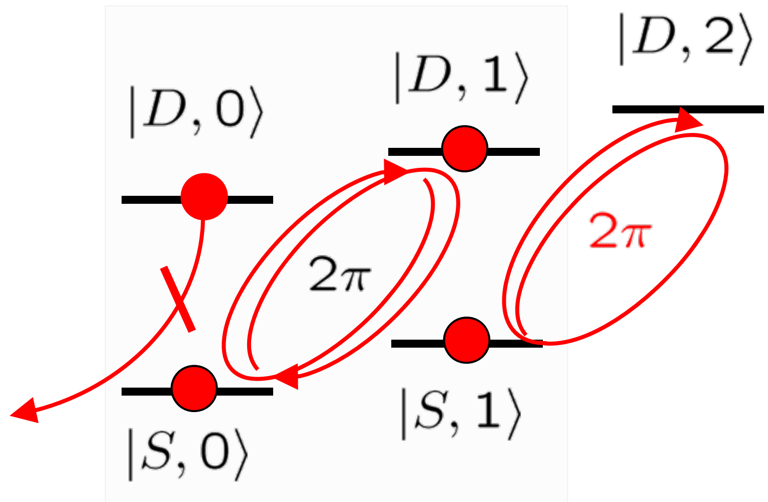
Conditional phase gate with a two-level system?

$$
\begin{array}{cccc}
 & |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \\
U_\Phi = & \multicolumn{4}{c}{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \boxed{?} \end{pmatrix}}
\end{array}
$$

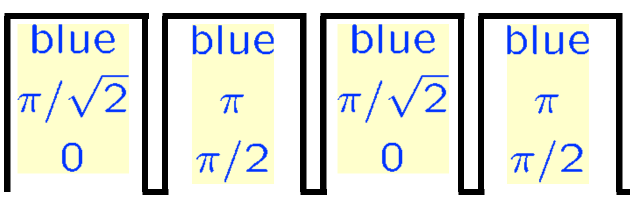


Conditional phase gate: composite pulses!

$$\begin{array}{cc} & \begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\ U_\Phi = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array}$$

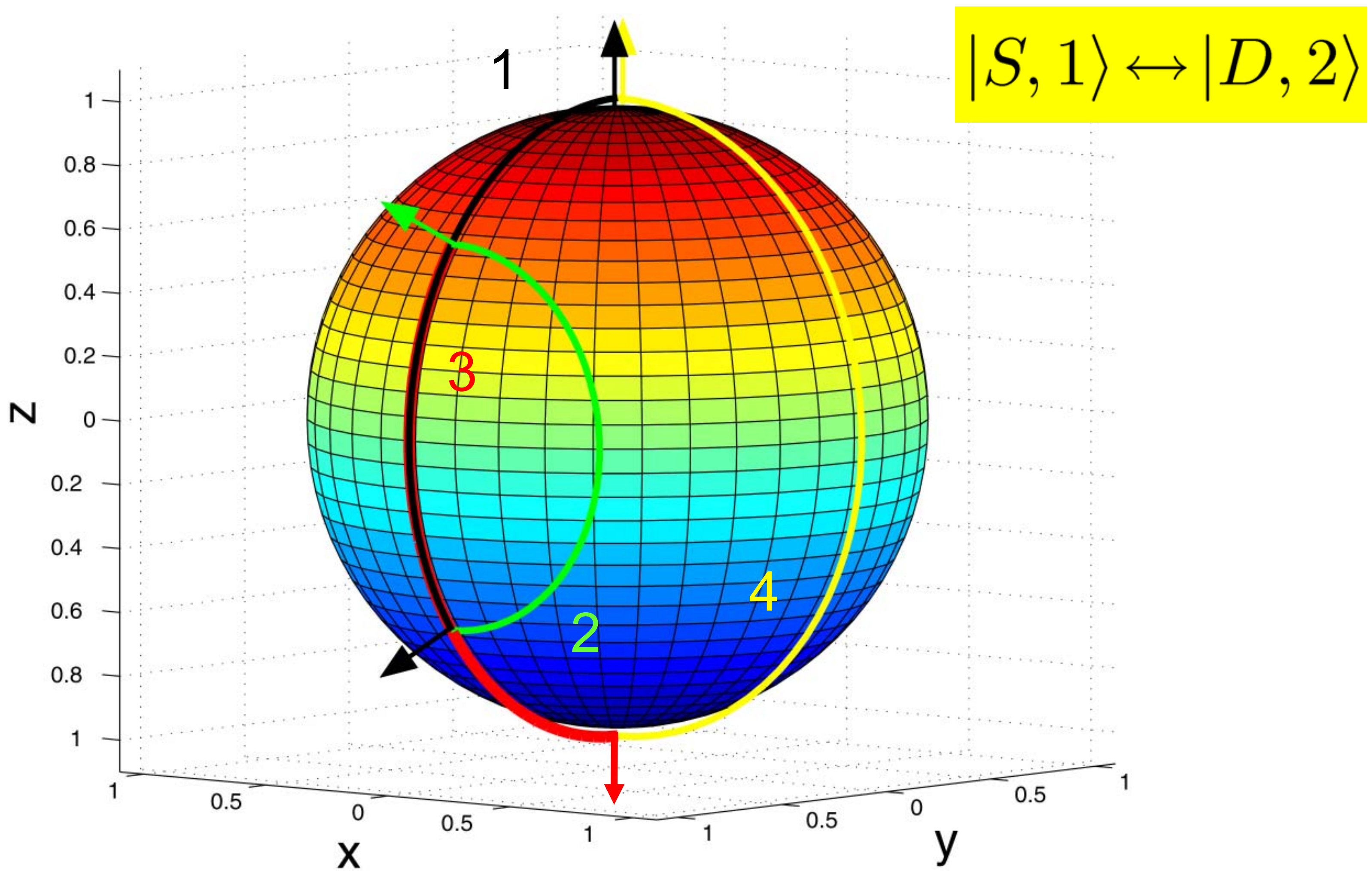


Composite 2π-rotation:



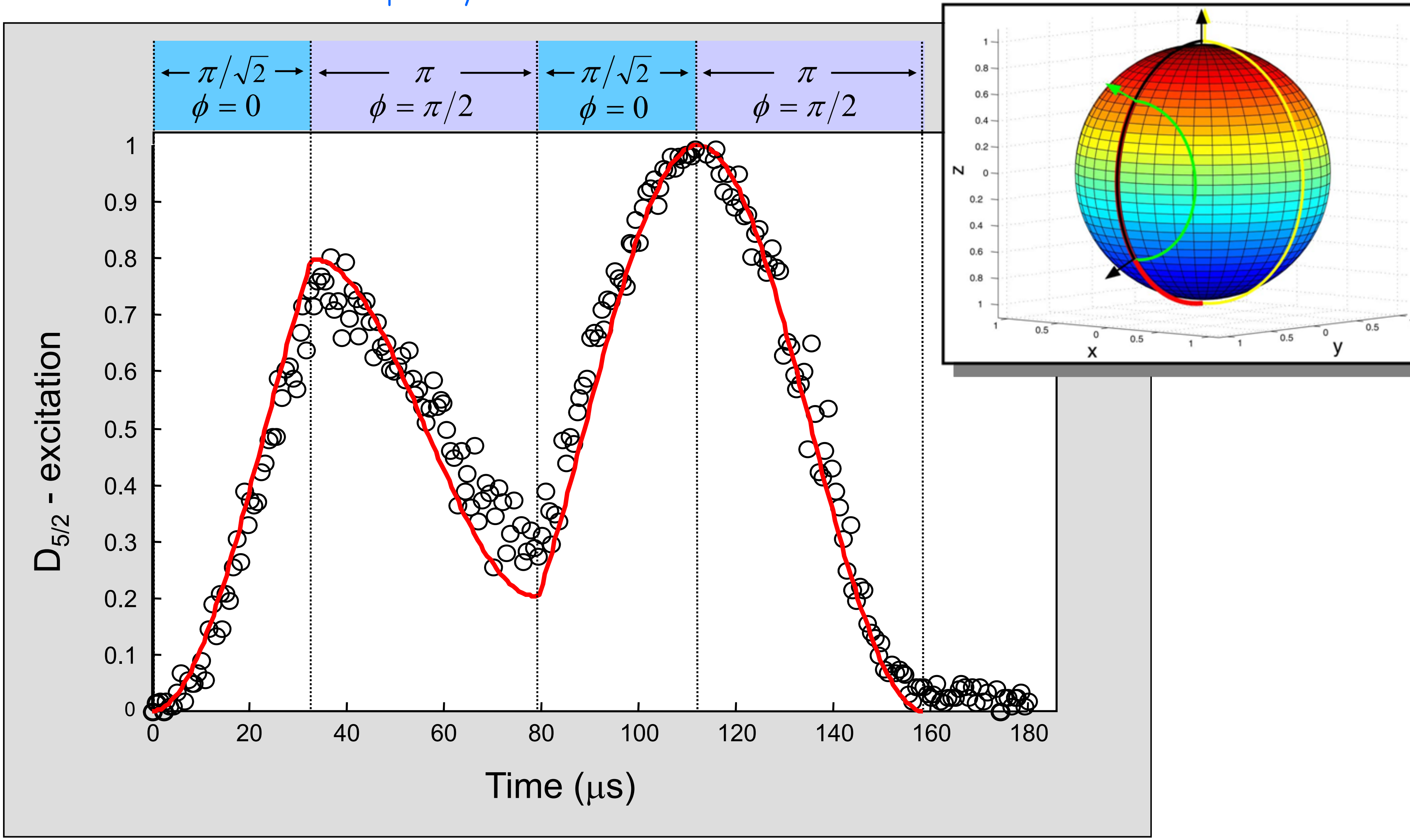
A phase gate with 4 pulses (2π rotation)

$$R(\theta,\phi) = R_1^+(\pi,\pi/2)\,R_1^+\left(\pi/\sqrt{2},0\right)R_1^+(\pi,\pi/2)\,R_1^+\left(\pi/\sqrt{2},0\right)$$



A single ion composite phase gate: Experiment

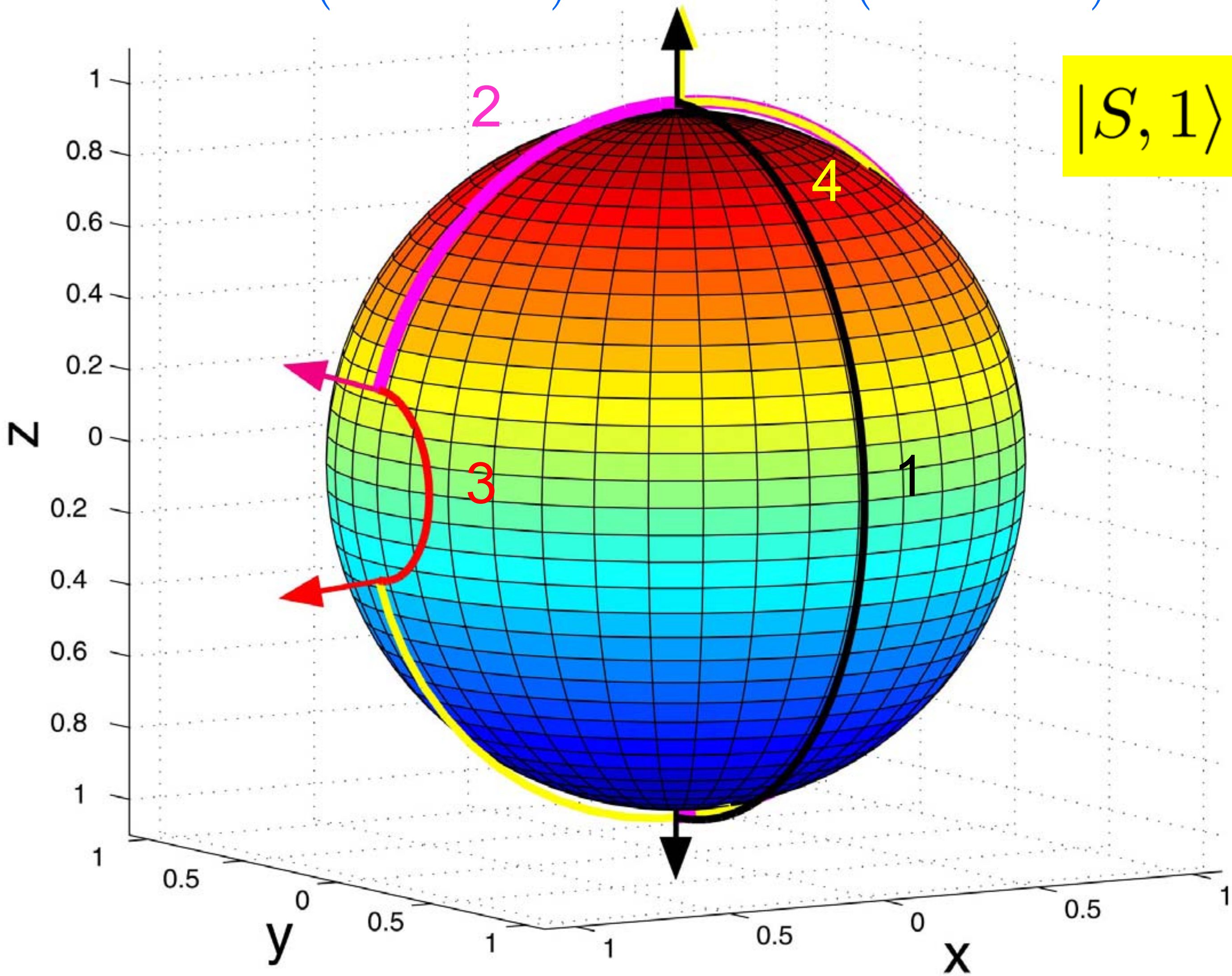
state preparation $|S,0\rangle$, then application of phase gate pulse sequence



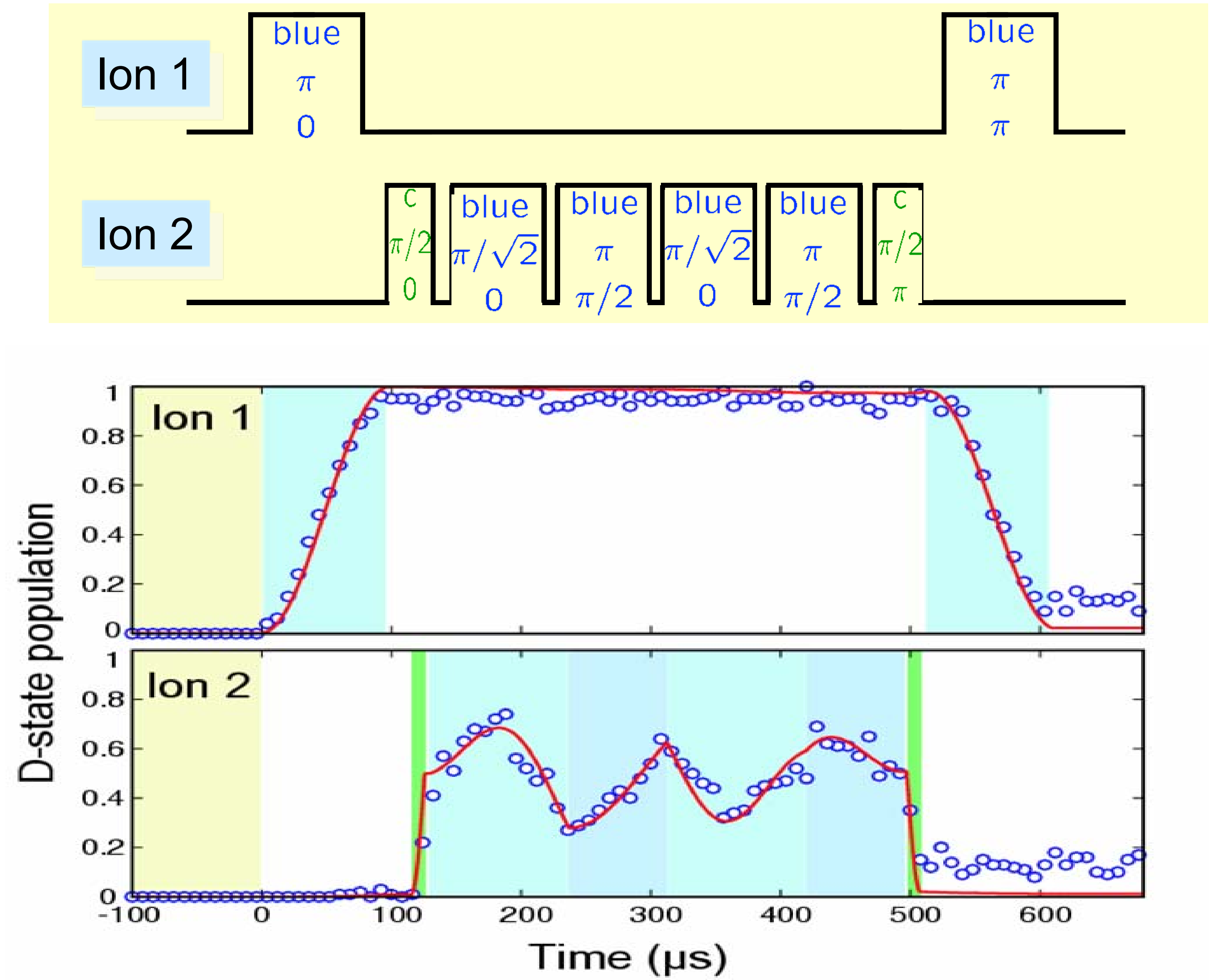
A phase gate with 4 pulses (2π rotation)

$$R(\theta,\phi) = R_1^+\left(\pi\sqrt{2},\pi/2\right) R_1^+\left(\pi,0\right) R_1^+\left(\pi\sqrt{2},\pi/2\right) R_1^+\left(\pi,0\right)$$

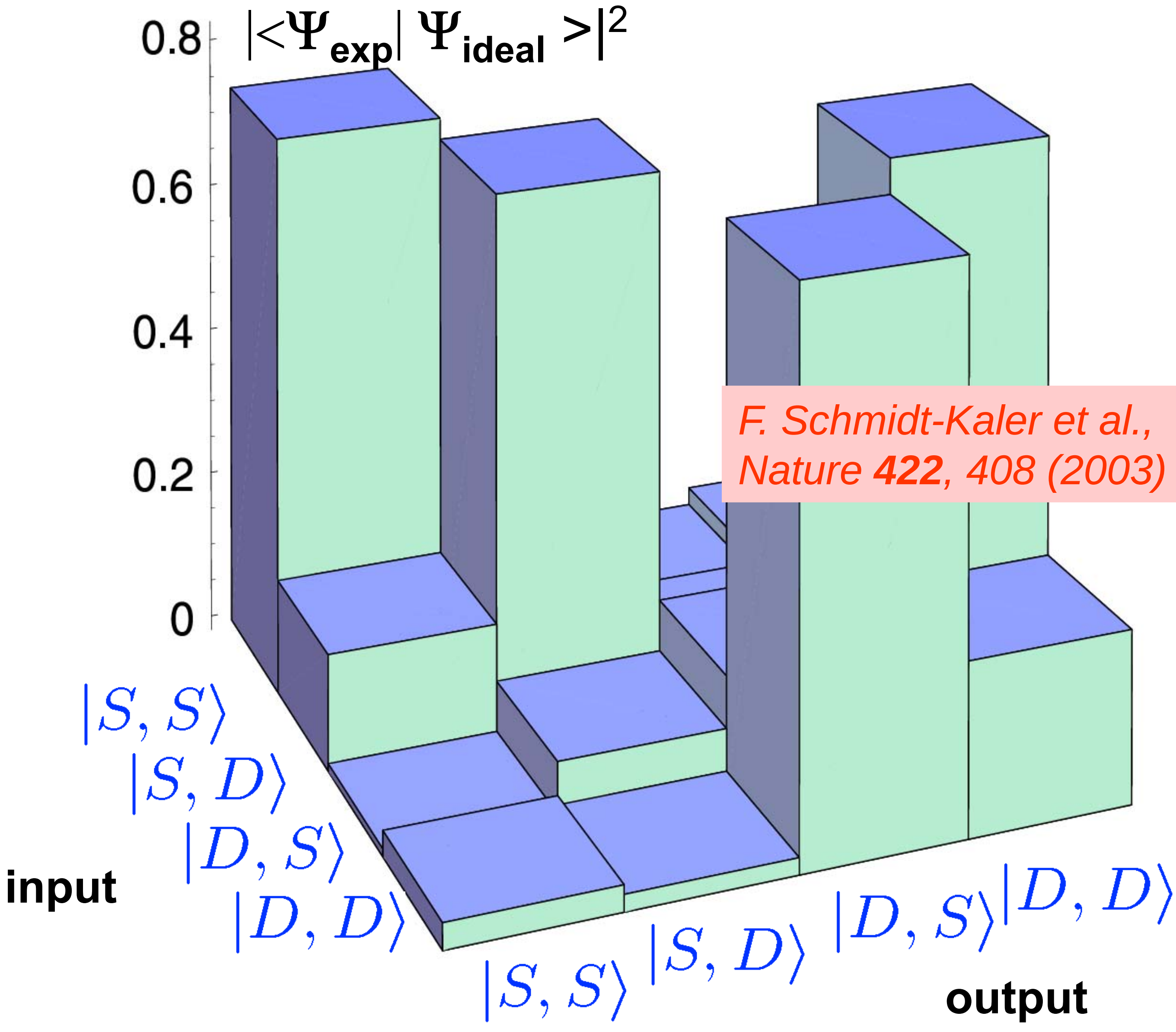
$$|S,1\rangle \leftrightarrow |D,2\rangle$$



CNOT gate: Complete pulse sequence



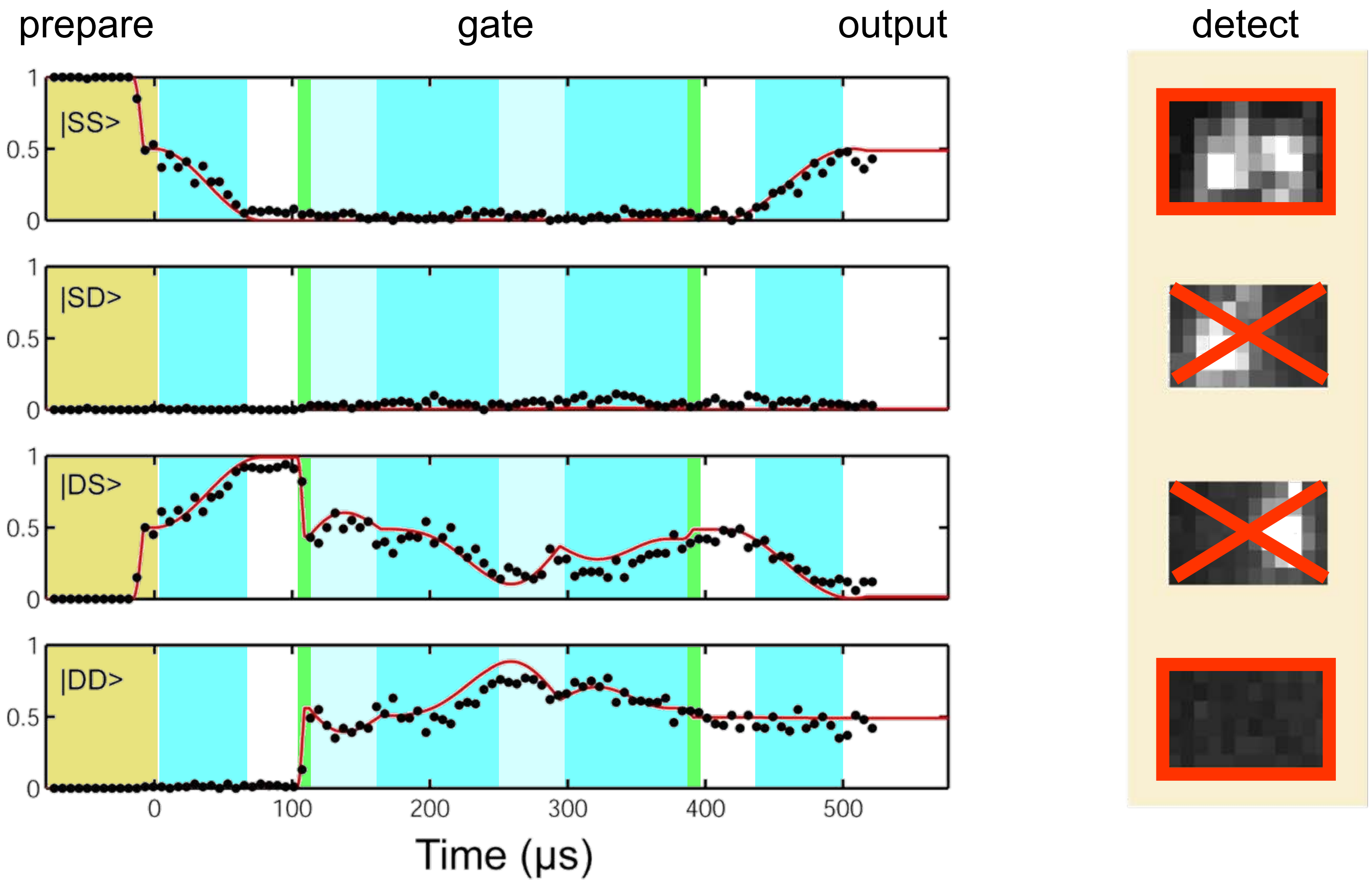
Fidelity of the CNOT operation



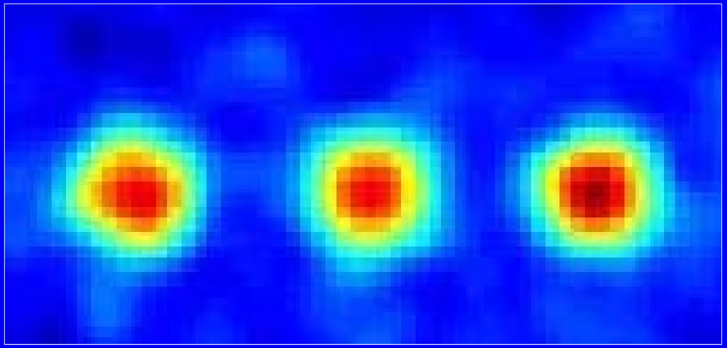
F. Schmidt-Kaler et al., Nature **422**, 408 (2003)

