Trapped ions, scalability, & quantum metrology

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Scottish Universities Summer School in Physics
29 July 2011

A neat experiment with ions…….

PHYSICAL REVIEW LETTERS

VOLUME 70 19 APRIL 1993 NUMBER 16

Young's Interference Experiment with Light Scattered from Two Atoms

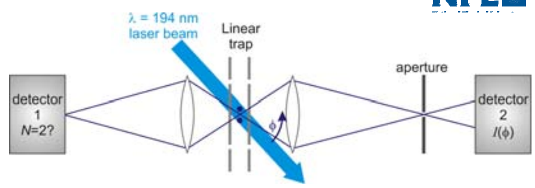
U. Eichmann,[a] J. C. Bergquist, J. J. Bollinger, J. M. Gilligan, W. M. Itano, and D. J. Wineland
National Institute of Standards and Technology, Boulder, Colorado 80303

M. G. Raizen
Department of Physics, University of Texas, Austin, Texas 78712

(Received 18 December 1992)

We report the first observation of interference effects in the light scattered from two trapped atoms. The visibility of the fringes can be explained in the framework of Bragg scattering by a harmonic crystal and simple "which path" considerations of the scattered photons. If the light scattered by the atoms is detected in a polarization-sensitive way, then it is possible to selectively demonstrate either the particle nature or the wave nature of the scattered light.

Young's interference with 2 ions



$^{198}Hg^+$ stored along axis of a linear trap

- laser is linearly polarised
- laser-cooled ions: strong localisation of ions in trap
- ion separation: 3.7 μm ≤ d_{ion} ≤ 4 μm
- detector 1: monitors ion number
- detector 2: measures $I(\phi)$: light intensity scattered from ions
- aperture suppresses background

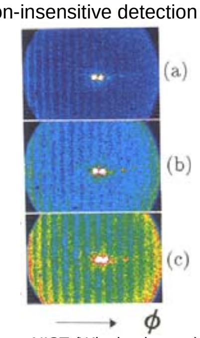
Young's interference fringes

with polarisation-insensitive detection……

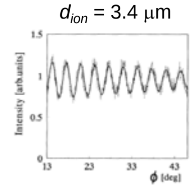
d_{ion} = 5.4 μm

d_{ion} = 4.3 μm

d_{ion} = 3.7 μm



d_{ion} = 3.4 μm



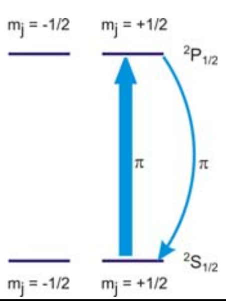
• fringe spacing ↑ as ion separation ↓

NIST (Wineland group):
U. Eichmann et al, Phys. Rev. Lett. 70 2359 (1993)

Scattered photons: 1

Excite $^{198}Hg^+$ ions with linearly (π) polarised light

- Scattered light has either linear (π) or circular (σ) polarisation
- Assume only one scattered photon at a time



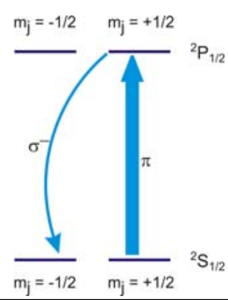
π-polarised scattered photons

- Δm_j=0
- ions' final states same as initial states
- can't determine which ion scattered the photon
- quantum mechanics predicts that interference arises from scattered light

Scattered photons: 2

Excite $^{198}Hg^+$ ions with linearly (π) polarised light

- Scattered light has either linear (π) or circular (σ) polarisation
- Assume only one scattered photon at a time



σ -polarised scattered photons:

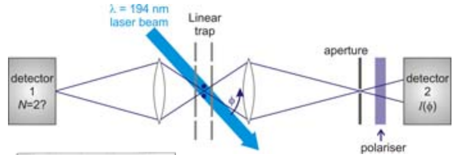
- Δm_j=±1
- final state of scattering ion differs from its initial state
- *in principle* distinguish scattering ion from "spectator"
- hence could determine which path the photon travelled

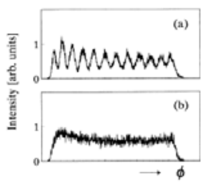
Polarisation-sensitive detection can switch between wave or particle nature of the scattered photon

“Which-way?” experiment

Polarisation-sensitive detection of scattered light

- assume one ion scattering at a time





detecting π-polarised photons:

- can’t determine which ion scattered the photon

detecting σ-polarised photons:

- *in principle* determine which ion scattered a photon

from: U. Eichmann et al, Phys. Rev. Lett. 70 2359 (1993)

Lecture 1 outline: some basics

Theoretical

- Ion traps
- Laser cooling
- Atom-laser interaction (confined 2-level atom)

Experimental

- Practical ion traps
- Elementary experimental techniques
- Qubit transitions and coherent spectroscopy