Superposition as input to CNOT gate

$$|D+S\rangle|S\rangle \longrightarrow |DD\rangle + |SS\rangle$$



Deterministic quantum teleportation with trapped ions



Quantum state teleportation

Phys. Rev. Lett. 70, 1895 (1993)

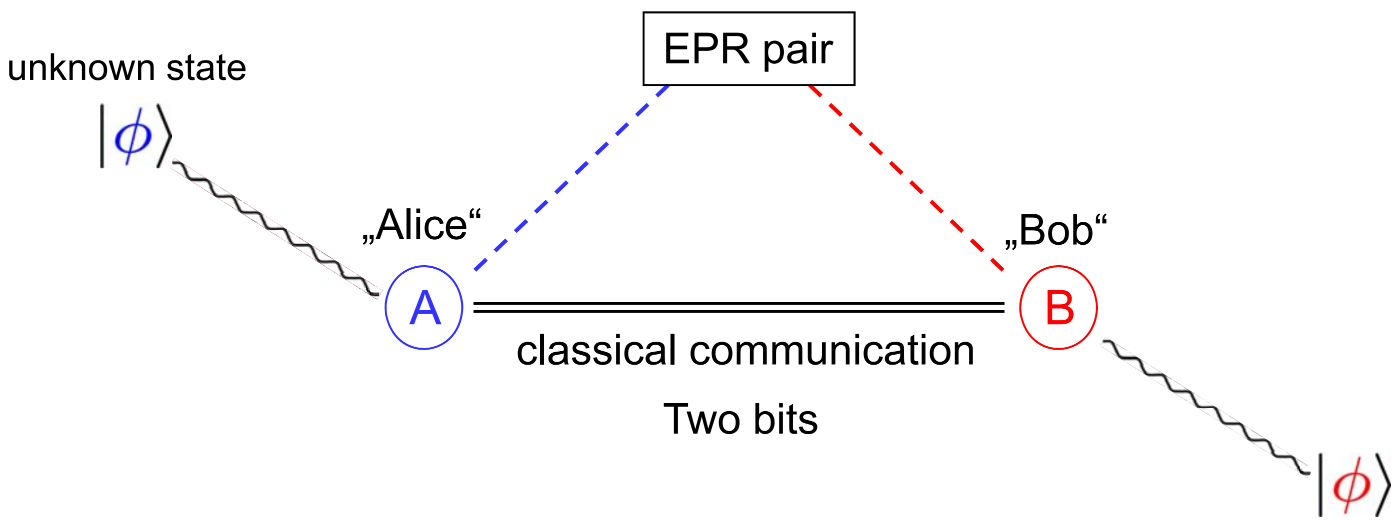
VOLUME 70 29 MARCH 1993 NUMBER 13

Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

Charles H. Bennett,[1] Gilles Brassard,[2] Claude Crépeau,[2],[3]
Richard Jozsa,[2] Asher Peres,[4] and William K. Wootters[5]

Is it possible to transfer an unknown quantum state $|\phi\rangle = \alpha|D\rangle + \beta|S\rangle$ from „Alice“ to „Bob“ by classical communication ?

Yes, if Alice and Bob share a pair of entangled particles !



Principle of teleportation

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|DS\rangle \pm |SD\rangle)$$

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|DD\rangle \pm |SS\rangle)$$

unknown state

$$|\phi\rangle = \alpha|D\rangle + \beta|S\rangle$$

EPR pair

$$|\Psi_-\rangle$$

$$(\alpha|D\rangle_1 + \beta|S\rangle_1)(|SD\rangle_{23} - |DS\rangle_{23}) = \frac{1}{2}[\ |\Psi_-\rangle_{12}(-\alpha|D\rangle_3 - \beta|S\rangle_3\ )$$

$$+|\Psi_+\rangle_{12}(-\alpha|D\rangle_3 + \beta|S\rangle_3\ )$$

$$+|\Phi_-\rangle_{12}(\ \beta|D\rangle_3 + \alpha|S\rangle_3\ )$$

$$+|\Phi_+\rangle_{12}(-\beta|D\rangle_3 + \beta|S\rangle_3\ )]$$

Bob: Apply appropiate inverse operation
to restore the original state

Principle of teleportation

$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|DS\rangle \pm |SD\rangle)$$

$$|\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|DD\rangle \pm |SS\rangle)$$

unknown state

$$|\phi\rangle = \alpha|D\rangle + \beta|S\rangle$$

EPR pair

$$|\Psi_-\rangle$$

$$(\alpha|D\rangle_1 + \beta|S\rangle_1)(|SD\rangle_{23} - |DS\rangle_{23}) = \frac{1}{2}\Big[\,|\Psi_-\rangle_{12}\big(-I|\phi\rangle_3\big) + |\Psi_+\rangle_{12}\big(i\exp(i\tfrac{\pi}{2}\sigma_z)|\phi\rangle_3\big) + |\Phi_-\rangle_{12}\big(-i\exp(i\tfrac{\pi}{2}\sigma_x)|\phi\rangle_3\big) + |\Phi_+\rangle_{12}\big(-\exp(i\tfrac{\pi}{2}\sigma_y)|\phi\rangle_3\big)\Big]$$

Alice: Measurement in Bell state basis

$$\{\Psi_-, \Psi_+, \Phi_+, \Phi_-\}$$

Bob: Apply appropiate inverse operation
to restore the original state

Mapping between product and Bell basis

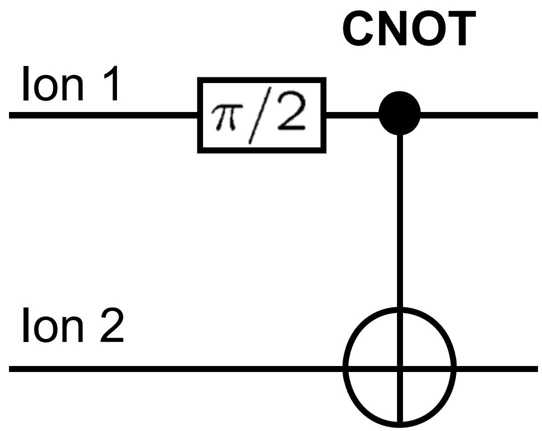
Product states

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

$|DD\rangle$



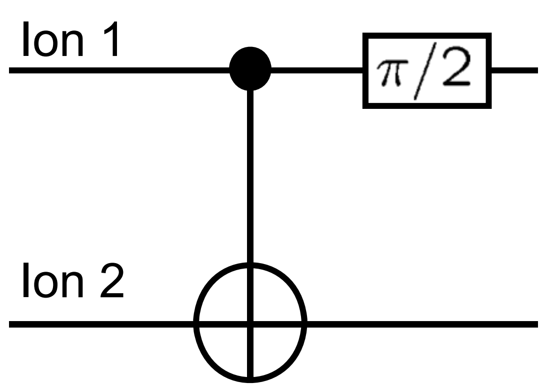
Bell states

$$\Psi_\pm = \frac{1}{\sqrt{2}}(|DS\rangle \pm |SD\rangle)$$

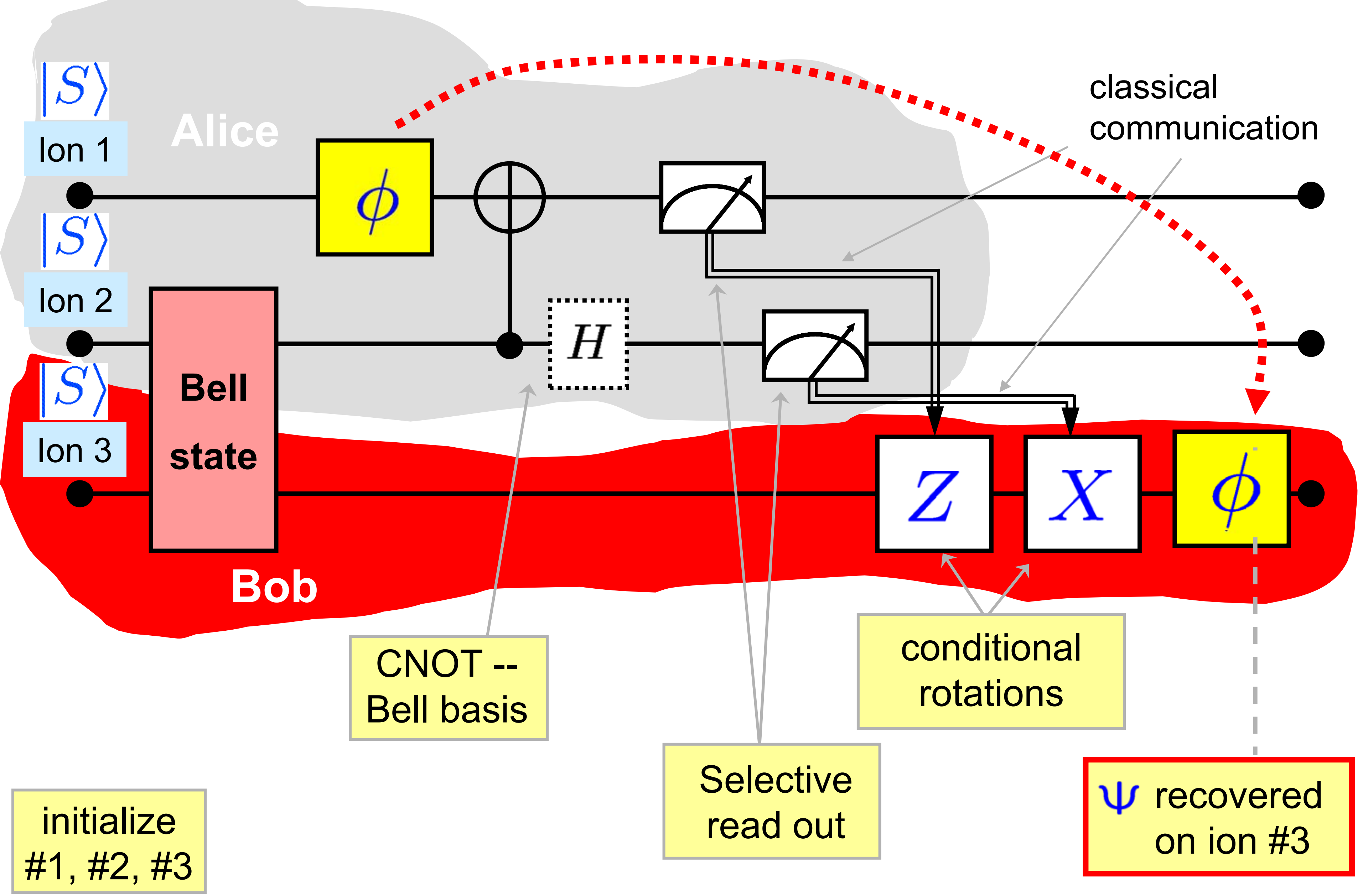
$$\Phi_\pm = \frac{1}{\sqrt{2}}(|DD\rangle \pm |SS\rangle)$$

Example $|SS\rangle \longrightarrow |S+D\rangle|S\rangle \longrightarrow |SS\rangle + |DD\rangle$

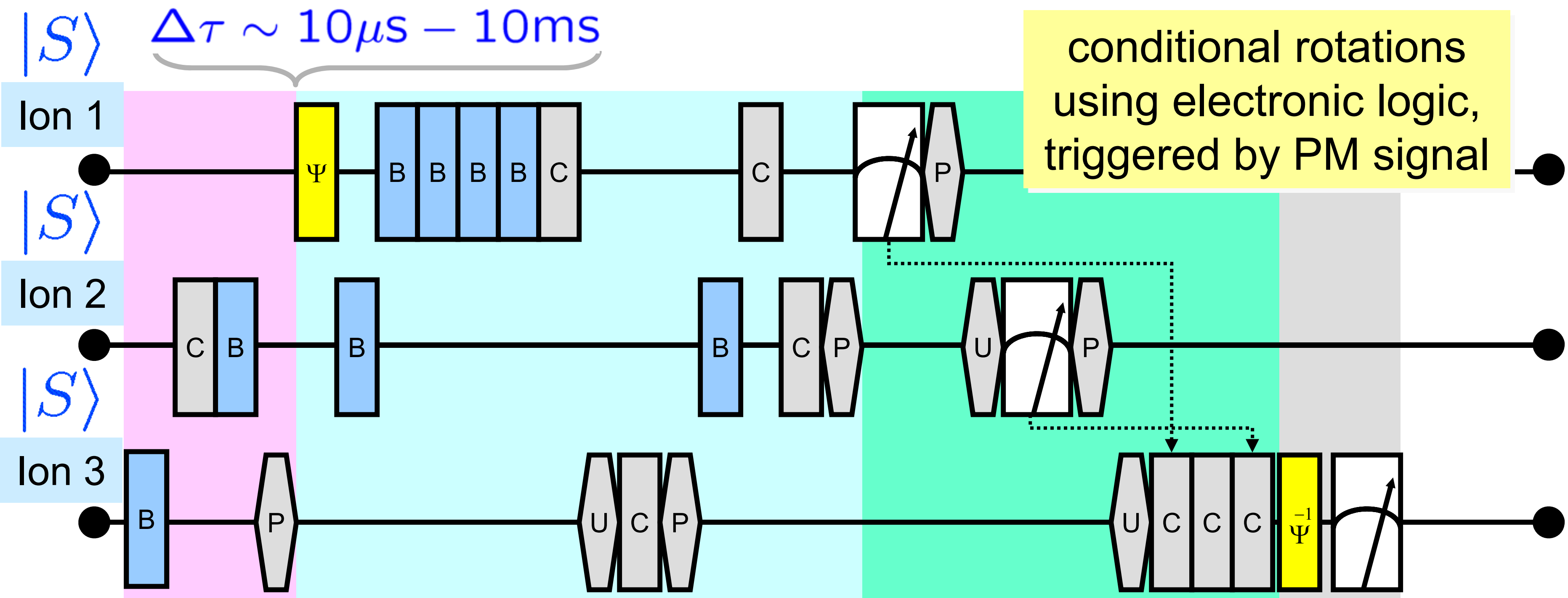
Inverse operation:



Teleportation protocol



Teleportation protocol, details



spin echo sequence

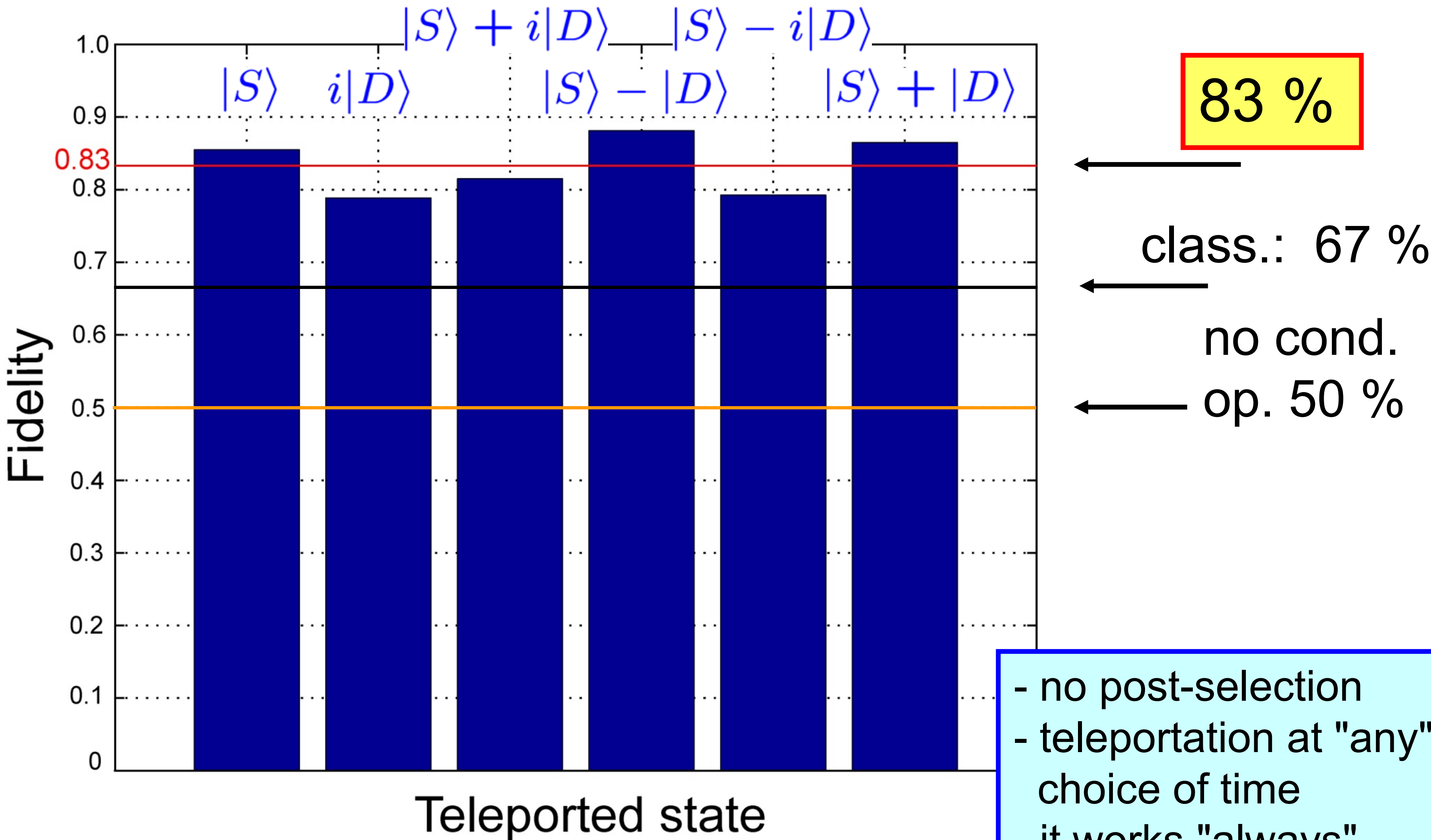
$\tau \approx 2$ ms

B blue sideband pulses $R^+(\theta, \varphi)$

C carrier pulses $R(\theta, \varphi)$

full sequence:
26 pulses + 2 measurements

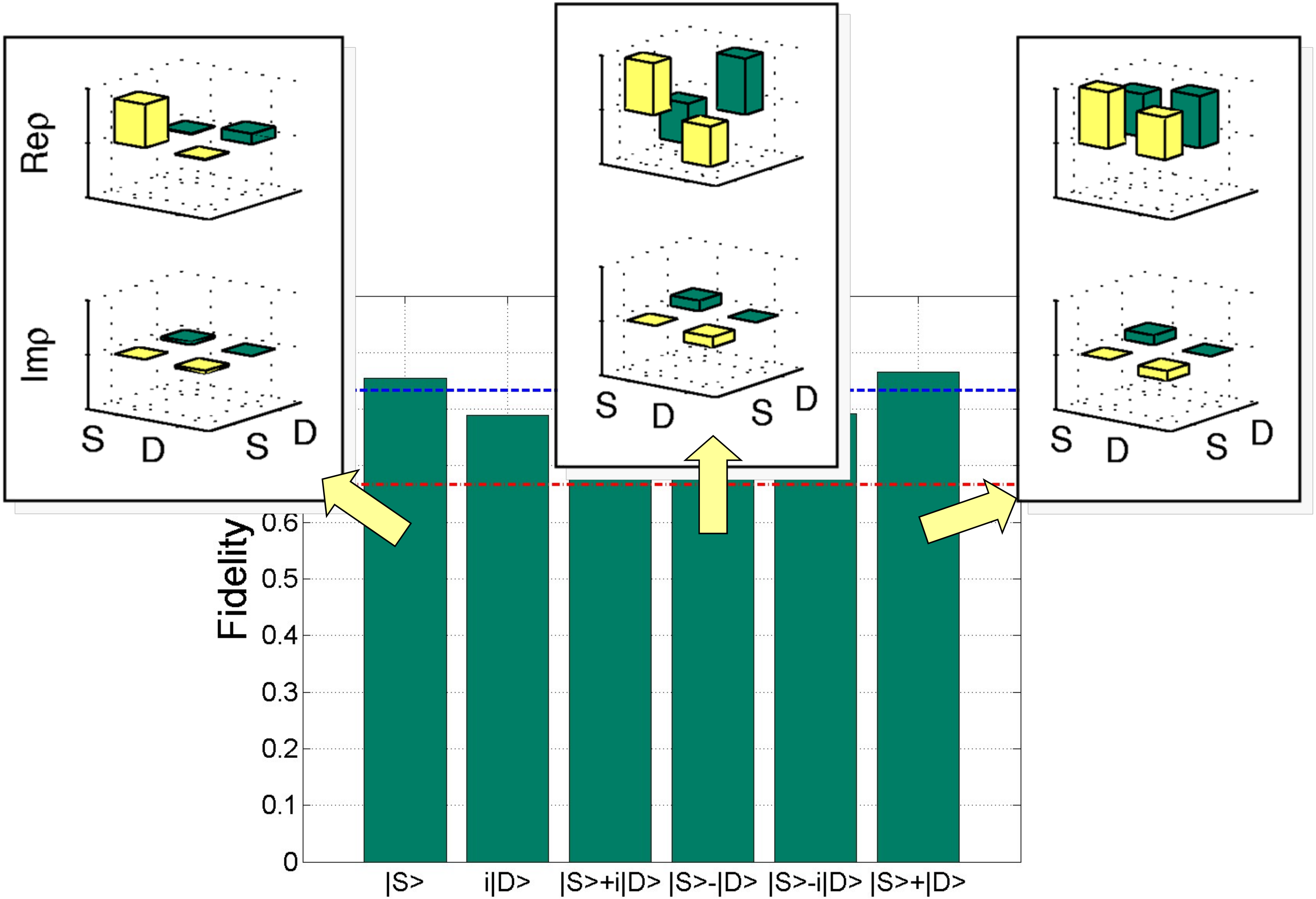
Quantum teleportation with atoms: result



- no post-selection
- teleportation at "any" choice of time
- it works "always"
- only 10 μm

M .Riebe et al., Nature **429**, 734 (2004)

Process tomography of quantum teleportation

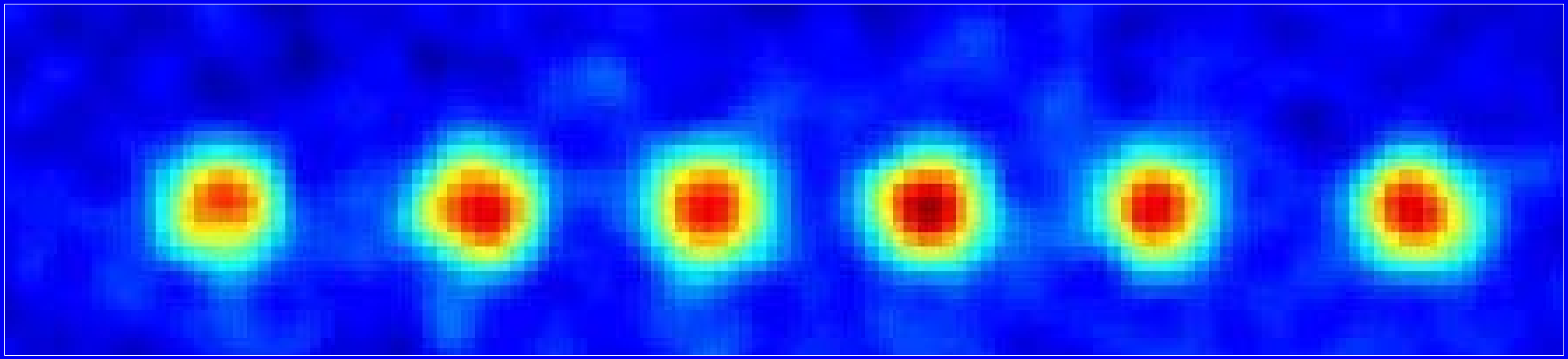


Mølmer-Sørensen gates

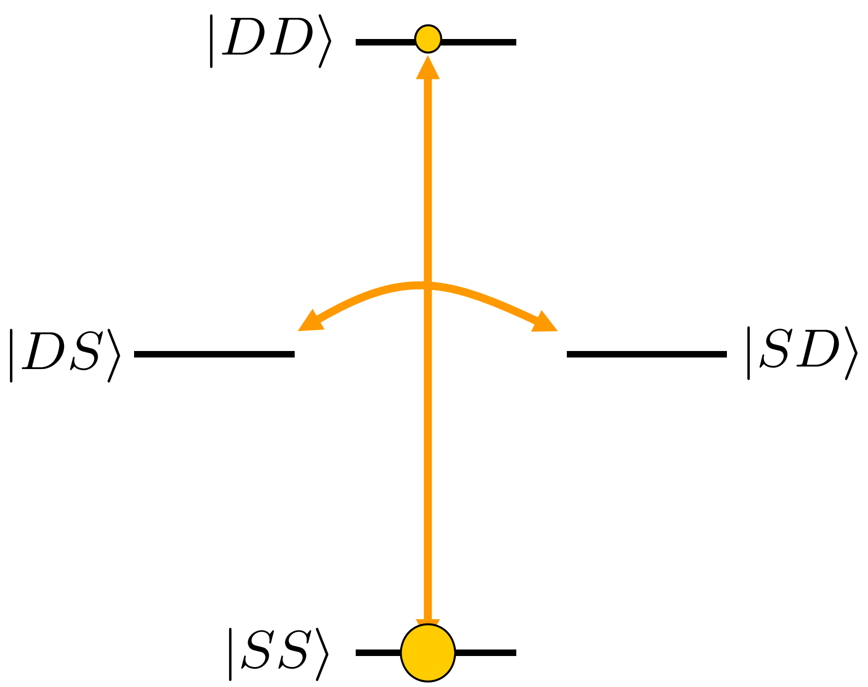
How does it work ?