…foundation for lecture 2 and for Christian Roos’ lectures

Lecture 2 outline

Microtraps

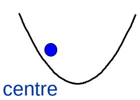
- The scalability challenge
- Various approaches
- A case study in trap design, creation, and operation

Precision measurements

- Atomic clocks
- Ion clocks
- Fundamental constants
- Advanced techniques: quantum logic clock

1. Ion trap theory basics

RF traps for charged particles



Desire harmonic potential

- Consider electric potential $\Phi(x,y,z,t)$
- Approximately quadrupole spatial shape at centre
- Potential composed of static and time varying (ω_{RF}) parts

$$\Phi(x,y,z,t) = U_{DC}\,\tfrac{1}{2}\left(\alpha x^2+\beta y^2+\gamma z^2\right) + U_{RF}\cos(\omega_{RF}t)\,\tfrac{1}{2}\left(\alpha' x^2+\beta' y^2+\gamma' z^2\right)$$

Need to fulfill Laplace’s equation $\nabla^2\Phi = 0$ for all t

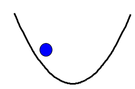
- constraints $\alpha+\beta+\gamma=0$
  $\alpha'+\beta'+\gamma'=0$

Can’t generate 3D potential minimum

→ Can only trap charged particle in dynamic fashion

RF traps for charged particles

$$\Phi(x,y,z,t) = U_{DC}\,\tfrac{1}{2}\left(\alpha x^2+\beta y^2+\gamma z^2\right) + U_{RF}\cos(\omega_{RF}t)\,\tfrac{1}{2}\left(\alpha' x^2+\beta' y^2+\gamma' z^2\right)$$



Constraints: $\alpha+\beta+\gamma=0$ $\quad\alpha'+\beta'+\gamma'=0$

Option 1 : $\alpha=\beta=\gamma=0$
$\alpha'+\beta'=-\gamma'$

- 3D dynamical confinement in pure oscillating field

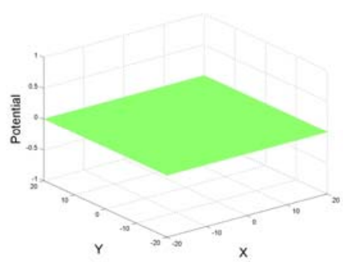
Option 2 : $-(\alpha+\beta)=\gamma>0$
$\alpha'=-\beta'$

- 2D dynamical confinement in oscillating field
- Static potential confinement in 3rd dimension

RF potential

- Linear trap
- $\alpha' = -\beta'$
- 2D oscillating potential

$$\Phi(x,y,z,t) = U_{DC}\,\tfrac{1}{2}\left(\alpha x^2 + \beta y^2 + \gamma z^2\right) + U_{RF}\cos(\omega_{RF} t)\,\tfrac{1}{2}\left(\alpha' x^2 + \beta' y^2 + \gamma' z^2\right)$$



Equations of motion

1. Consider 1D motion of ion with mass m, and charge +e

$$m\ddot{x} = -e\frac{\partial \Phi}{\partial x} \qquad \ddot{x} = -\frac{e}{m}\left[U_{DC}\alpha + U_{RF}\cos(\omega_{RF}t)\alpha'\right]$$

2. Transform using subsititions:

$$\xi = \frac{\omega_{RF}t}{2} \qquad a_x = \frac{4eU_{DC}\alpha}{m\omega_{RF}^2} \qquad q_x = \frac{2eU_{RF}\alpha'}{m\omega_{RF}^2}$$

3. Transform to Mathieu differential equation

$$\frac{d^2x}{d\xi^2} + \left[a_x - 2q_x\cos(2\xi)\right]x = 0$$

- differential equation with periodic coefficients

Solutions to Mathieu equation: 1

$$\frac{d^2x}{d\xi^2} + \left[a_x - 2q_x\cos(2\xi)\right]x = 0 \qquad (1)$$

Stable **solutions….**

- …to ensure the ion's motion is bound
- general form is

$$x(\xi) = Ae^{i\beta_x\xi}\sum_{n=-\infty}^{\infty}C_{2n}e^{i2n\xi} + Be^{-i\beta_x\xi}\sum_{n=-\infty}^{\infty}C_{2n}e^{-i2n\xi} \qquad (2)$$

- β_x, C_{2n} are functions of a_x, q_x only
- A, B are constants

(2) into (1) obtains recursion relations connecting a_x, q_x

$$C_{2n+2} - D_{2n}C_{2n} + C_{2n-2} = 0 \qquad D_{2n} = \frac{1}{q_x}\left[a_x - (2n+\beta_x)^2\right]$$

Solutions to Mathieu equation: 2

$$C_{2n+2} - D_{2n}C_{2n} + C_{2n-2} = 0 \qquad D_{2n} = \frac{1}{q_x}\left[a_x - (2n+\beta_x)^2\right]$$