Bell states: creation and verification

GHZ states



Entangling ions by correlated spin flips



Gate action: correlated spin flips

$\boxed{|DS\rangle \leftrightarrow |SD\rangle}$ $\boxed{|SS\rangle \leftrightarrow |DD\rangle}$

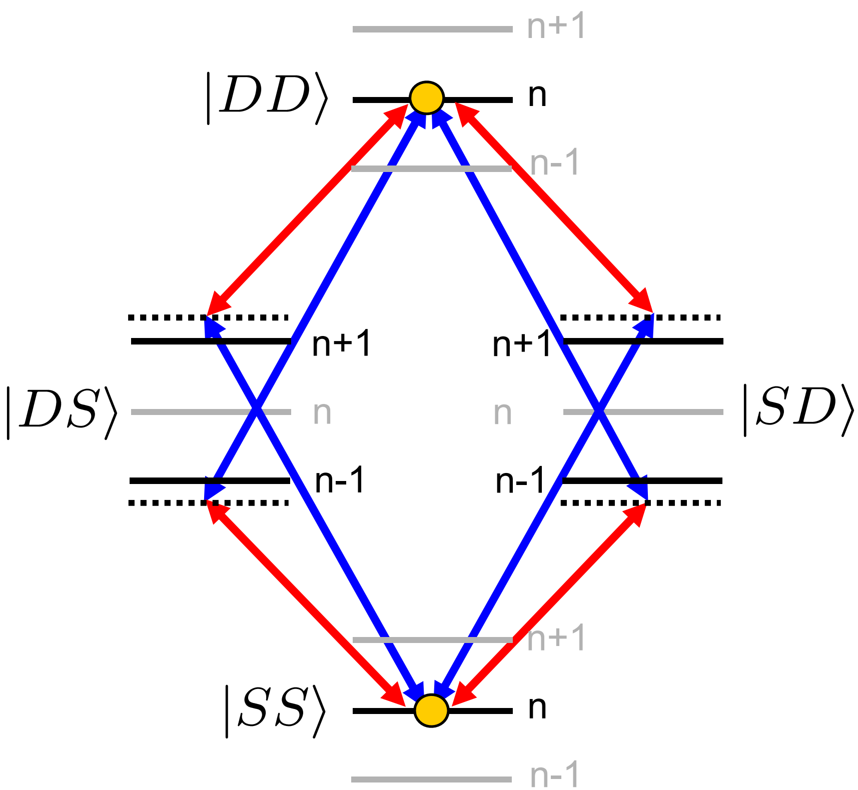
Entanglement generation:

$$|SS\rangle \longrightarrow |SS\rangle + |DD\rangle$$

How do we create correlated spin flips ?

$\longrightarrow$ Couple the ions via the vibrational mode !

Coupling to motional states: Two-photon transition



Gate action: correlated spin flips

$|DS\rangle \leftrightarrow |SD\rangle$ $\boxed{|SS\rangle \leftrightarrow |DD\rangle}$

Bichromatic laser field coupling to

upper motional sideband

lower motional sideband

$$\boxed{H_{eff} = J\sigma_x \otimes \sigma_x}$$

Theory:

A. Sørensen, K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1999)

A. Sørensen, K. Mølmer, Phys. Rev. A **62**, 022311 (2000)

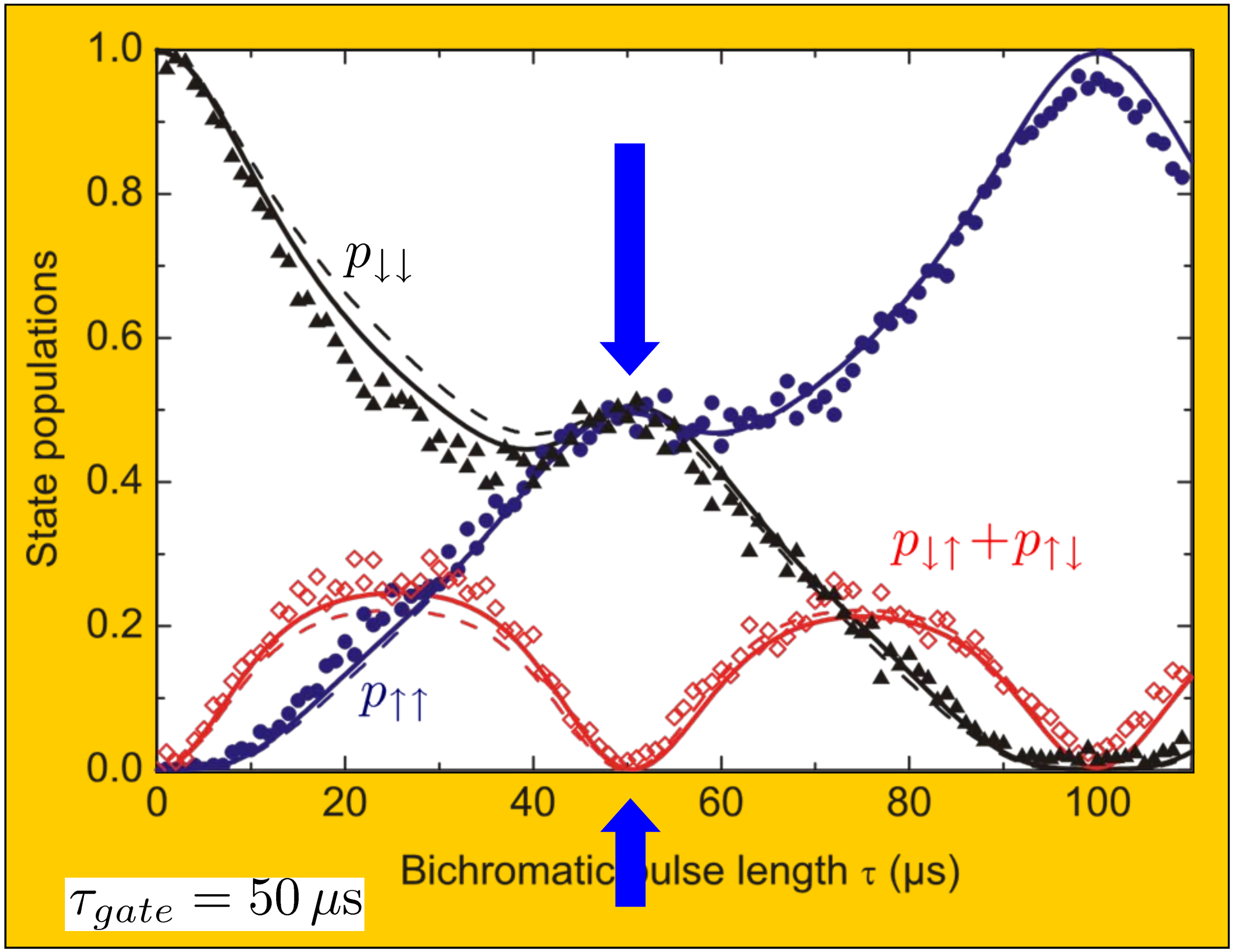
Experiments (Boulder + Ann Arbor):

C. A. Sackett et al, Nature **404**, 256 (2000)

P. Haljan et al., Phys. Rev. A **72**, 062316 (2005)

Creating Bell states

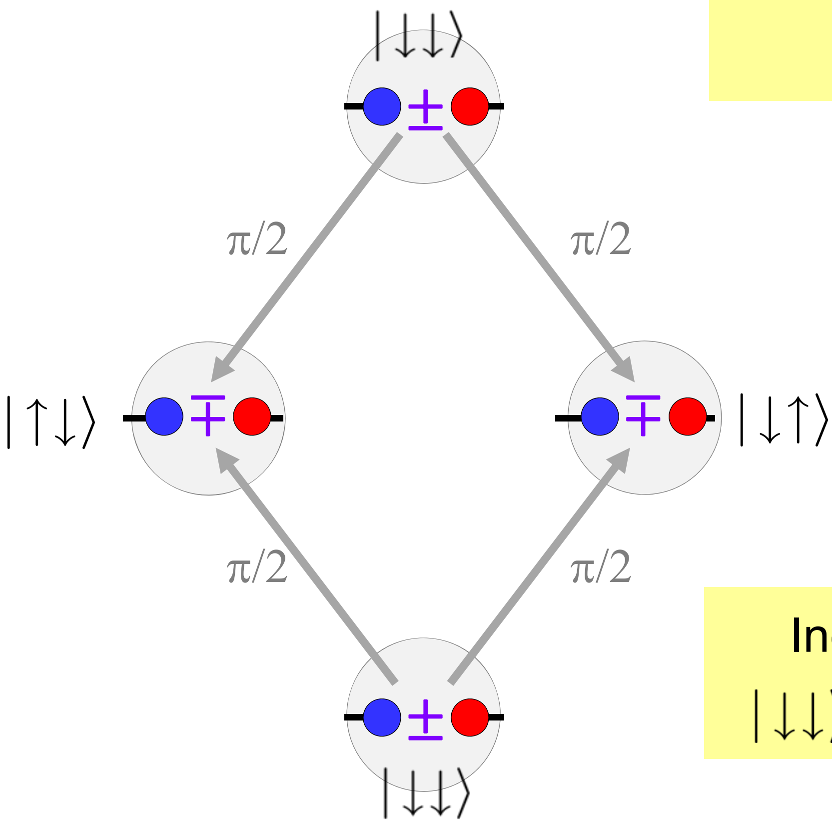
$$|\downarrow\downarrow\rangle \longrightarrow |\downarrow\downarrow\rangle + i|\uparrow\uparrow\rangle$$



$p_{\downarrow\downarrow}+p_{\uparrow\uparrow}$ = 0.9965(4) 13,000 measurements

J. Benhelm *et al.*, Nature Phys. **4**, 463 (2008)

Entanglement check : interference



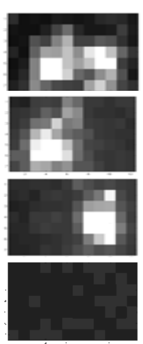
Maximally entangled state:

$$|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$$

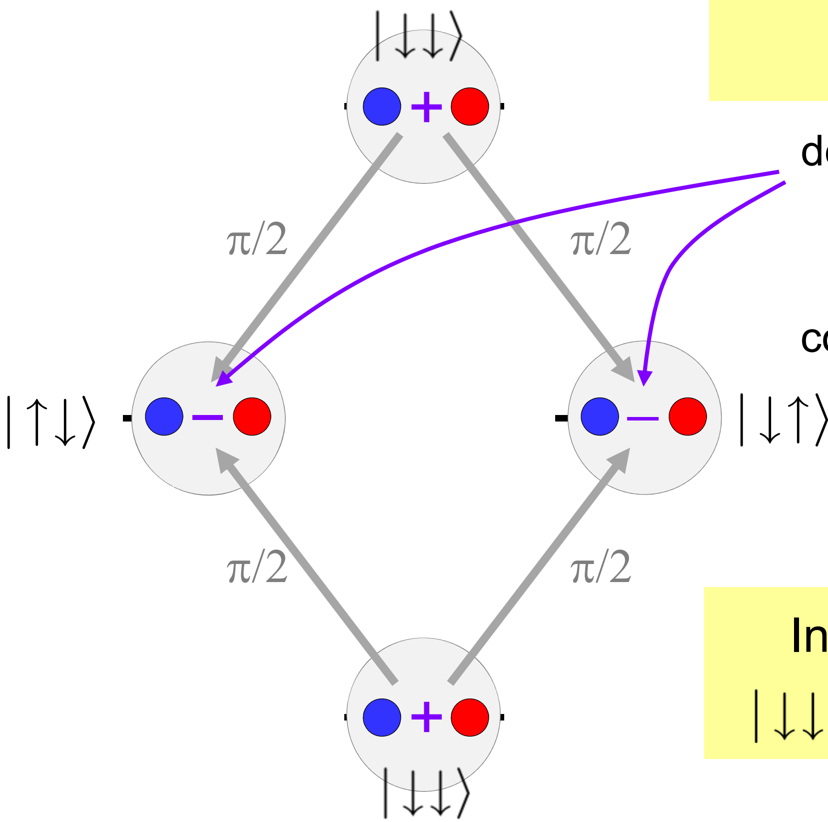
Incoherent mixture:

$$|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$

$\longrightarrow$ equal population in all states



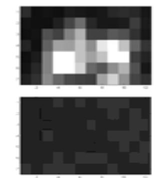
Entanglement check : interference



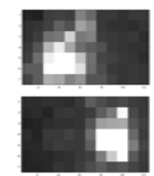
Maximally entangled state:

$$|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$$

destructive interference $\longrightarrow$ $|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$



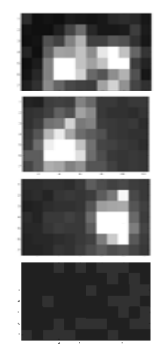
constructive interference $\longrightarrow$ $|\downarrow\uparrow\rangle + |\downarrow\uparrow\rangle$



Incoherent mixture:

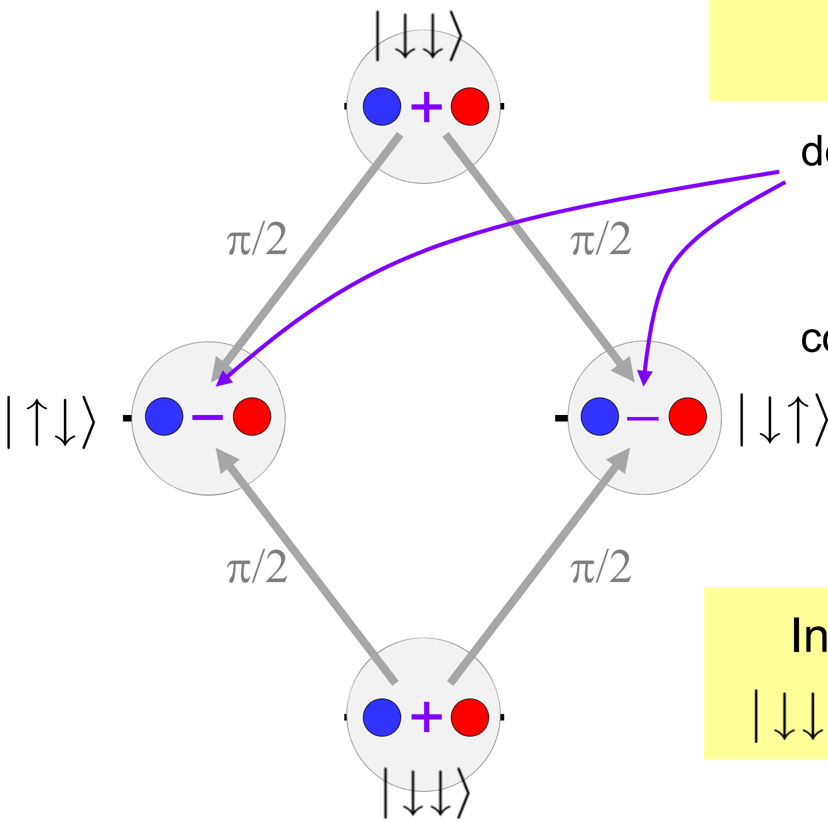
$$|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$

$\longrightarrow$ equal population in all states



Entanglement check: Scan laser phase ϕ and measure parity

Entanglement check : interference



Maximally entangled state:

$$|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$$

destructive interference $\longrightarrow$ $|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$

constructive interference $\longrightarrow$ $|\downarrow\uparrow\rangle + |\downarrow\uparrow\rangle$

Incoherent mixture:

$$|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$

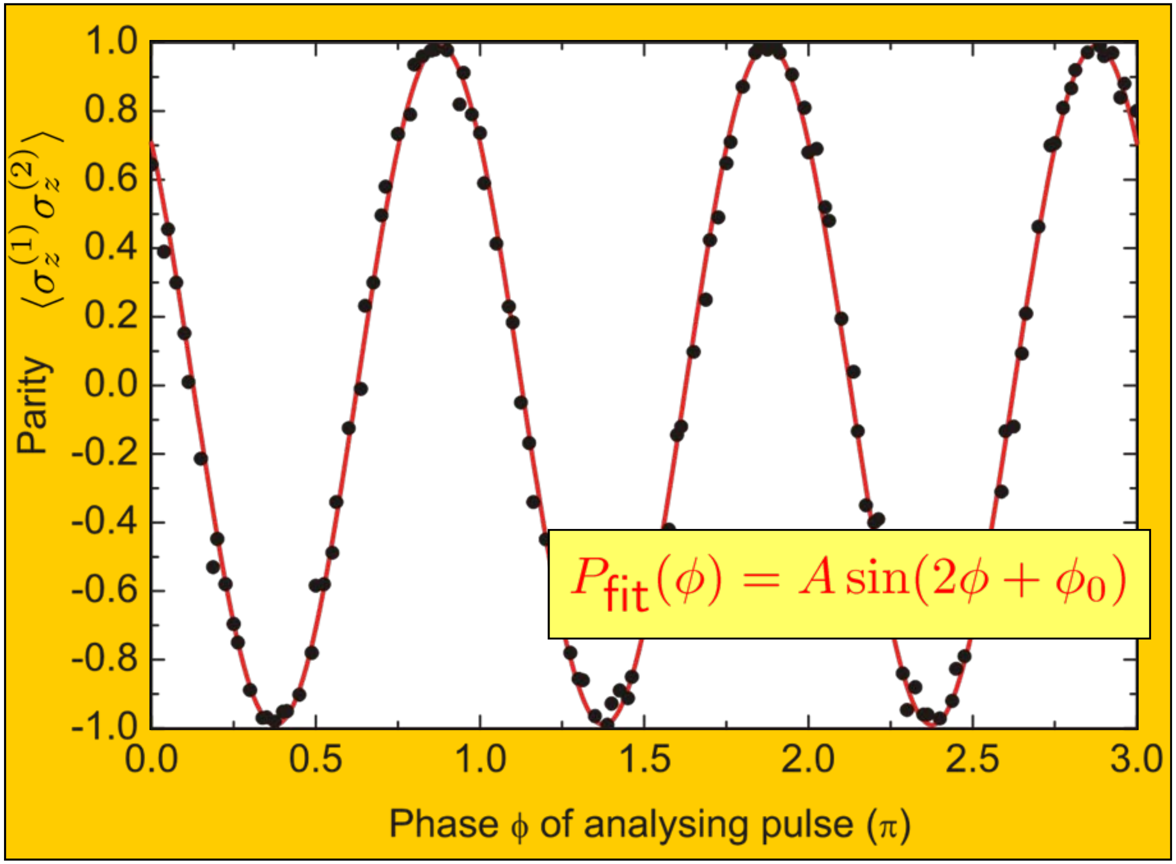
$\longrightarrow$ equal population in all states

Parity $\langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle$
+ 1
- 1
0

Entanglement check: Scan laser phase ϕ and measure parity

Mølmer-Sørensen gate: parity oscillations

Parity oscillation contrast: $|\langle SS|\rho_\Psi|DD\rangle|$



Bell state:

$$\Psi = |SS\rangle + i|DD\rangle$$

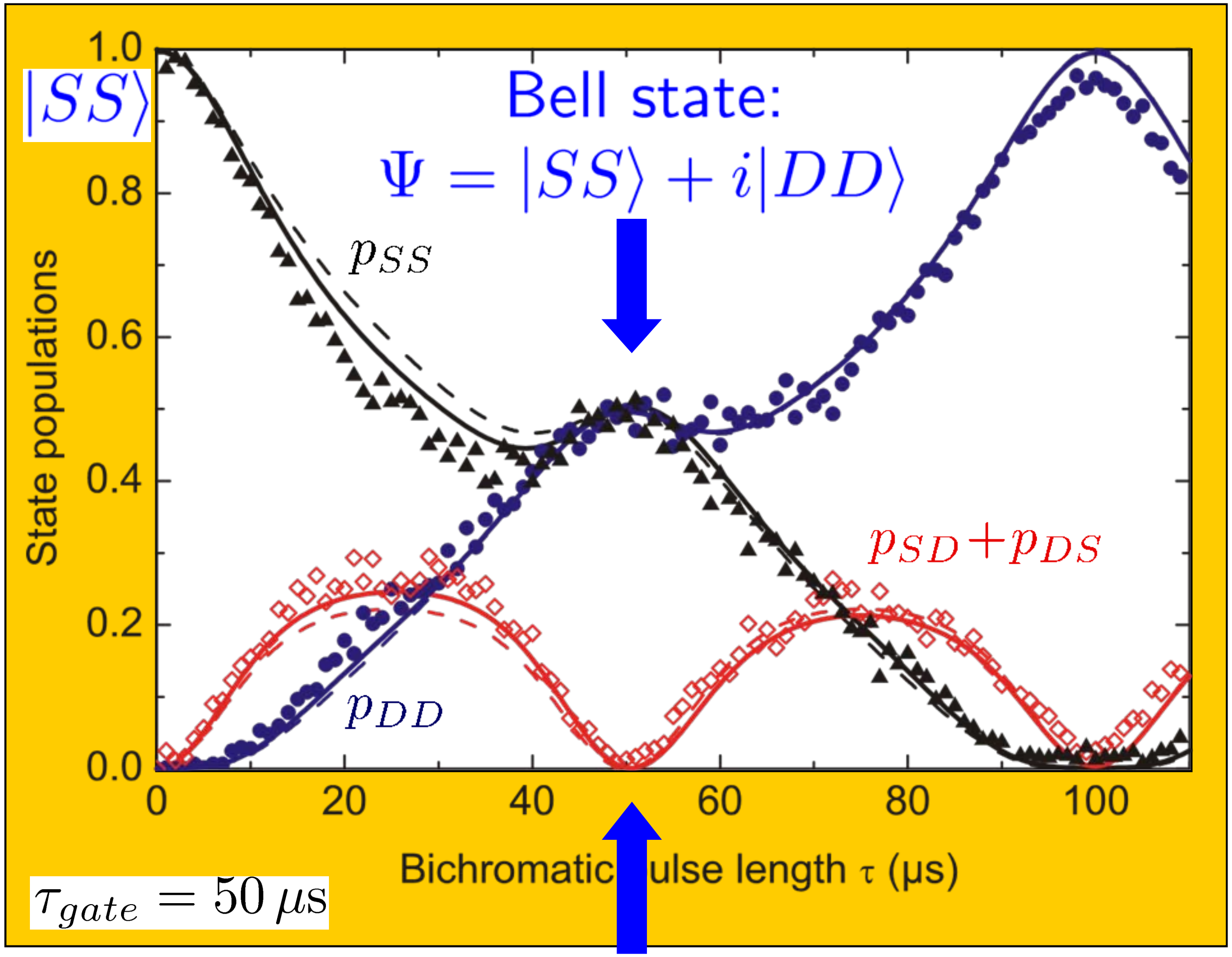
A = 0.990(1)   29,400 measurements

$p_{SS}+p_{DD}$ = 0.9965(4)   13,000 measurements

Bell state fidelity

F=99.3(1)%

Creating Bell states



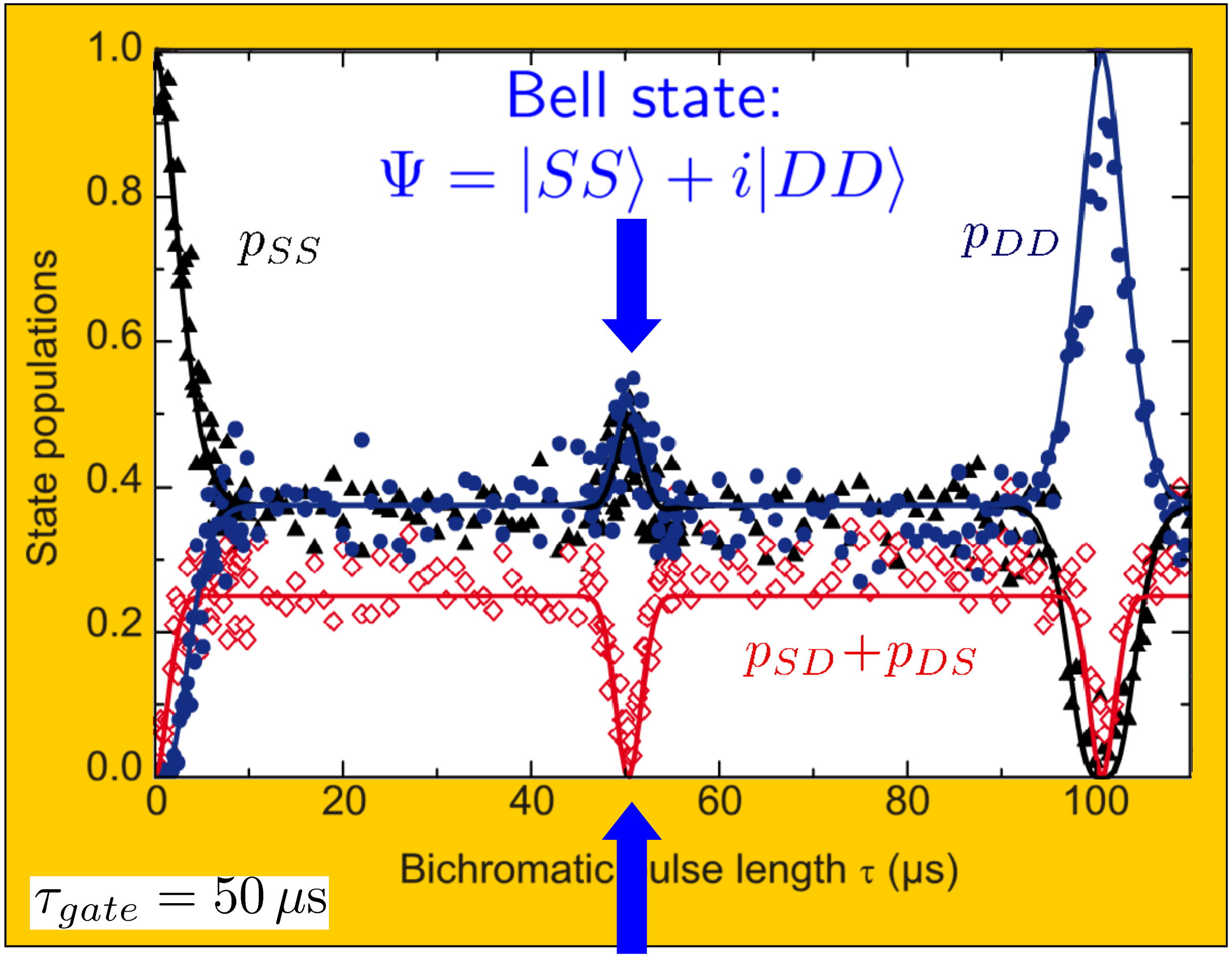
Fidelity

F=99.3(1)%

$\langle \bar{n} \approx 0 \rangle$

J. Benhelm *et al.*, Nat. Phys. **4**, 463 (2008)

‘Hot’ Bell states



Fidelity

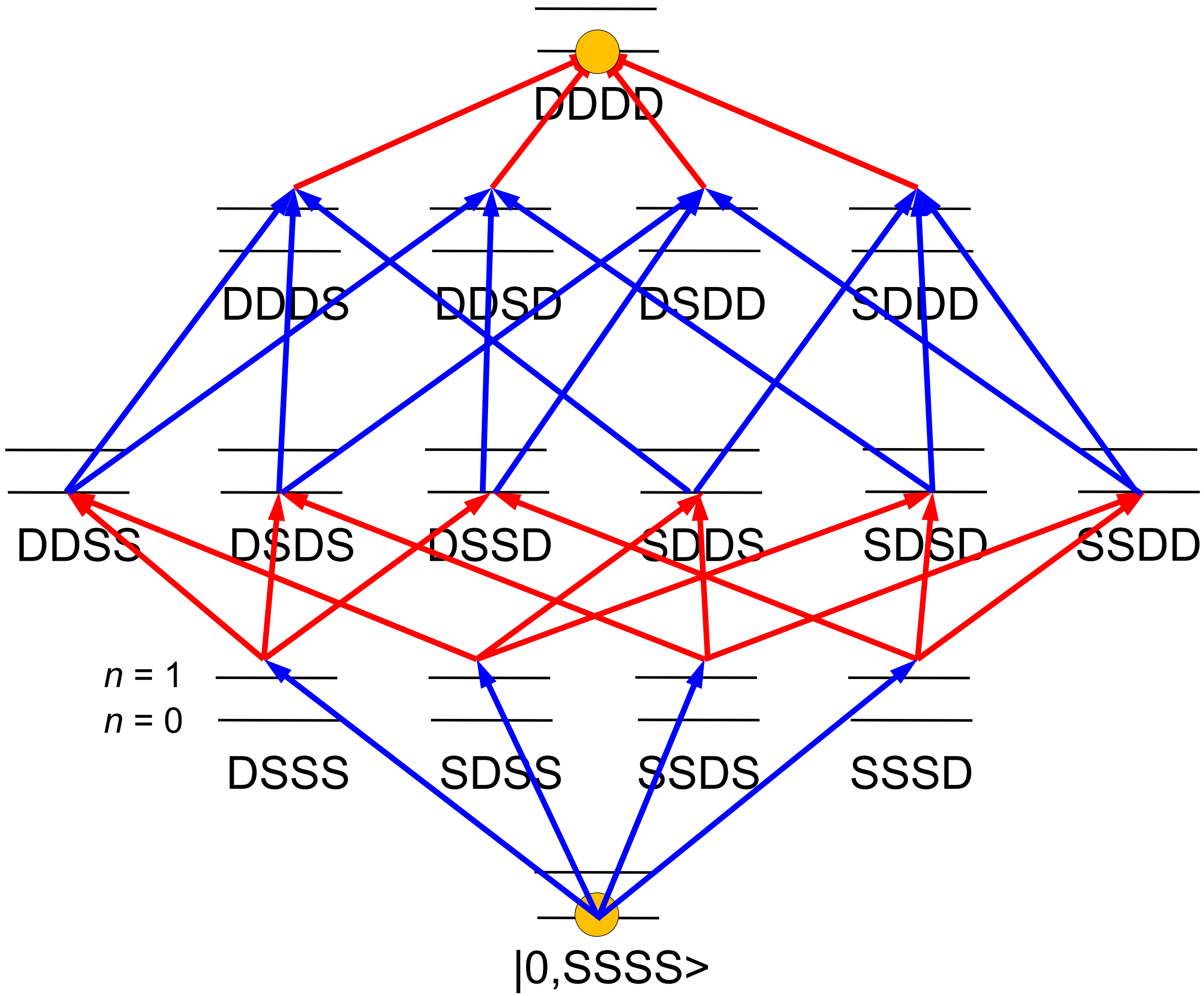
F ≈ 98%

Doppler-cooled ions !

$\langle \bar{n} \approx 18 \rangle$

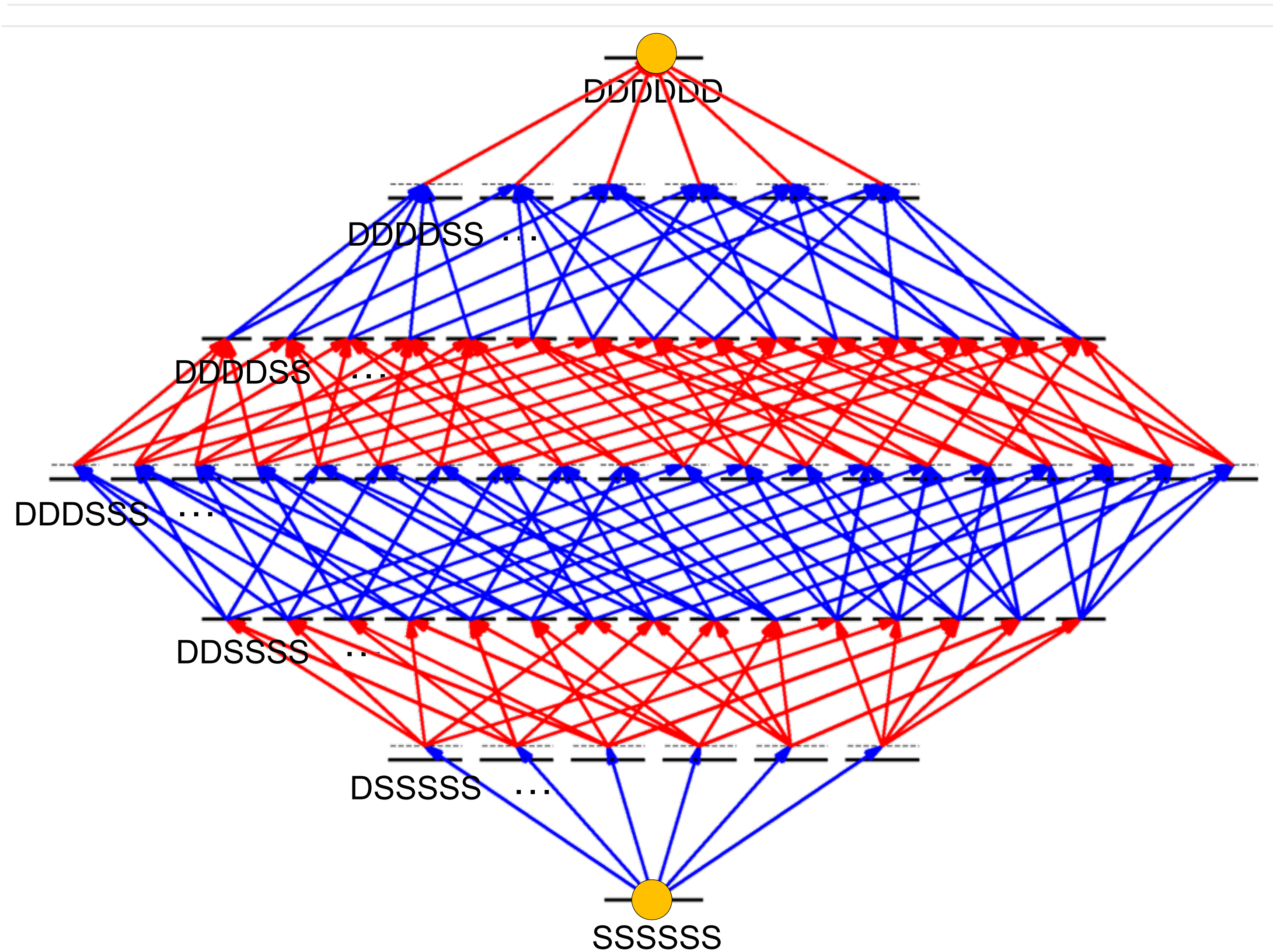
G. Kirchmair *et al.*, New J. Phys **11**, 023002 (2009)

Creating GHZ-states with 4 ions



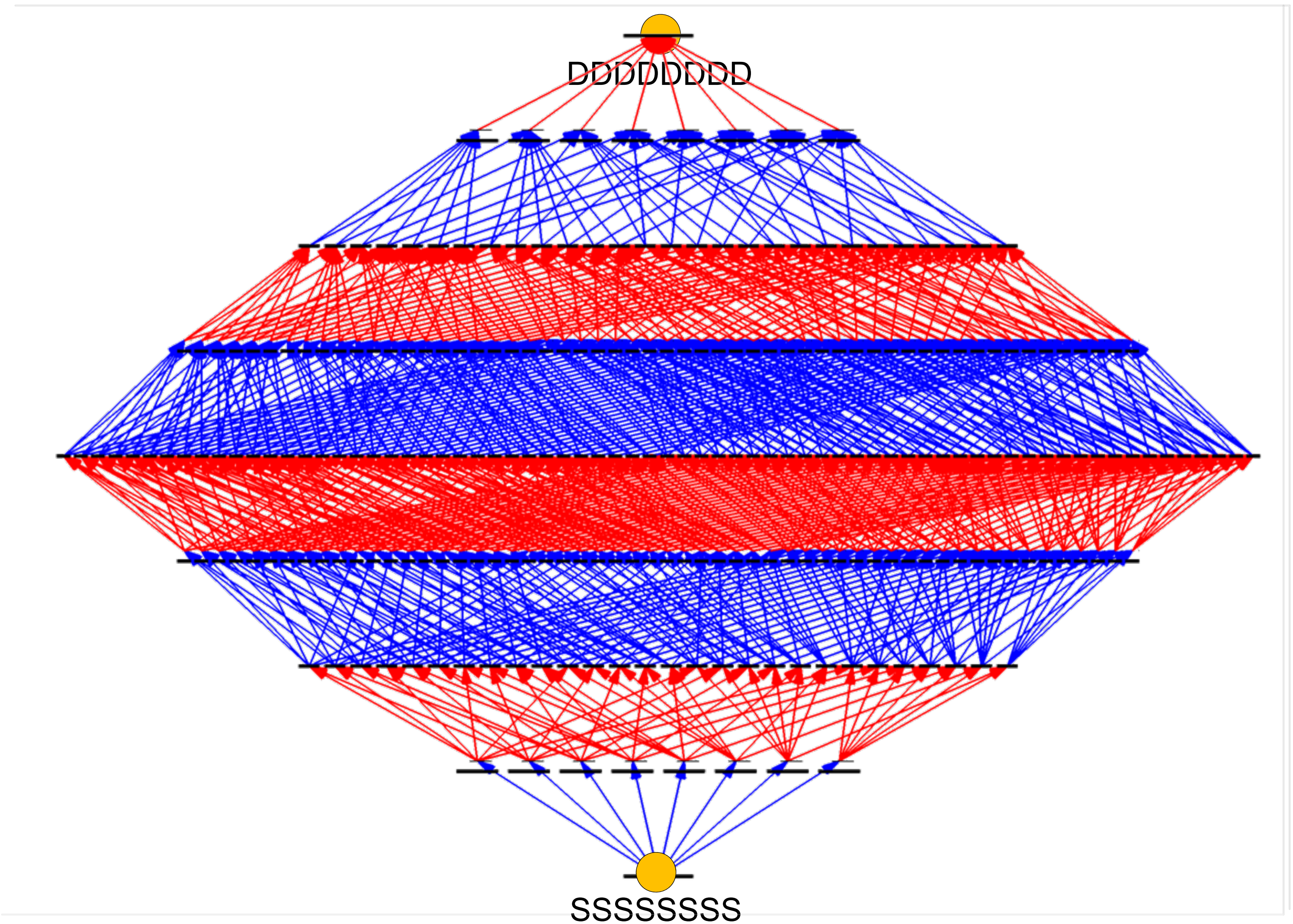
$$|SSSS\rangle \longrightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

Creating GHZ-states with 6 ions



$$|SSSSSS\rangle \longrightarrow (|SSSSSS\rangle + |DDDDDD\rangle)/\sqrt{2}$$

Creating GHZ-states with 8 ions

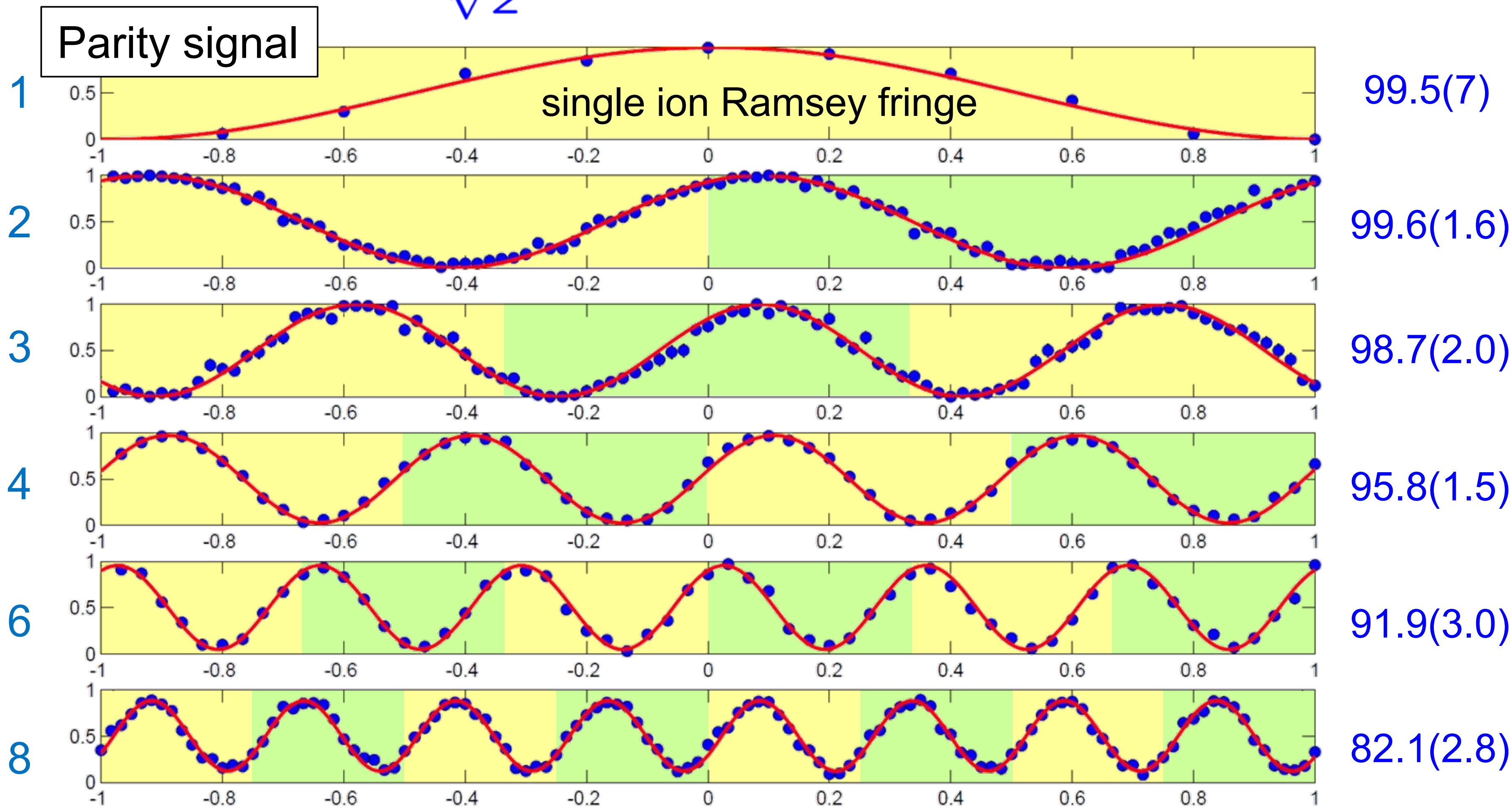


$$|SSSSSSSS\rangle \longrightarrow (|SSSSSSSS\rangle + |DDDDDDDD\rangle)/\sqrt{2}$$

N - qubit GHZ state generation

$$\Psi = \frac{1}{\sqrt{2}}(|SS\ldots S\rangle + |DD\ldots D\rangle)$$

$N =$ Fidelity (%)



T. Monz et al., PRL 106, 130506 (2011)

Entangling quantum gates

Sequential laser-ion interaction with strongly focussed beam

Collective laser-ion interaction induced by a wide beam

The entangling interaction is mediated by phonons

Lasers are used for entangling the ions

Gates mediated by photons

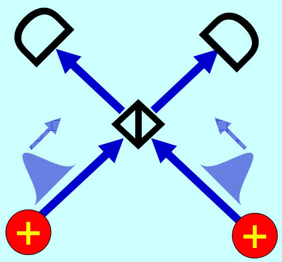
Creation of long-distance entanglement

Magnetic gradient gates

Creation entanglement without laser light

Entangling quantum gates

Photon-mediated two-ion entanglement



- Heralded probabilistic entanglement
- Long-distance entanglement

C. Simon and W. Irvine, PRL **91**, 110405 (2003)
D. Moehring, *et al.*, Nature **449**, 68 (2007)
D. Matsukevich, *et al.*, PRL **100**, 150404 (2008)

Ion-photon entanglement

Transparencies borrowed from:

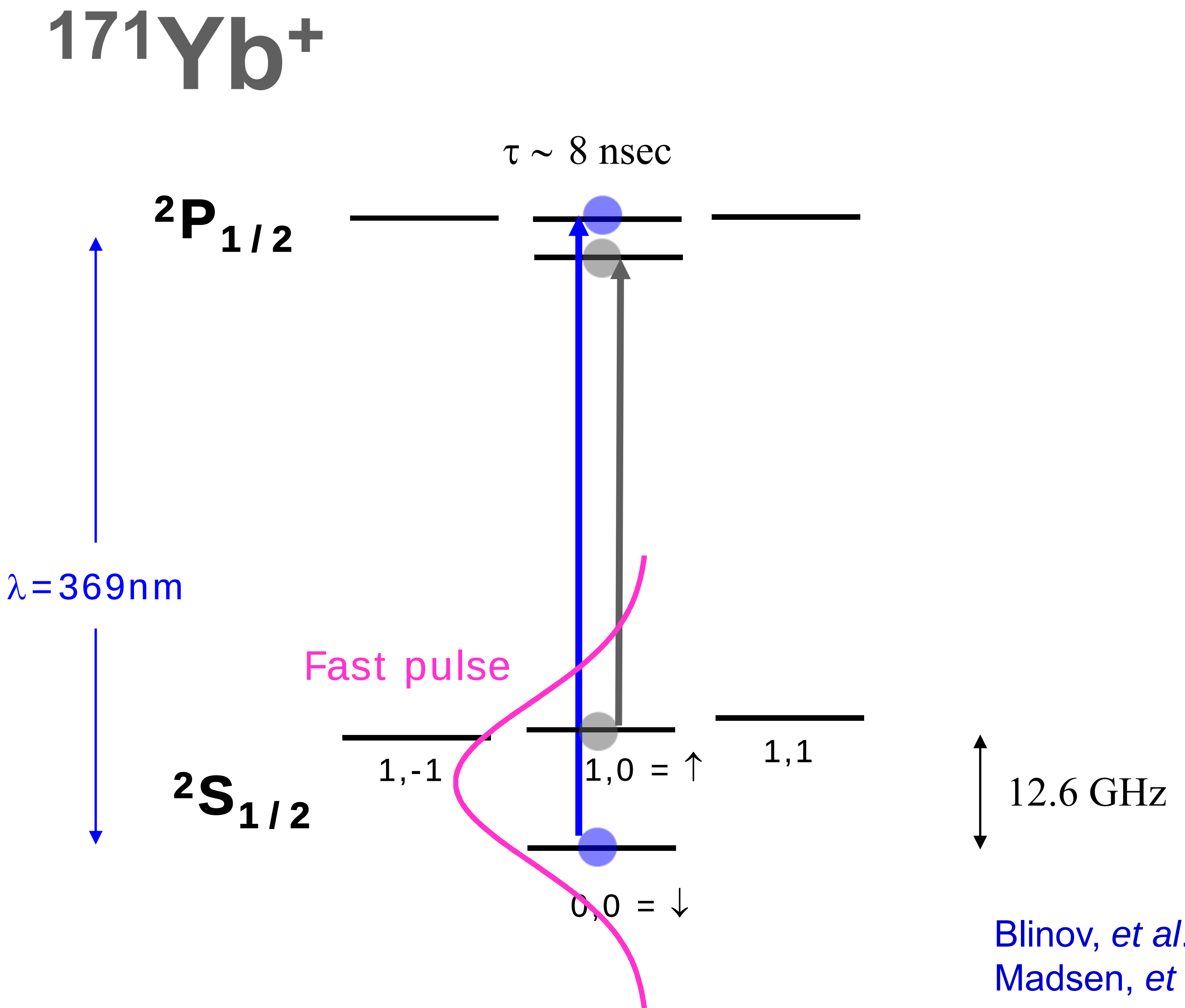
Trapped ion quantum networks

Christopher Monroe

University of Maryland
Department of Physics
and
Joint Quantum Institute

www.iontrap.umd.edu

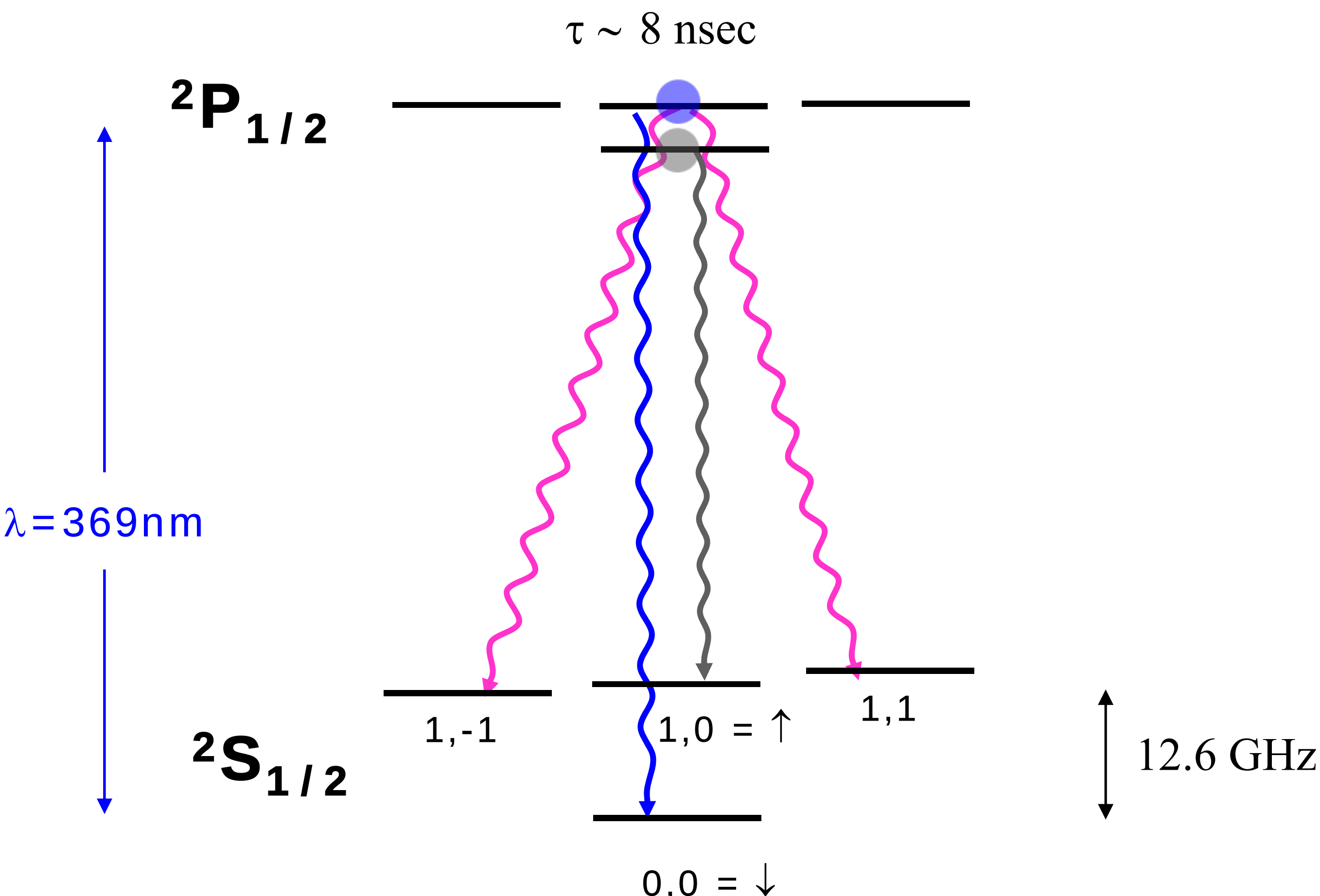
Linking atoms with ~~phonons~~ photons



Blinov, *et al.*, Nature **428**, 153 (2004)
Madsen, *et al.*, PRL **97,** 040505 (2006)

Linking atoms with ~~phonons~~ photons

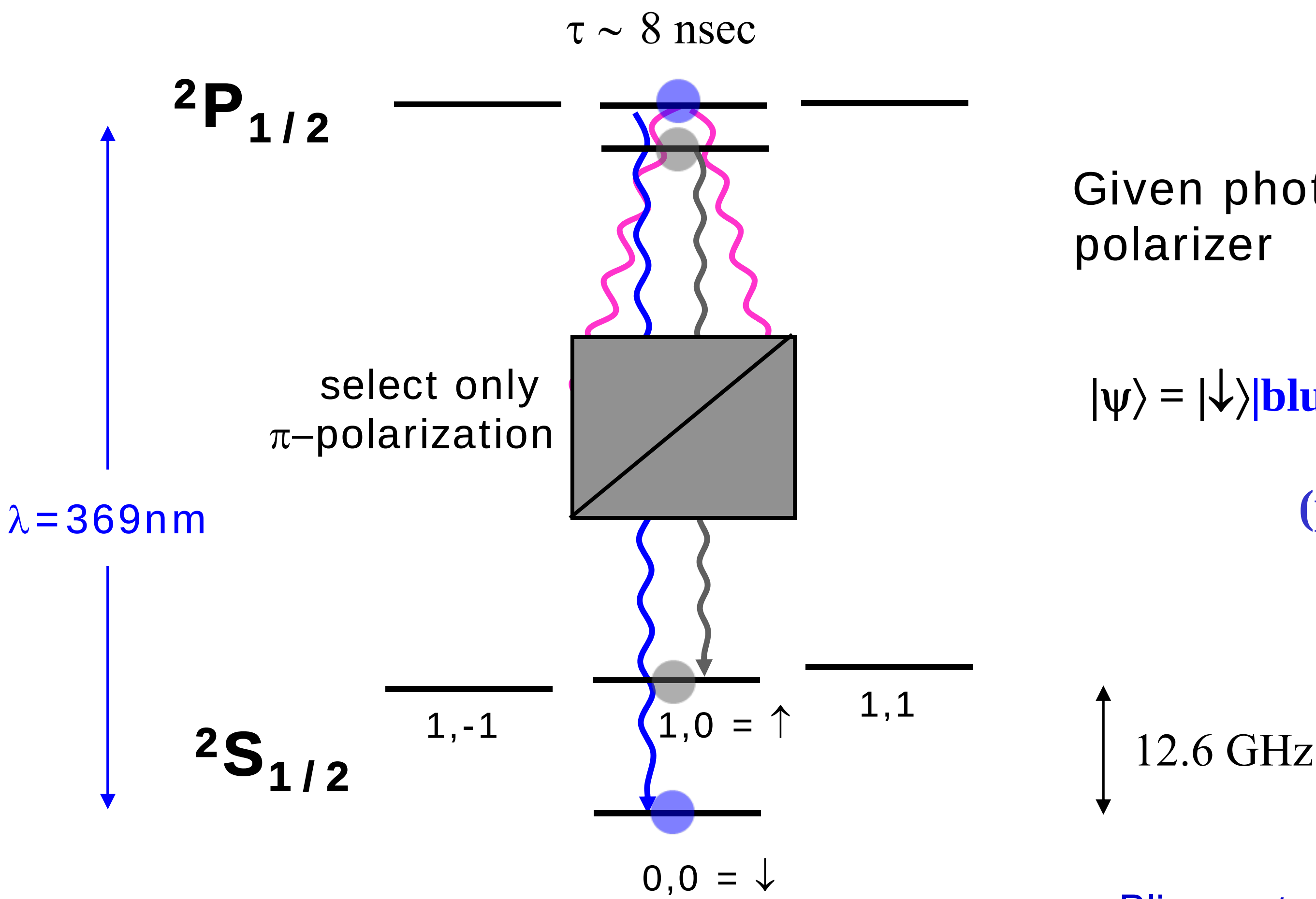
$^{171}Yb^+$



Blinov, *et al.*, Nature **428**, 153 (2004)
Madsen, *et al.*, PRL **97,** 040505 (2006)

Linking atoms with ~~phonons~~ photons

$^{171}Yb^{+}$



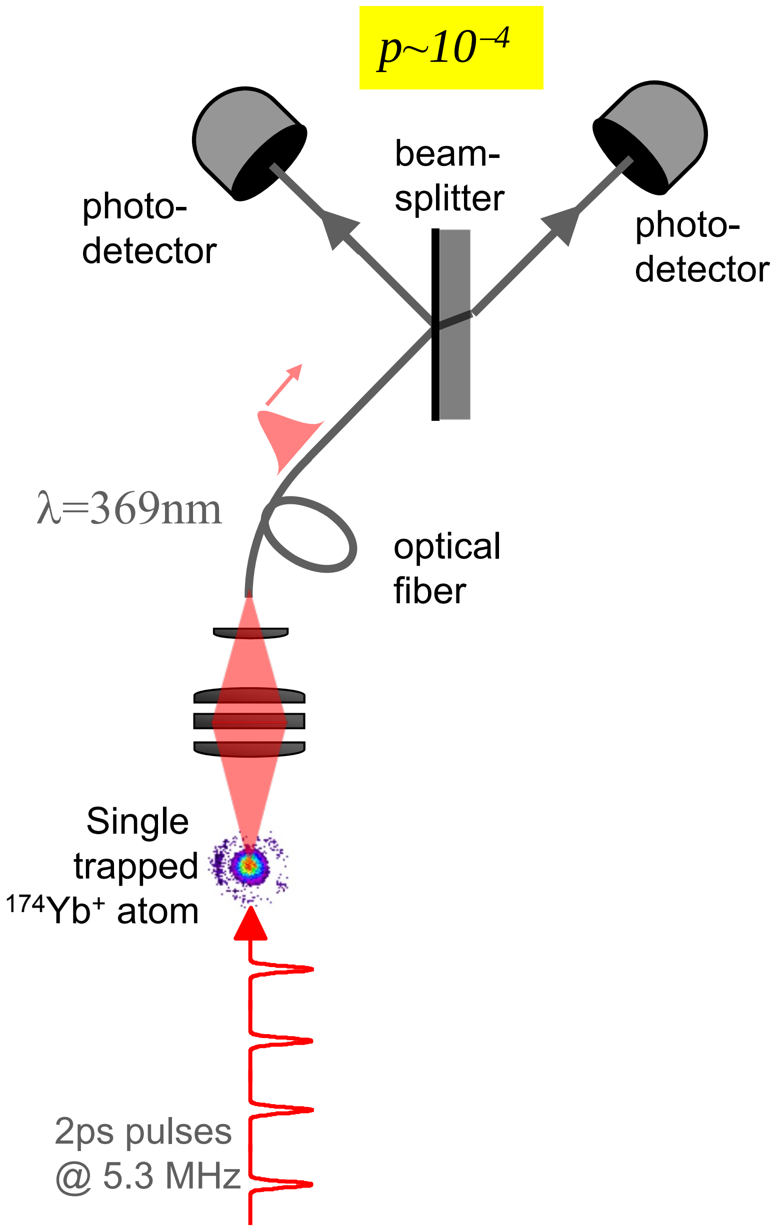
Given photon emerges from polarizer

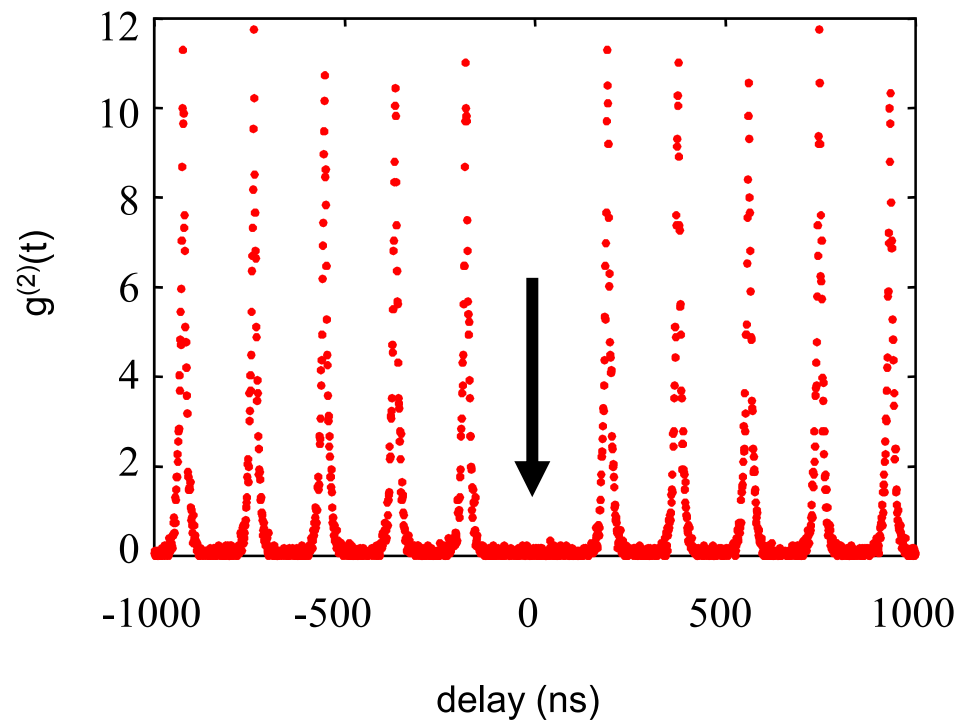
$|\psi\rangle = |\downarrow\rangle|\text{blue}\rangle + |\uparrow\rangle|\text{red}\rangle$

(post-selected)

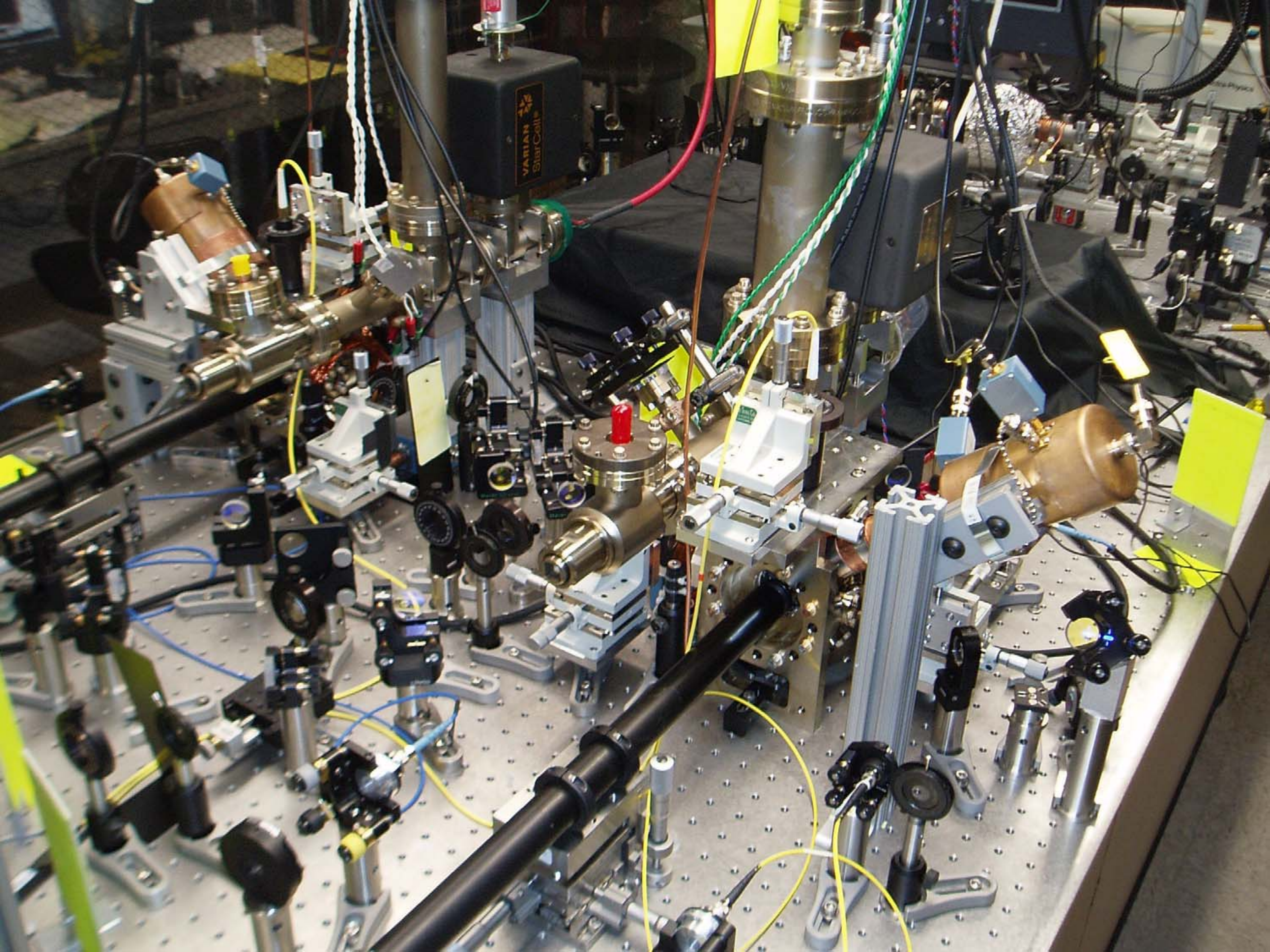
Blinov, *et al.*, Nature **428**, 153 (2004)
Madsen, *et al.*, PRL **97,** 040505 (2006)

excellent probabilistic
single photon source

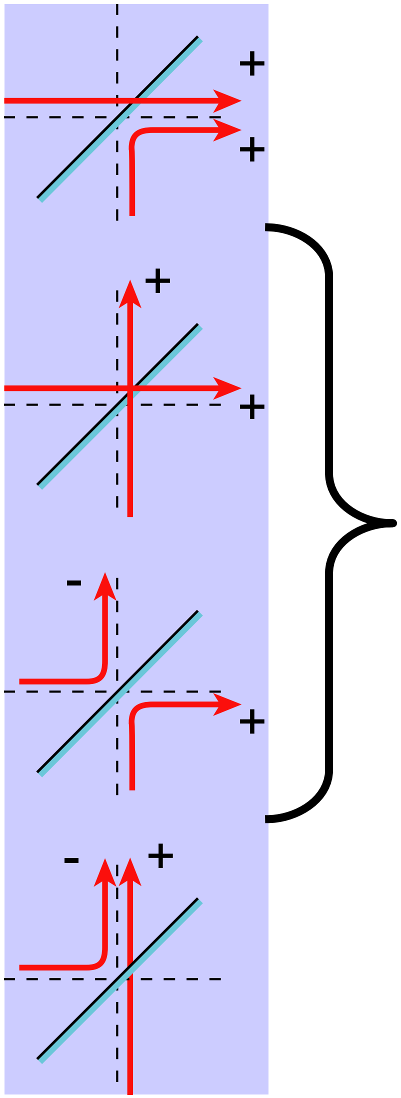




P. Maunz, *et al.*, Nature Physics **3**, 538 (2007)

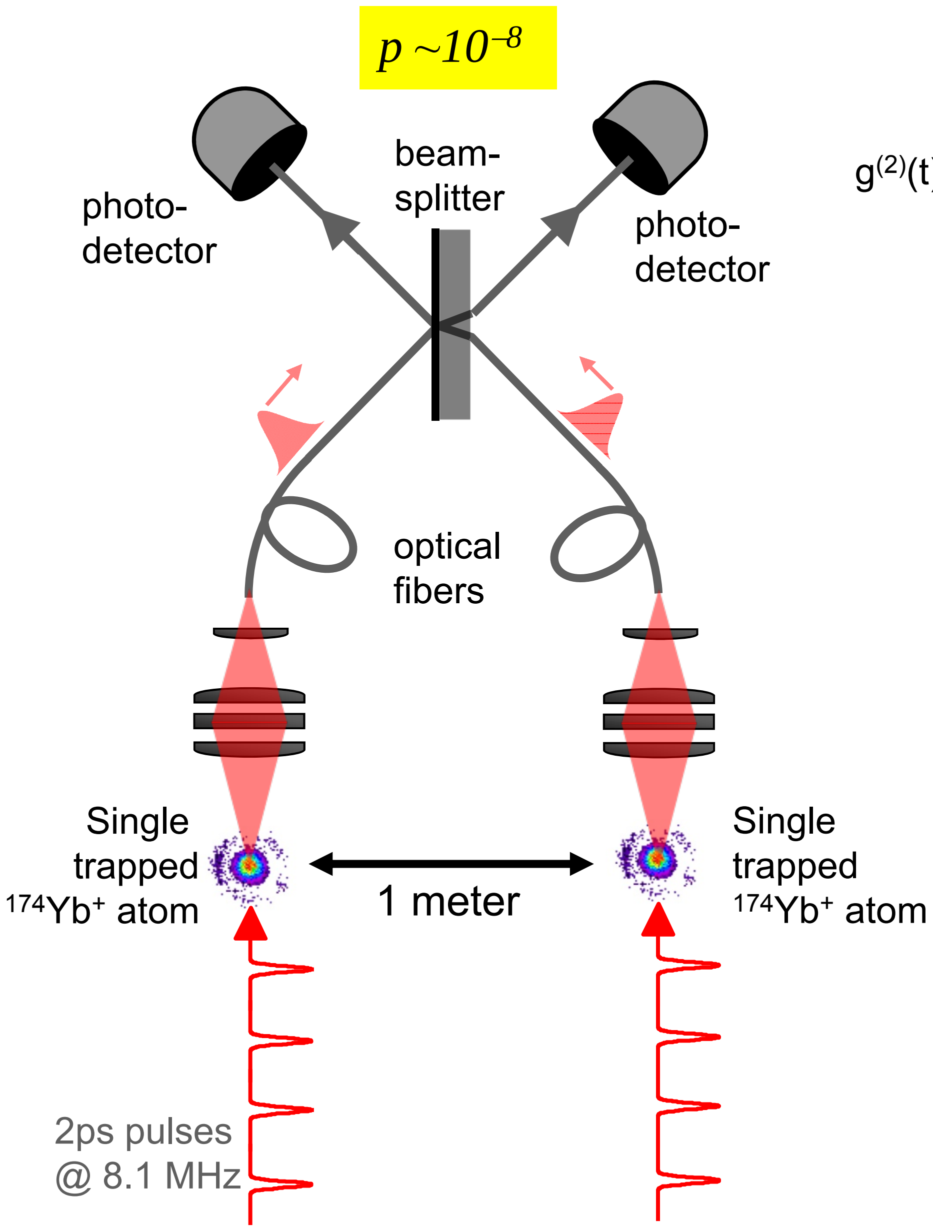

VARIAN
StarCell®

Two-photon Interference

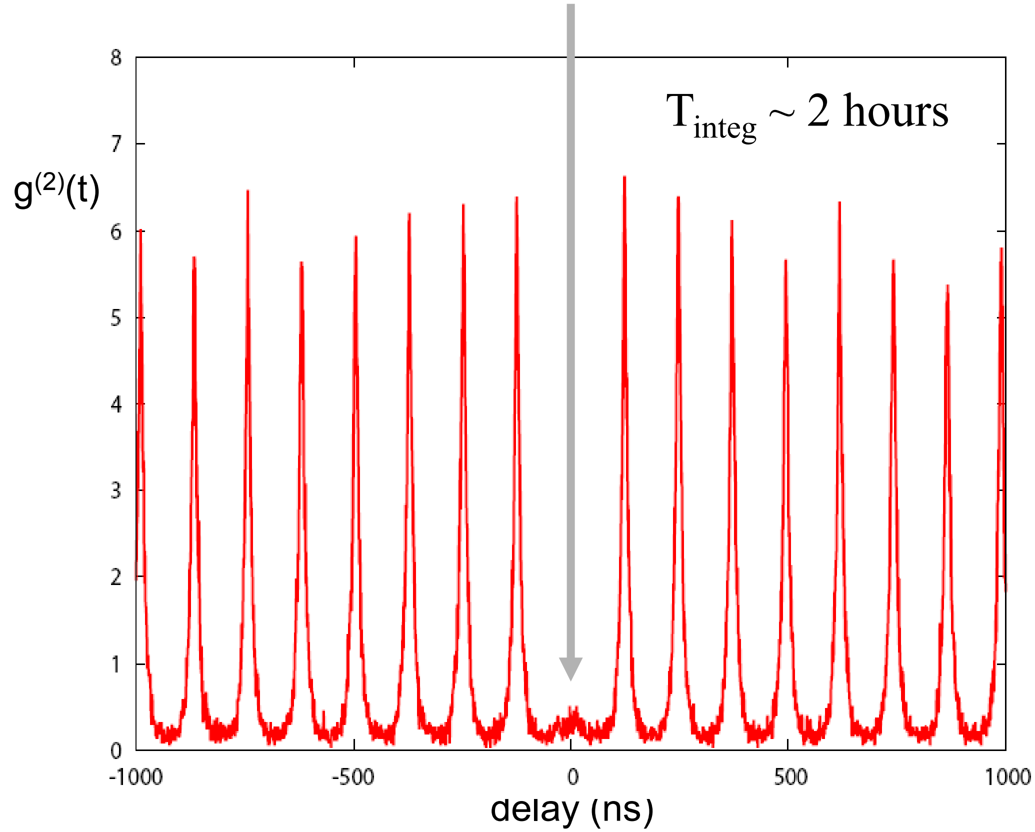


destructive interference of these paths

Y.H. Shih and C.O. Alley, Proc. 2nd Int'l Symp. Found. Quant. Mech, Tokyo (1986)
Hong, Ou, and Mandel, *Phys. Rev. Lett.*, **59**, 2044 (1987)
Y.H. Shih and C.O. Alley, *Phys. Rev. Lett.* **61**, 2921 (1988)

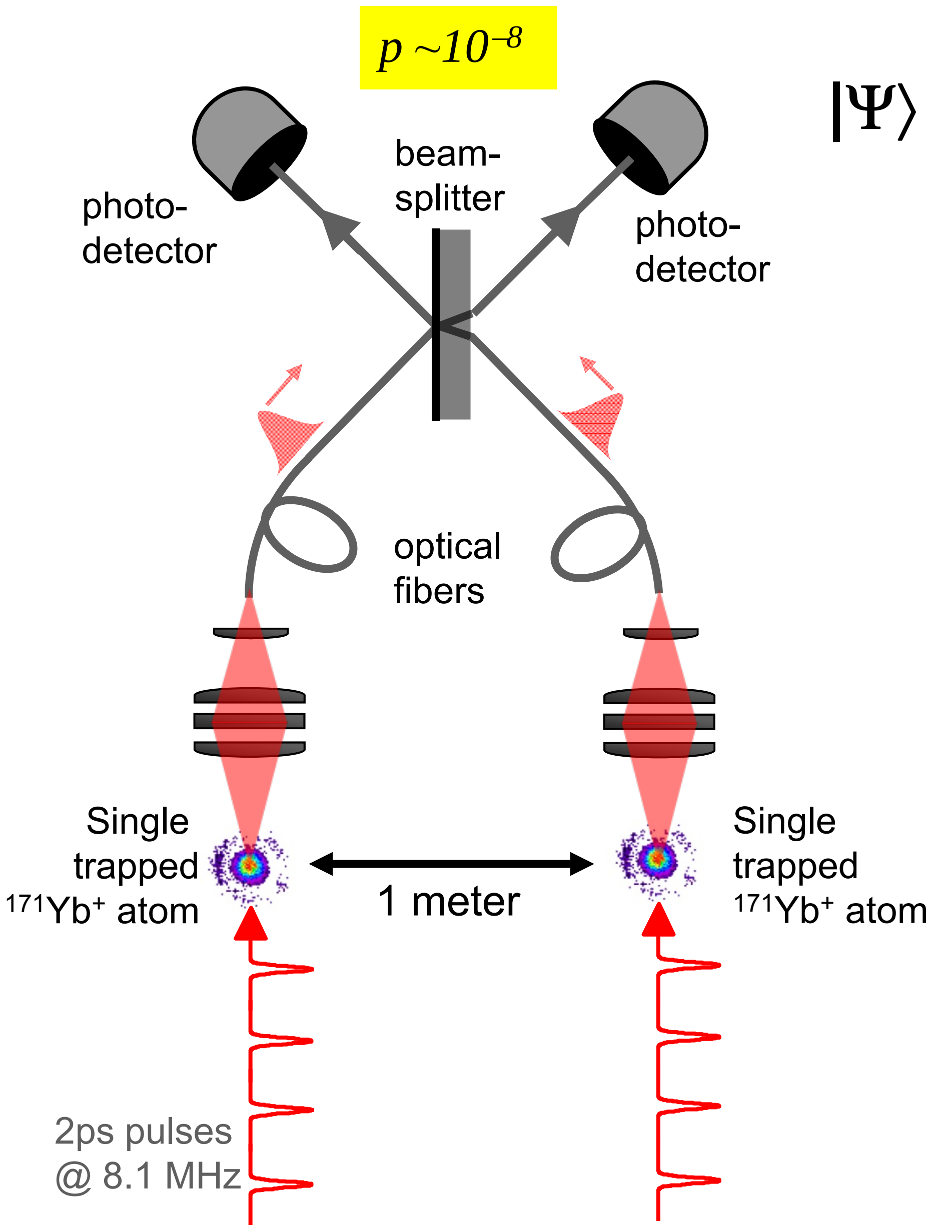


Quantum interference from two independent photons



Hong, Ou, Mandel, PRL 59, 2044 (1987)
Y.H. Shih & C. O. Alley, PRL 61, 2921 (1988)
Santori, et al., Nature, 419, 594 (2002)
Kaltenbaek, et al, PRL, 96, 240502 (2006)
Legero, et al., PRL, 93, 070503 (2004).
Thompson, et al., Science, 313, 74 (2006).
Felinto, et al. Nature Physics, 2, 844 (2006).
Beugnon, et al. Nature, 440, 779 (2006).
P. Maunz, et al., Nature Physics 3, 538 (2007)

Now with odd isotopes (having nuclear spin)



$$|\Psi\rangle = (|\downarrow\rangle_1 |\text{blue}\rangle_1 + |\uparrow\rangle_1 |\text{red}\rangle_1) \otimes (|\downarrow\rangle_2 |\text{blue}\rangle_2 + |\uparrow\rangle_2 |\text{red}\rangle_2)$$

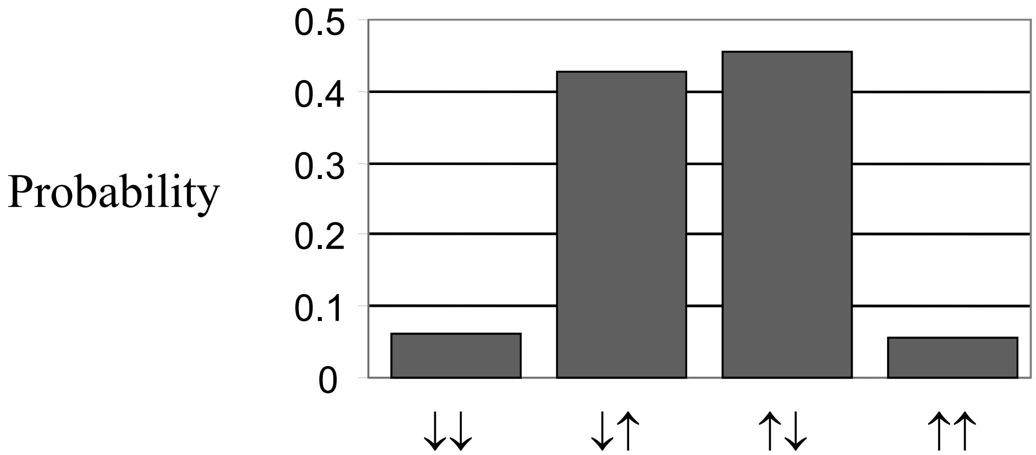
$$\Rightarrow |\downarrow\rangle_1 |\uparrow\rangle_2 - |\uparrow\rangle_2 |\downarrow\rangle_2$$

…upon coincidence
photon detection

insensitive to
- interferometric phase noise
- ion motion

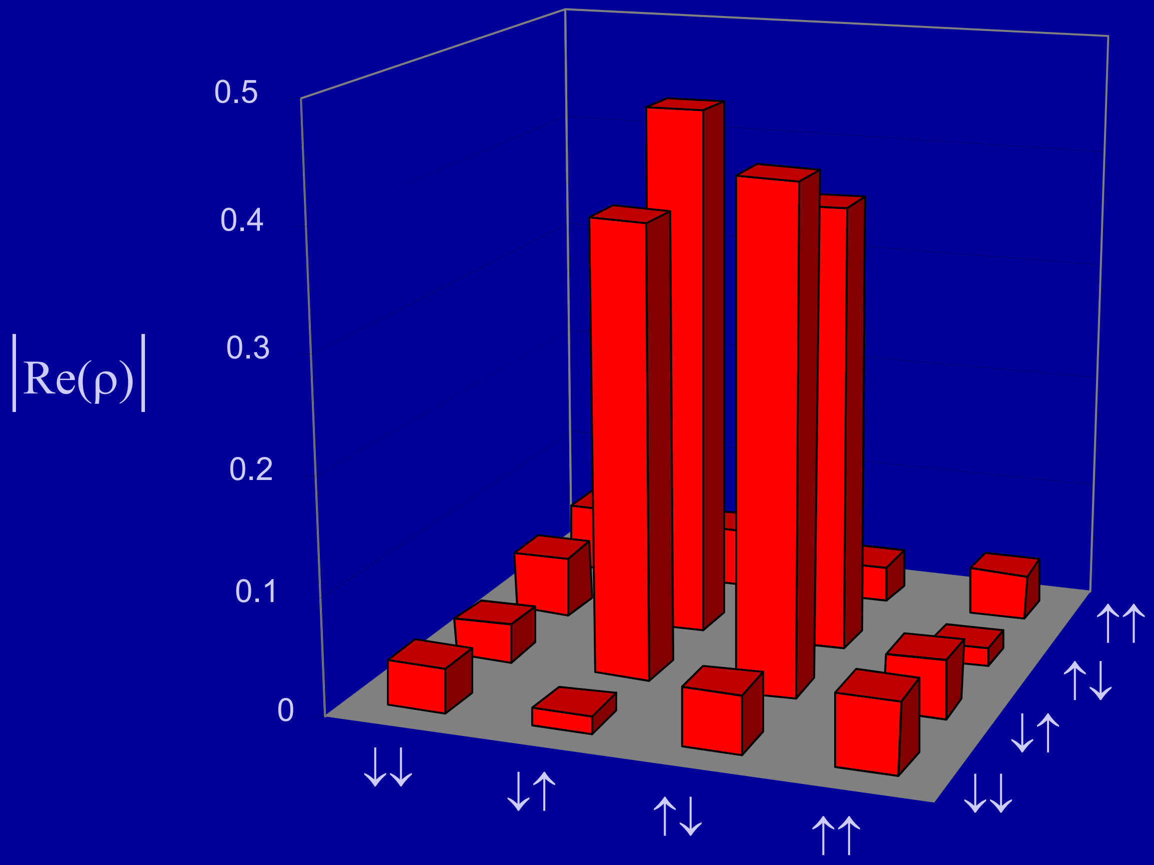
C. Simon and W. Irvine, PRL **91**, 110405 (2003)
L.-M. Duan, *et. al*., Quant. Inf. Comp. **4**, 165 (2004)

Measured qubit correlations
(given two photons detected)



D. Moehring, et al., Nature **449**, 68 (2007)

Full tomography of entangled state
(rotate qubits before measurement)



Fidelity **F = 0.82**

Concurrence **C = 0.67**

Entanglement of Formation **E = 0.56**

Bell Signal **S = 2.22 ± 0.07**

D. Matsukevich, et al., PRL **100**, 150404 (2008)

D. Moehring, et al., Nature **449**, 68 (2007)

Probability of heralding per attempt

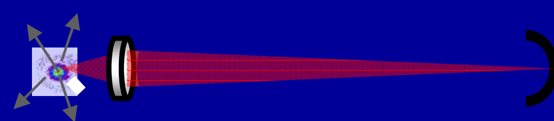
$$p = \underbrace{\frac{1}{4}}_{\text{Prob. of having singlet Bell state}}\Big[\overbrace{\underbrace{(0.995)}_{\text{branching to } {}^2D_{3/2} \text{ state}}\underbrace{(0.8)}_{\text{losses}}\underbrace{(0.95)}_{\text{excitation probability}}\underbrace{(0.2)}_{\text{fiber coupling efficiency}}\underbrace{(0.15)}_{\text{detector efficiency}}\underbrace{(0.02)}_{\text{solid angle}}}^{\text{probability of 1 photon emitted and detected}}\Big]^2 = 6\times 10^{-8}$$

Rate of heralded entanglement

$$R = \Gamma p = 0.04/sec$$

Increase p with cavity ?

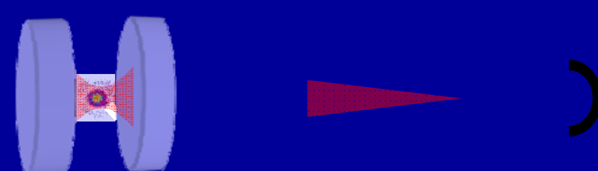
- Free space



$$\frac{d\Omega}{4\pi} \approx 0.02$$

- "Decent" cavities

$$C = \frac{g^2}{\kappa\gamma} \approx 1$$

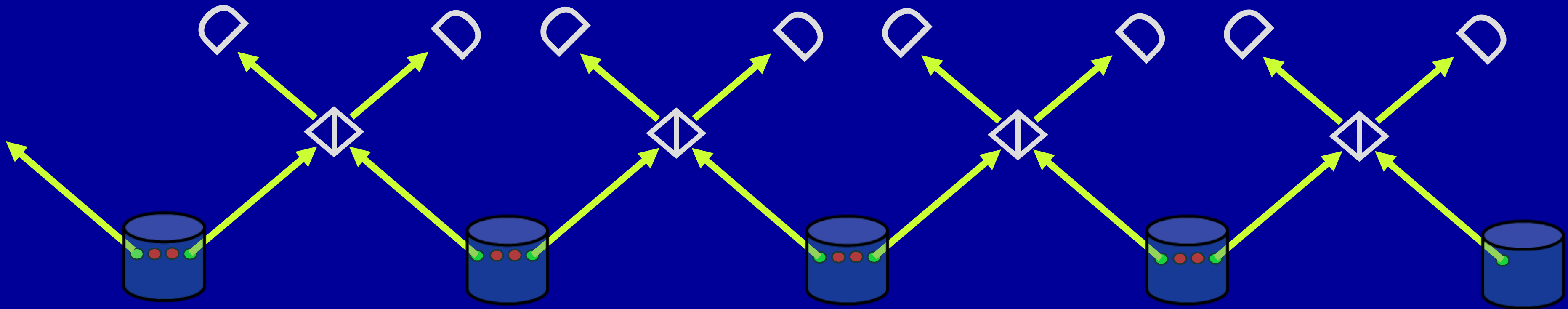


$$g = \frac{\mu E}{\hbar} \sim \frac{1}{\sqrt{Vol}}$$

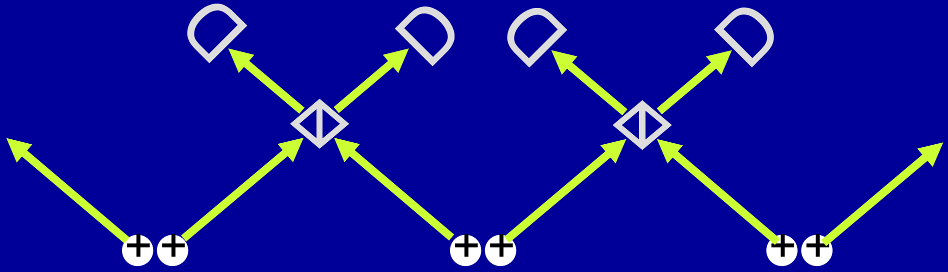
$$\frac{d\Omega}{4\pi} = \frac{C}{2C+1} \approx 0.1 - 0.2$$

G. Guthorlein, *et al.*, Nature **414**, 49 (2001)
A. Mundt, *et al.*, Phys. Rev. Lett. **89**, 103001 (2002)
W. Keller, *et al.*, Nature **431**, 1075 (2004)

Quantum networking with probabalistic entanglement

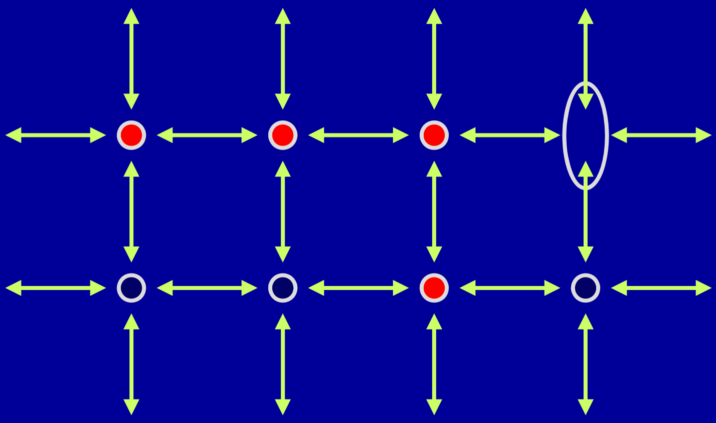


1. Quantum repeaters Briegel *et al.*, PRL **81**, 5932 (1998)



2. Distributed quantum computing with probabilistic gates

Duan, et al., Quant. Inf. Comp. 4, 165 (2004)



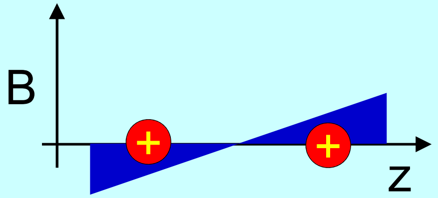
3. Cluster state quantum computing

Raussendorf and Briegel, PRL **86**, 910 (2001)

Duan and Raussendorf, PRL 95, 080503 (2005)

Entangling quantum gates

Trapped-ion entangling gates without lasers



- coupling: phonons +magnetic field gradients
- No laser-ion interactions
- No laser-cooling (?)

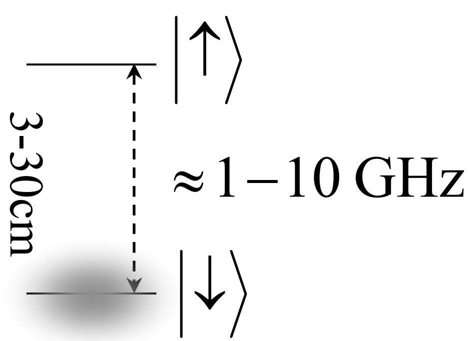
Mintert+Wunderlich, PRL **87**, 257904 (2001)
C. Ospelkaus *et al.*, PRL **101**, 090502 (2008)
M. Johanning *et al.*, PRL **102**, 073004 (2009)

Experiments underway at: Boulder, Siegen, MIT

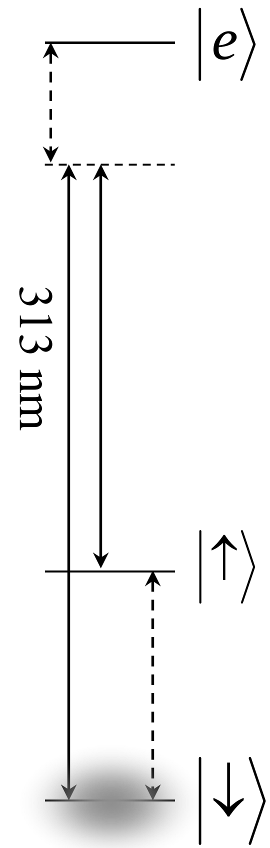
Manipulating hyperfine qubits with lasers

Transparencies borrowed from:

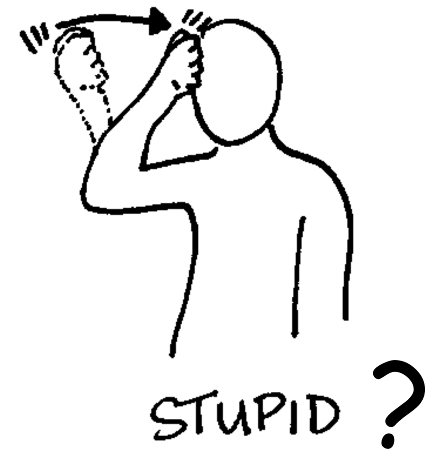
C. Ospelkaus, NIST Boulder



Hyperfine qubit



Raman process



Why would you not couple the states directly?

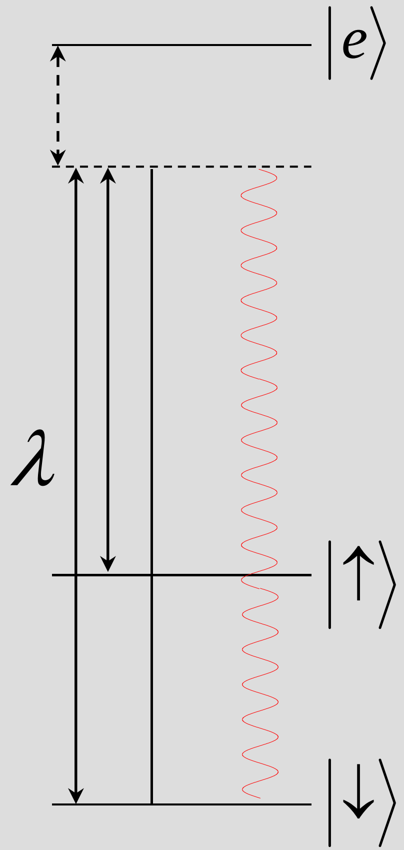
$\eta \approx 0$

$\eta \approx 0.1$

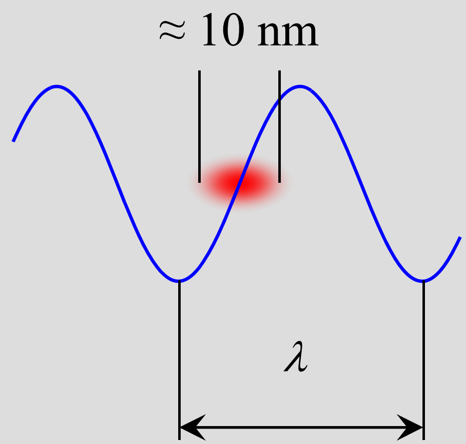
Infidelities of laser phase gates

Spontaneous emission

(for Raman transitions)



Motional state sensitivity



Coupling depends on motional state

Thermal occupation of states leads to a fluctuating coupling

Fluctuation is a gate error

Classical control

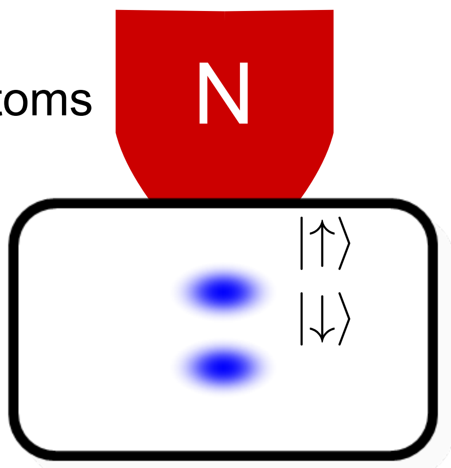
- Laser intensity noise
- Laser beam pointing instabilities
- Timing (AOs, thermal drifts)

Current best two-qubit gate 99.3% fidelity

Benhelm *et al.*, Nature Physics **4**, 463 (2008)

Coupling internal states to the motion

Ag Atoms



Spin quantization

Walter Gerlach

Otto Stern

$$F \propto \nabla |B| \sigma_z \longrightarrow H^{(i)} = \hbar \mu_B \Delta g B' x_0 (a + a^\dagger) \sigma_z$$

Magnetic gradient gates

Microwave excitation of a hyperfine qubit:

$\eta \ll 1$ $\longrightarrow$ Negligible coupling to sidebands, no entangling gates possible

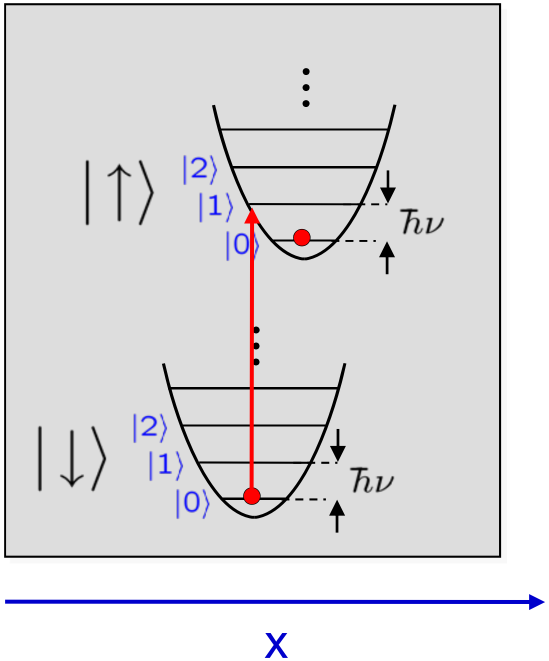
Microwave excitation of a hyperfine qubit in a magnetic field gradient:

$$H^{(i)} = \hbar \mu_B \Delta g B' x \sigma_z$$

Coupling to sidebands possible
if g-factors of qubit state unequal.

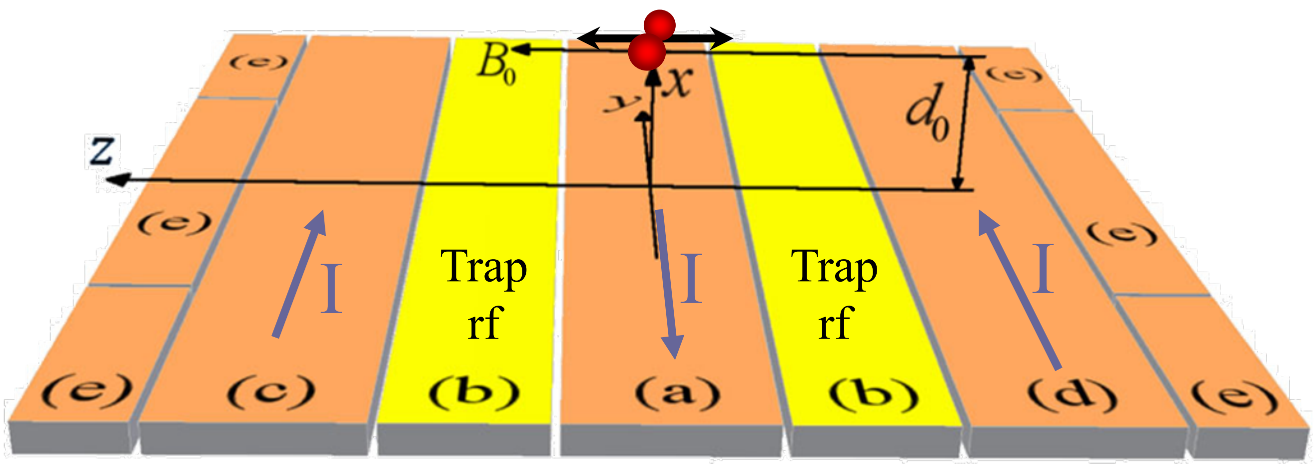
High field gradients needed !

$\rightarrow$ Microtraps needed !

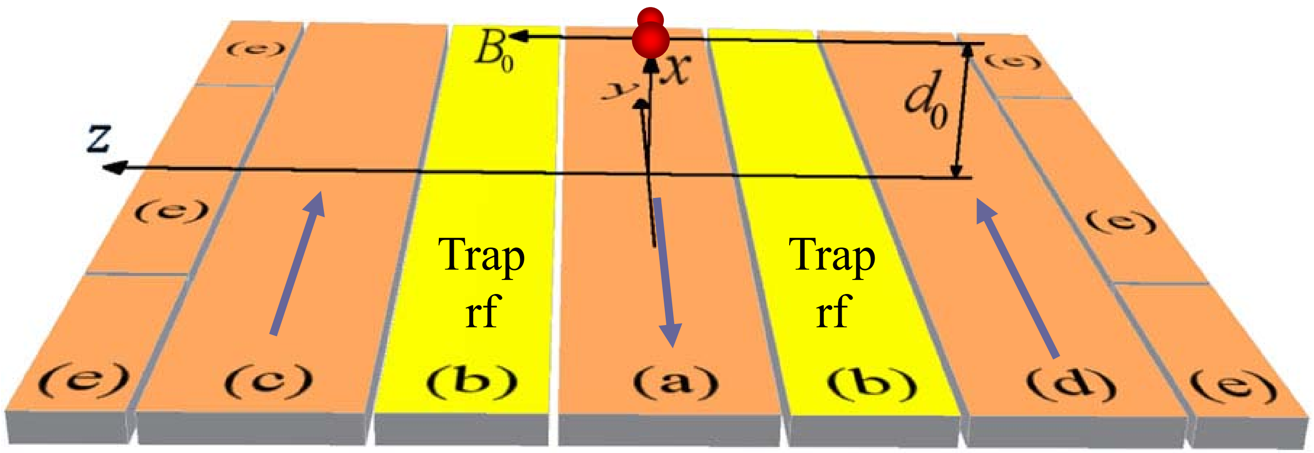


Mintert+Wunderlich, PRL **87**, 257904 (2001)

An AC Stern-Gerlach experiment



Integrate the gate and the trap



$^9Be^+$ $\omega_t = 2\pi \cdot 5\,\mathrm{MHz}$

$d_0 = 30\,\mu\mathrm{m}$

$|\uparrow\rangle = |F=1, m_F=1\rangle$

$|\downarrow\rangle = |F=2, m_F=0\rangle$

$\Delta E = h \cdot 1.2$ GHz

Multi-qubit gate

Center current 1.7A

$\tau_G = 20\,\mu\mathrm{s}$ $\propto d_0^2$

- Similar *cw* currents realized in neutral atom "chip traps"
- Speed comparable to current laser gates

Single-qubit gate

(c) or (d) 15 mA

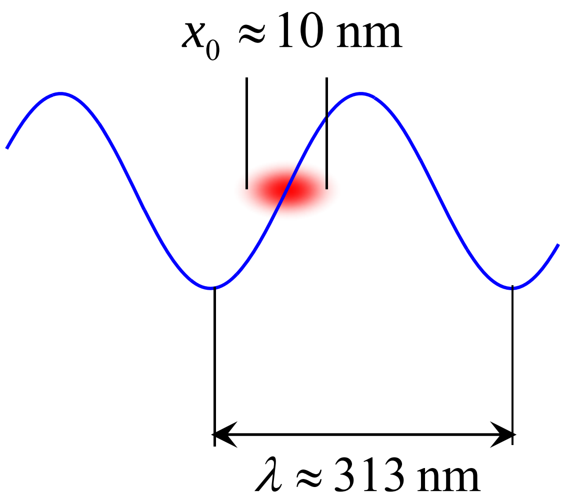
$\tau_G = 1\,\mu\mathrm{s}$ $\propto d_0$

- Relatively low current required
- Rotations much faster than currently achieved with lasers should be possible

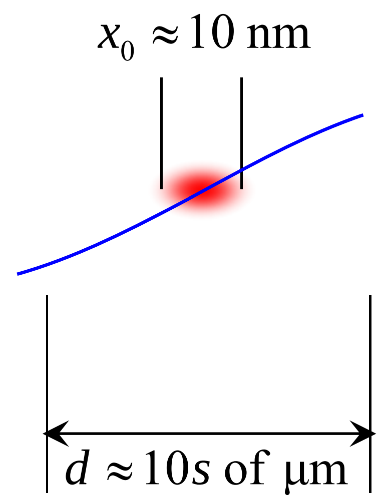
C. Ospelkaus *et al.*, PRL **101**, 090502 (2008)

Motional state insensitivity

Laser gates



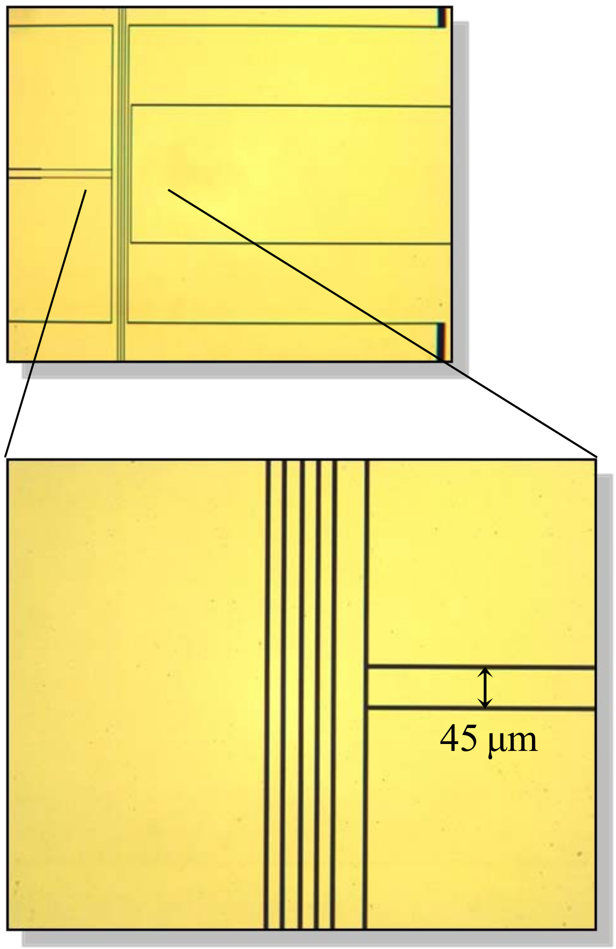
Magnetic gates



If ions are Doppler cooled only,
resulting gate infidelity is 10^{-6}!!!

Eliminate Raman lasers completely
Only low power Doppler lasers

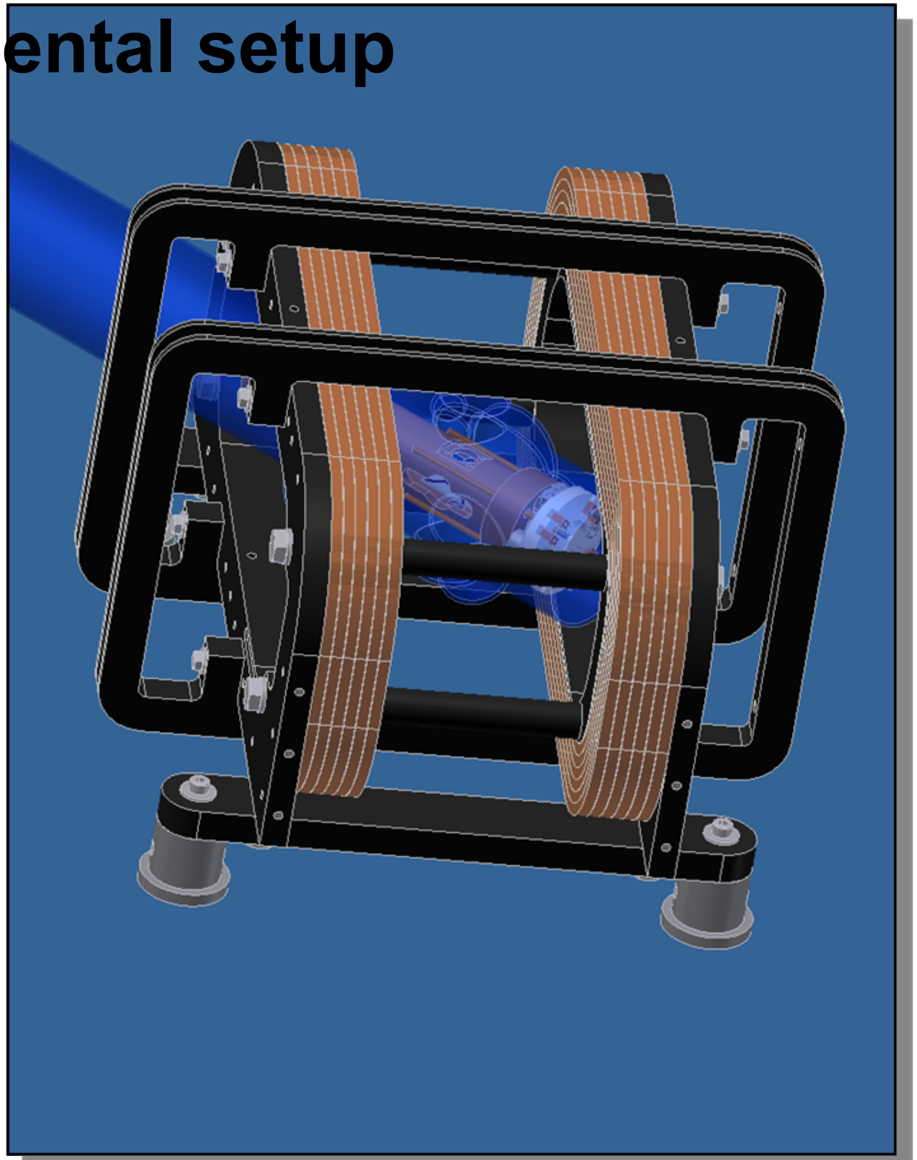
Experimental setup

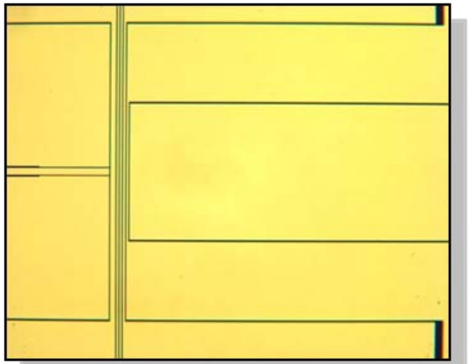


Evaporated gold
on fused silica

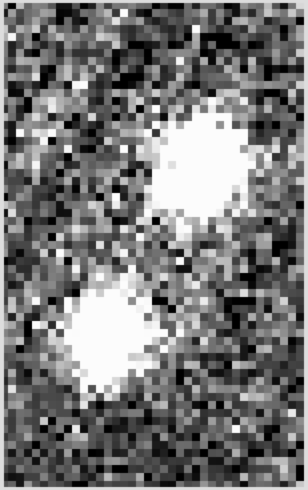
Chip fabrication in NIST cleanroom

Experimental setup



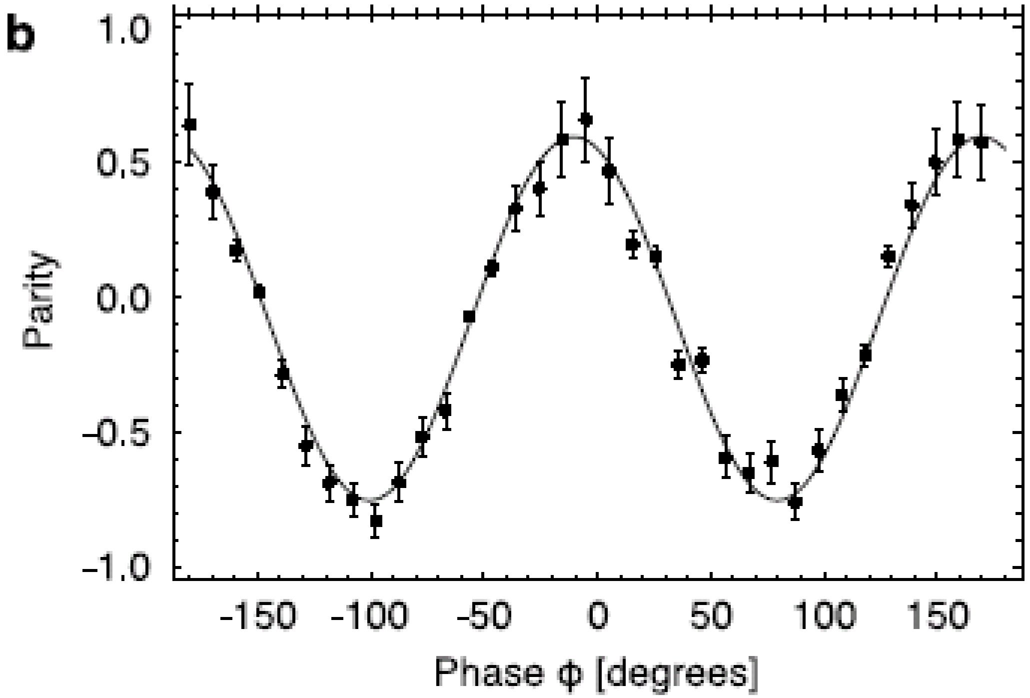


Trapped
$^{24}Mg^+$ and $^{25}Mg^+$
30 µm
from surface!



Entangling gate with microwaves

Parity oscillations of the created entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$



C. Ospelkaus et al, arXiv:1104.3573, accepted by Nature.

Noise in trapped-ion experiment

Decoherence in trapped ion experiments

Gate times for one- and two-qubit gates: $\tau \sim 10\text{-}100\ \mu\text{s}$

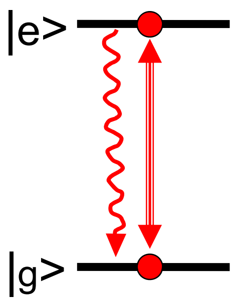
Motional decoherence

Dephasing by fluctuating trap voltages $\tau \sim 100$ ms

$$\alpha|0\rangle + \beta|1\rangle \rightarrow |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$$

Anomalous heating of ion motion by fluctuating electric fields $\tau \sim 1\text{-}100$ ms

Decoherence of optical qubits



- Spontaneous decay of metastable state — τ_{sp} ~ 1 s

  irrelevant in most experiments

phase noise dominated by low-frequency noise

- Magnetic field noise — τ_B ~ 1-100 ms

$$\Delta\nu = \mu_B(g_e - g_g)\Delta B/\hbar$$

('clock states' have much longer coherence times)

- Frequency noise of phase reference — τ_L ~ 1-100 ms

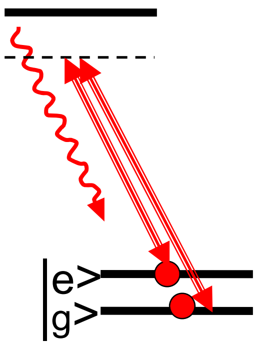
$$\Psi(0) = \alpha|g\rangle + \beta|e\rangle$$

$$\longrightarrow \Psi(t) = \alpha|g\rangle + \beta e^{-i\omega_0 t}|e\rangle$$

→ spin-echo techniques !

requires ultrastable laser !

Decoherence of hyperfine qubits



- Spontaneous decay of excited state mediating the Raman coupling

  τ_{sp} ~ 1-10 ms

  requires large detuning -> high laser power!

phase noise dominated by low-frequency noise

→ spin-echo techniques !

- Magnetic field noise

  τ_B ~ 1-100 ms

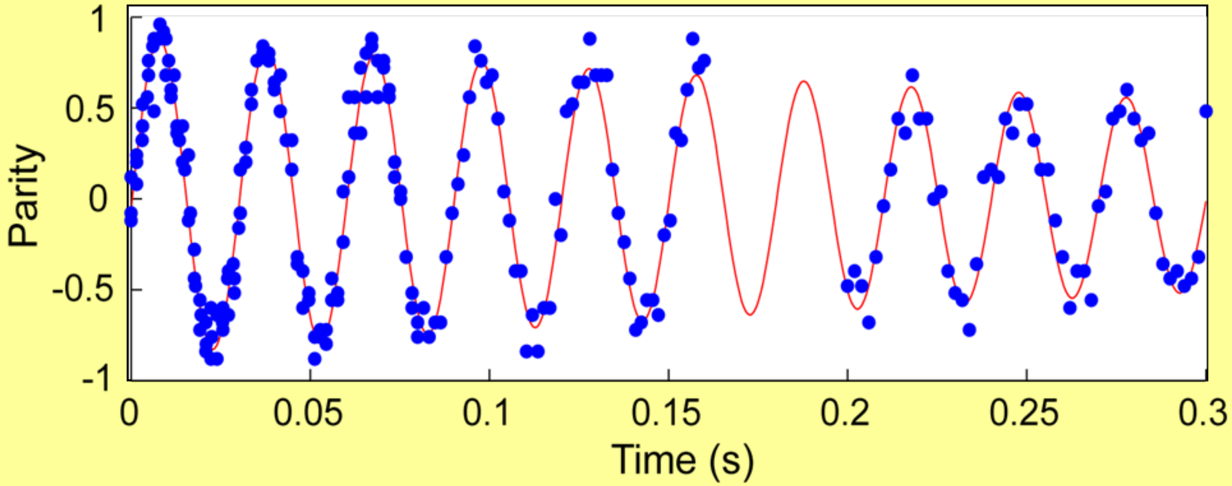
$$\Delta\nu = \mu_B(g_e - g_g)\Delta B/\hbar$$

('clock states' have much longer coherence times)

- Frequency noise of phase reference

  negligible

Spectroscopy with entangled states



- Improving the signal/noise ratio with entangled states
- High-resolution spectroscopy in decoherence-free subspace:

  Measurement of the $D_{5/2}$ quadrupole moment of $^{40}Ca^{+}$

Experiments with single trapped ions

Precision spectroscopy / Optical frequency standards

$$\frac{\delta\omega_0}{\omega_0} \approx 10^{-17}$$

Quantum information processing

- Entangling quantum gates
- Elementary quantum algorithms
- Multi-particle entangled states

Quantum metrology with entangled states

- Improved S/N ratio with entangled states

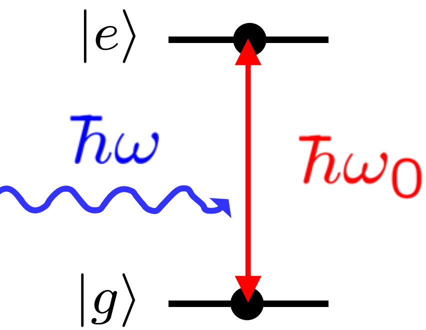
  J. J. Bollinger et al, Phys. Rev. A **54**, 4649 (1996)
- Quantum logic for clock state read-out

  P.O.Schmidt et al, Science **309**, 749 (2005)
- Spectroscopy in decoherence-free subspaces

  C. F. Roos et al., Nature **443**, 316(2006)

Clock measurement with single atom

Ramsey experiment with interrogation time τ



$$|g\rangle + |e\rangle \longrightarrow |g\rangle + e^{-i\Delta T}|e\rangle, \qquad \Delta = \omega - \omega_0$$

Phase measurement:

π/2 pulse + state measurement

N uncorrelated atoms: Measurement uncertainty ~ $N^{-1/2}\,T^{-1}$

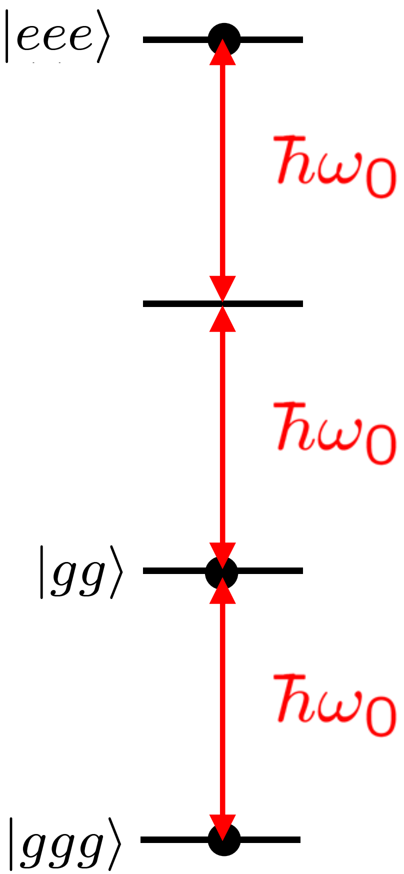
$$\frac{\Delta\omega}{\omega_0} \propto \frac{1}{\omega_0 T\sqrt{NM}} = \frac{1}{\omega_0\sqrt{NTT_{tot}}}$$

Ion clocks with entangled states

How does noise affect this scheme?

Clock experiments with maximally entangled states

(Wineland, PRA 46,R6797(`92), Bollinger, PRA 54, R4649 (`96))



$$|gg\rangle + |ee\rangle \xrightarrow{T} |gg\rangle + e^{-i2\Delta\tau}|ee\rangle,$$

$$|ggg\rangle + |eee\rangle \xrightarrow{T} |ggg\rangle + e^{-i3\Delta\tau}|eee\rangle,$$

$$\vdots$$

Phase measurement:
π/2 pulses + parity measurement

Measurement uncertainty ~ $N^{-1}\,T^{-1}$

$$\frac{\Delta\omega}{\omega_0} \propto \frac{1}{\omega_0 N T \sqrt{M}} = \frac{1}{\omega_0 N \sqrt{T T_{tot}}}$$

(Demonstration experiment: D. Leibfried et al, Science 304, 1476 (2004))

Decoherence of entangled states

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|SSSSSSSS\rangle + |DDDDDDDD\rangle)$$

Laser frequency and magnetic field noise: $H = f(t)\sum_{i=1}^{N}\sigma_z^{(i)}$

with $f(t) = \int_{-\infty}^{\infty} d\omega A(\omega)\cos(\omega t)$ describing the noise

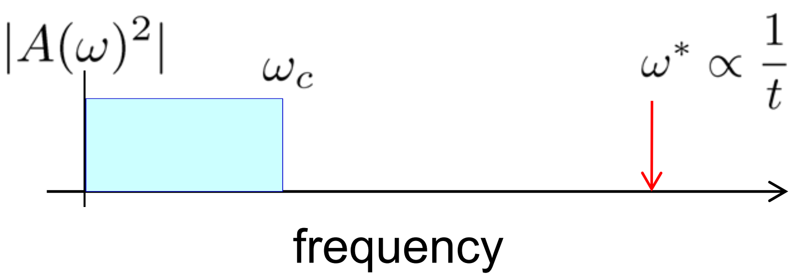
How does a GHZ state decohere in our experiments: $|\psi\rangle \xrightarrow{\text{time}} \rho(t)$?

Decoherence of entangled states

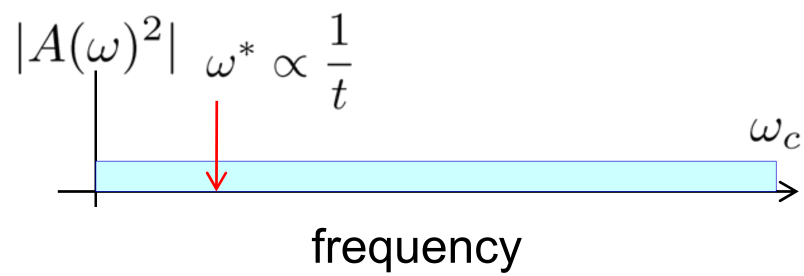
How does a GHZ state decohere in our experiments: $|\psi\rangle \xrightarrow{\text{time}} \rho(t)$?

Example: White noise with high-frequency cut-off

Long time limit $\omega_c \ll \omega^*$



Short time limit $\omega_c \gg \omega^*$



Fidelity decay:

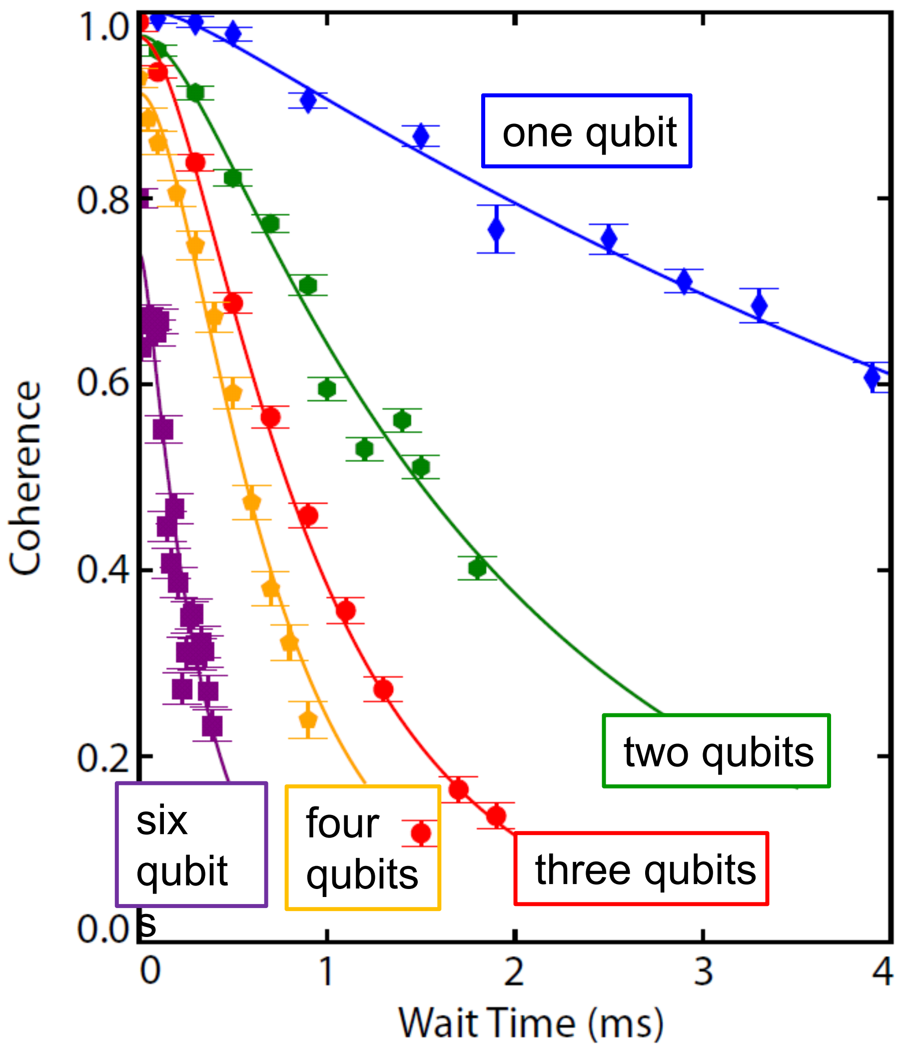
$$F(t) \cong \tfrac{1}{2}\left(1 + \exp\left[-(\tfrac{Nt}{\tau})^2\right]\right),$$

Fidelity decay:

$$F(\tau) = \tfrac{1}{2}\left(1 + \exp\left[-\tfrac{N^2 t}{\tau}\right]\right)$$

GHZ-states: Coherence of large-scale entanglement

T. Monz et al., Phys. Rev. Lett. **106**, 130506 (2011)



correlated noise
-> faster decay!

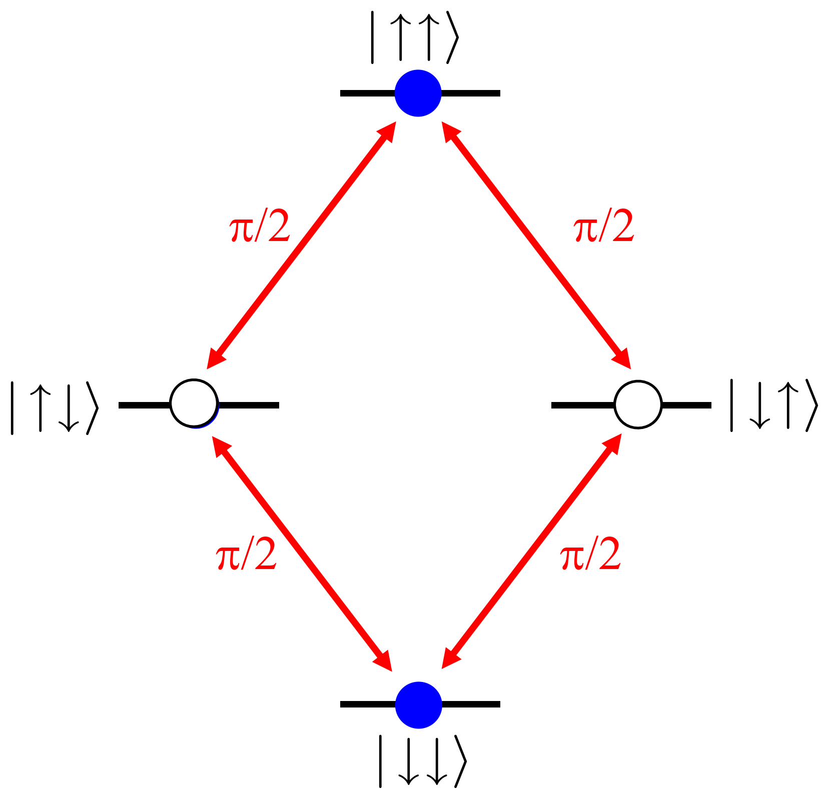
$$\tau_{coh} \propto \frac{1}{N}$$

or even

$$\tau_{coh} \propto \frac{1}{N^2}$$

Entangled states in decoherence-free subspaces

$$|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$$

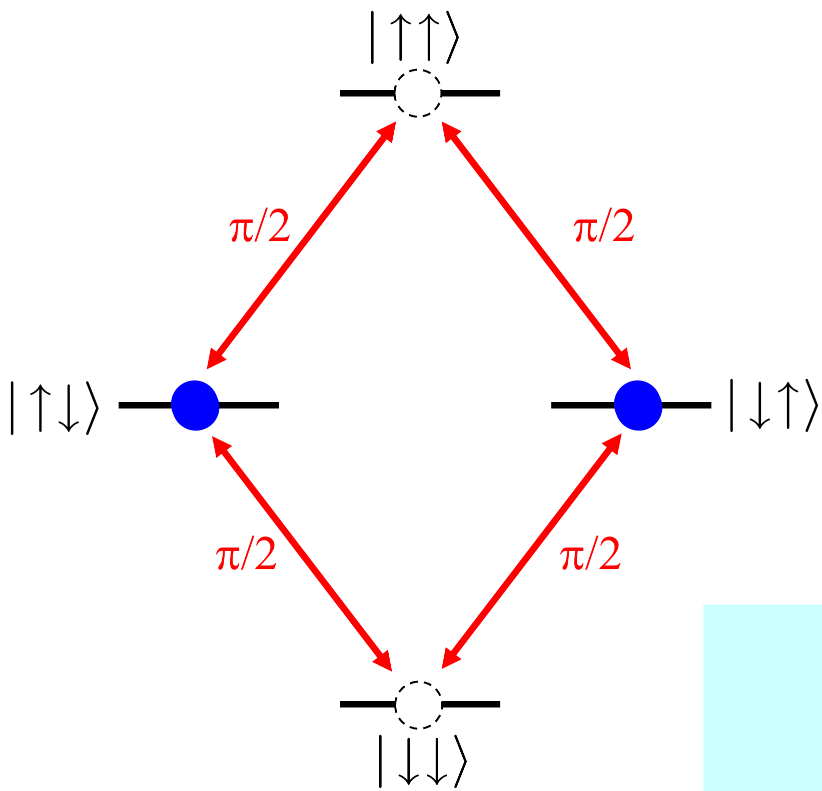


$$|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle \quad \longrightarrow |\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle$$

constructive interference

Entanglement check : interference

$$|\downarrow\uparrow\rangle + e^{i\phi}|\uparrow\downarrow\rangle$$



Parity:

$$|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle \longrightarrow |\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle \qquad +1$$

constructive interference

$$|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle \longrightarrow |\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle \qquad -1$$

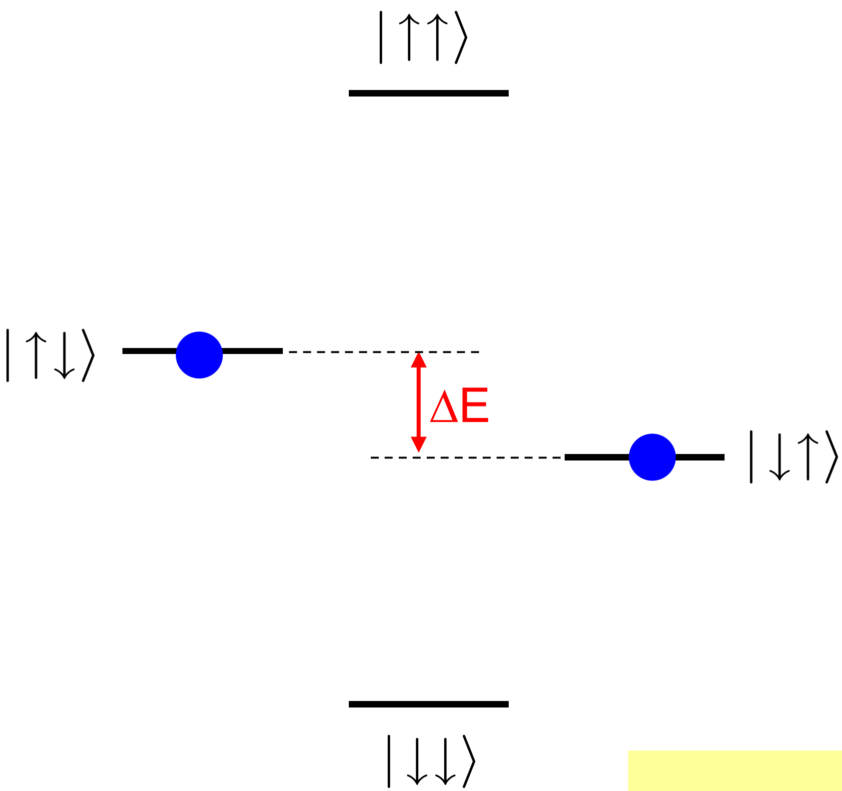
destructive interference

Parity measurement :
Are both ions in the same quantum state ?

$$P = (\rho_{\downarrow\downarrow} + \rho_{\uparrow\uparrow}) - (\rho_{\downarrow\uparrow} + \rho_{\uparrow\downarrow})$$

$$= \cos\phi$$

Detection of differential energy shifts

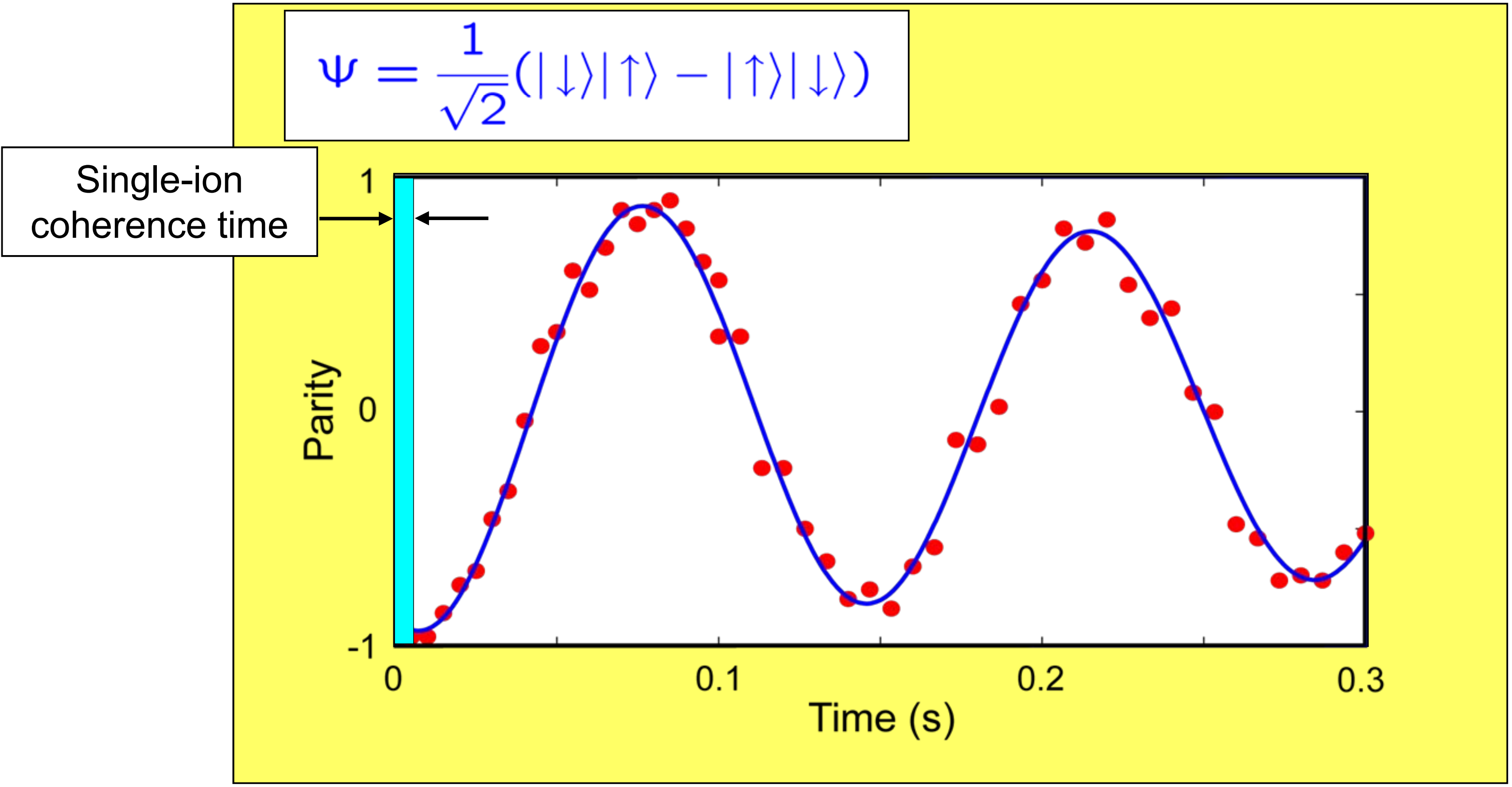


$$|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle \xrightarrow{\text{Time}} |\downarrow\uparrow\rangle - e^{i\phi(t)}|\uparrow\downarrow\rangle$$

$$\phi(t) = \frac{\Delta E}{\hbar} t$$

- Measurement of phase evolution $\phi(t)$ detects ΔE .
- Generalized Ramsey experiment !

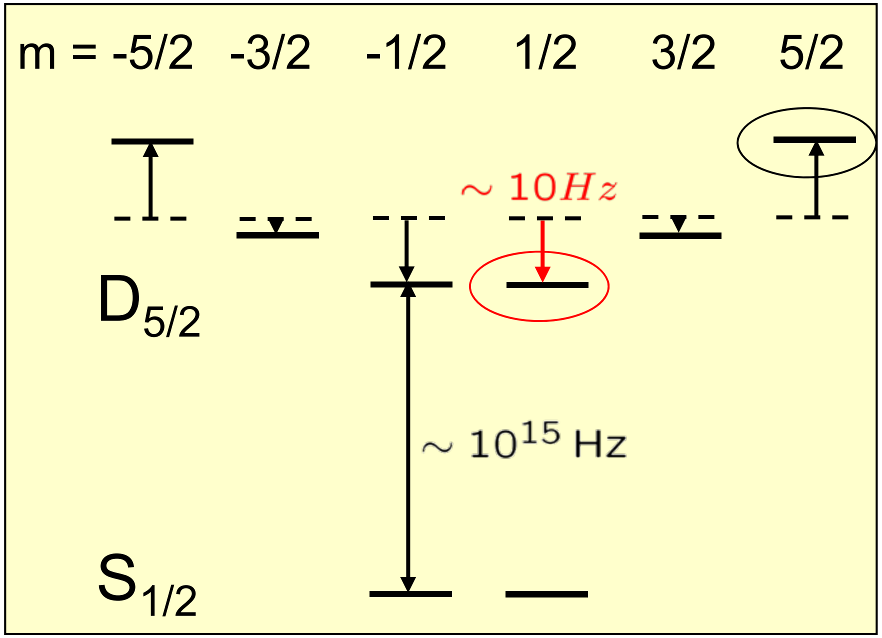
Parity oscillations in decoherence-free state-space



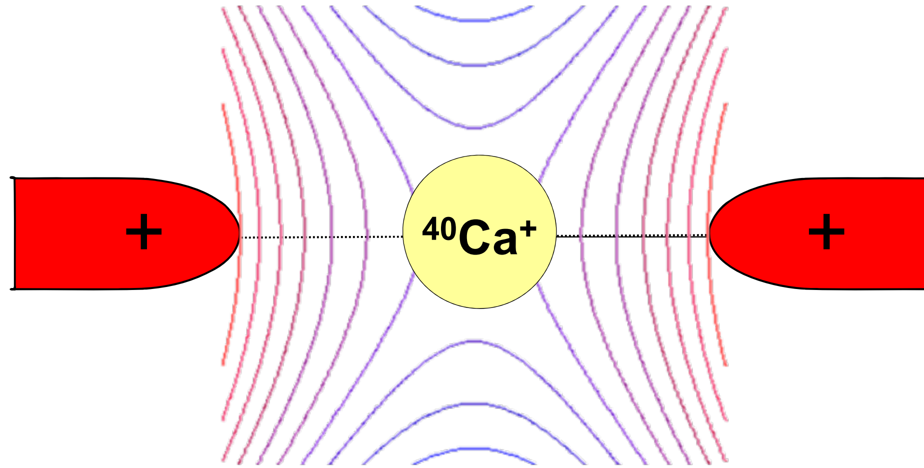
- sensitive to differential energy shifts $\longrightarrow$ noise rejection
- Long coherence times $\longrightarrow$ high spectral resolution

C. F. Roos *et al*., Phys. Rev. Lett. **92**, 220402 (2004)

Quantum metrology: Quadrupole moment of the D-state



Electric quadrupole field



$D_{5/2}$ – Quadrupole shift

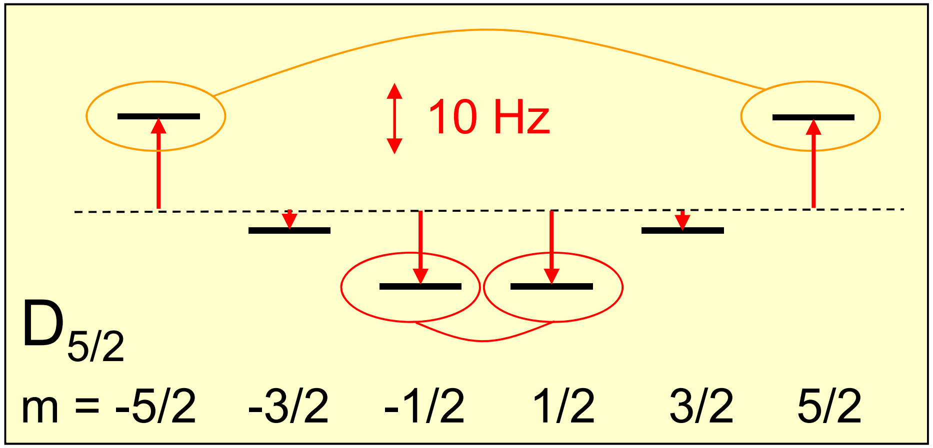
$$\Delta E_{quad} \propto \frac{dE_z}{dz}\,\Theta(D,5/2)$$ ~ 10 Hz

$D_{5/2}$ – Zeeman shift (B=1G) ~ 1 MHz

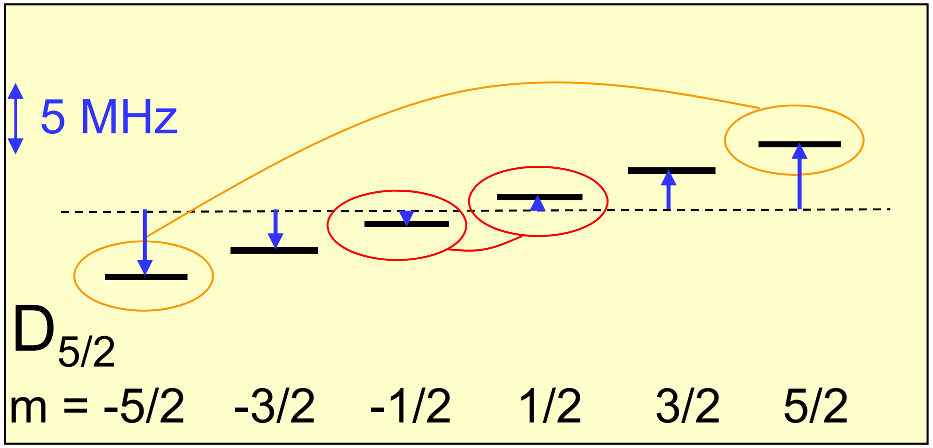
Spectroscopy with entangled states: measurement of the quadrupole moment

Step 1: Prepare $\Psi = |-5/2\rangle|5/2\rangle + |-1/2\rangle|1/2\rangle$

sensitive to quadrupole shift



insensitive to linear Zeeman shift



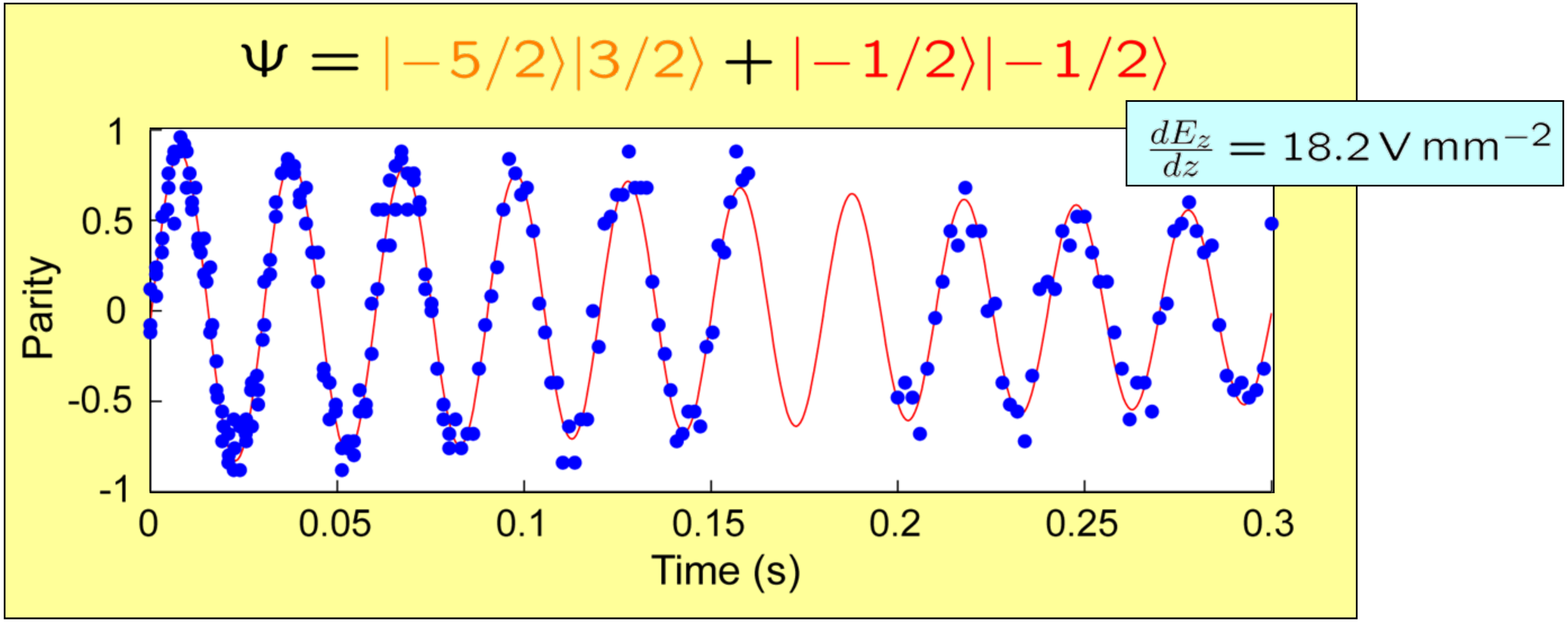
Decoherence-free subspace !

Step 2: Measure the state's phase evolution as a function of time

$$\Psi(t=0) \xrightarrow{\tau} \Psi(\tau) = |-5/2\rangle|5/2\rangle + e^{i\Delta\tau}|-1/2\rangle|1/2\rangle$$

C. F. Roos, arXiv: quant-ph/0508148

Quadrupole-induced phase oscillations



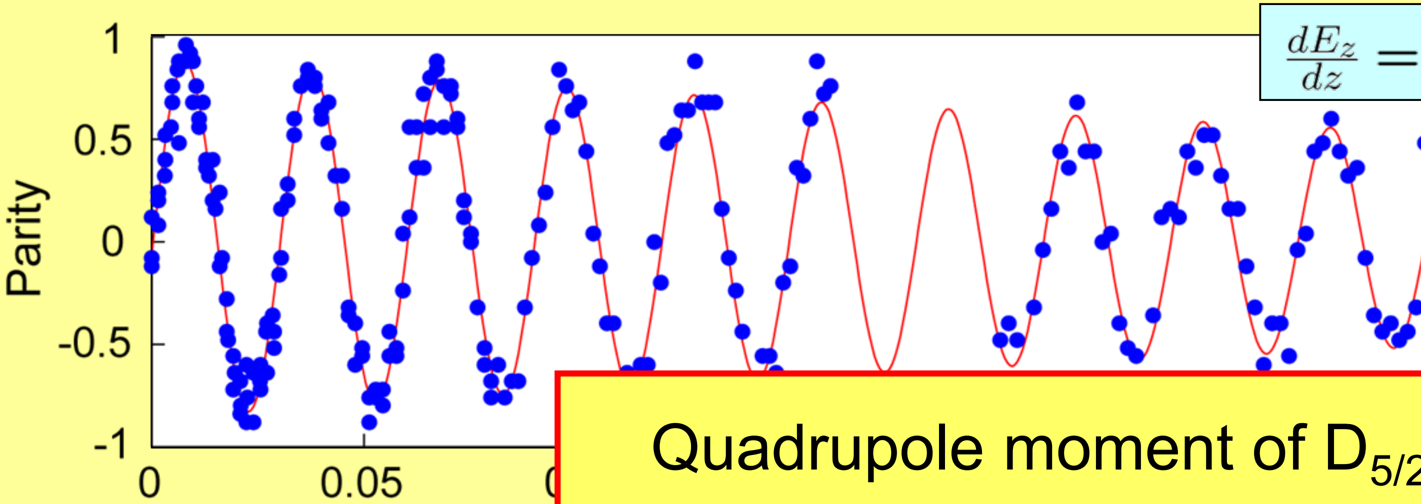
oszillation frequency:

Δ =(2π) 33.35(3) Hz

C. F. Roos *et al*., Nature **443**, 316 (2006)

Quadrupole-induced phase oscillations

$$\Psi = |-5/2\rangle|3/2\rangle + |-1/2\rangle|-1/2\rangle$$



$$\frac{dE_z}{dz} = 18.2\,\mathrm{V\,mm^{-2}}$$

Quadrupole moment of $D_{5/2}$ level

$^{40}Ca^{+}$: $\Theta(D,5/2) = 1.83(1)\,ea_0^2$

Oscillation frequency:

$\Delta = (2\pi)\ 33.35(3)$ Hz

Theory:

$\Theta = 1.819\,ea_0^2$

J. Mitroy: EPJD **46**, 415 (2008)

$\Theta = 1.85(2)\,ea_0^2$

M. Safronova: PRA **78**, 022514 (2008)

C. F. Roos *et al*., Nature **443**, 316 (2006)

Current status and outlook: QI with trapped ions

Status:

- Strings of 1-10 ion qubits
- Operational fidelities ≈ 99%
- Simple quantum algorithms requiring up to 50 laser pulses

  Quantum teleportation, entanglement swapping,
  entanglement purification, basic quantum error correction

Challenges:

- More ions $\longrightarrow$ Microfabricated segmented ion traps
- Even higher gate fidelities
- System integration

Other directions:

- Quantum simulations
- Tests of quantum theory
- Quantum metrology