- Rearranging yields continued fractions for C_{2n}, β_x^2

$$\beta_x^2 = a_x - q_x\left[\frac{1}{D_0 - \frac{1}{D_2 - \frac{1}{\cdots}}} + \frac{1}{D_0 - \frac{1}{D_{-2} - \frac{1}{\cdots}}}\right]$$

- Contributions of higher orders drop off rapidly for typical experiments (i.e. typical a_x, q_x)
- Stability region bounds: pairs of (a_i, q_i) yielding $\beta_i = 0,1$

The stability region: 1

Exact shape of potential depends on parameters in Φ(*x,y,z,t*)

- trap electrodes are often cylindrically symmetric about *z*-axis
- RF (dynamical) confinement in 3D
- α = α′ = β = β′ = -2γ = -2γ′

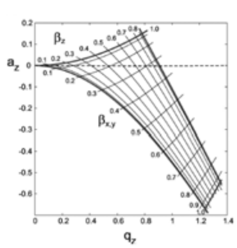
Mathieu eqn. Parameters:

$$a_z = -2a_x = -2a_y$$
$$q_z = -2q_x = -2q_y$$

$$a_x = \frac{4eU_{DC}\alpha}{m\omega_{RF}^2} \qquad q_x = \frac{2eU_{RF}\alpha'}{m\omega_{RF}^2}$$

Trap dimensions: r_o, z_o

$$\alpha = 2\big/\left(r_o^2 + 2z_o^2\right)$$



From: Leibfried *et al*, Rev Mod Phys, 281 75 (2003)

The stability region: 2

Linear trap

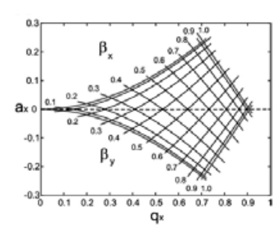
- Dynamic confinement in 2D
- Static confinement in *z*
- (α + β) = γ, α′ = β′, γ′ = 0

$$q_y = -q_x \qquad q_z = 0$$

$$a_x = \frac{4eU_{DC}\alpha}{m\omega_{RF}^2} \qquad q_x = \frac{2eU_{RF}\alpha'}{m\omega_{RF}^2}$$

Trap dimensions: r_o, z_o

$$\alpha = 2\big/\left(r_o^2 + 2z_o^2\right)$$



From: Leibfried *et al*, Rev Mod Phys, 281 75 (2003)

Ion trajectory

Lowest order approximation to *x*(*t*)

- $\left(|a_x|, q_x^2\right) << 1$
- Solutions approximated by:

$$r_i(t) = r_{oi}\cos(\omega_i t + \phi_i)\left[1 + \frac{q_i}{2}\cos(\omega_{RF} t)\right]$$

secular motion — Micromotion

$$\omega_i = \beta_i \frac{\omega_{RF}}{2}$$

$$\beta_i = \sqrt{a_i + \frac{q_i^2}{2}}$$

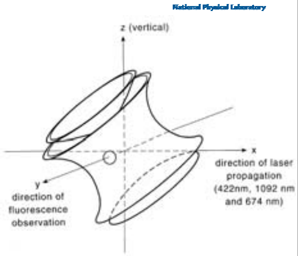
Neglect:
Approximate potential as that of harmonic oscillator
"Pseudopotential approximation"

2. Practical ion traps

First RF Paul traps

Cylindrically symmetric

- Hyperbolic electrodes
- Large ion-electrode distance
- Limited optical access
- Not ideal for single ions



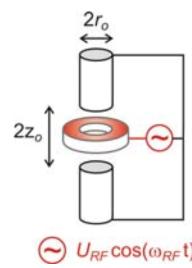
Operating parameters for $^{88}Sr^+$

- $r_o = z_o\sqrt{2} = 5$ mm
- $U_{RF} = 320$ V
- $\omega_{RF} = 1.778$ MHz
- $\omega_r = \sim 100$ kHz

Ring trap

Cylindrically symmetric

- Formed of ring and endcaps
- Good optical access
- Single ion storage
- ~MHz motional frequencies



Operating parameters for $^{88}Sr^+$

- $r_o = 500$ μm
- $U_{RF} = 450$ V
- $\omega_{RF}/2\pi = 14$ MHz
- $(\omega_r, \omega_z)/2\pi = (0.72, 1.16)$ MHz

Ring traps

NPL, UK
$^{88}Sr^+$

$r_o = 500$ μm

PTB, Germany
$^{171}Yb^+$

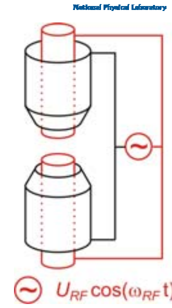
$r_o = 650$ μm

Endcap trap

Cylindrically symmetric

- Co-axial electrodes
- Excellent optical access
- Single ion storage
- ~MHz motional frequencies

C A Schrama *et al*, Opt. Comm. 101, 32 (1993)

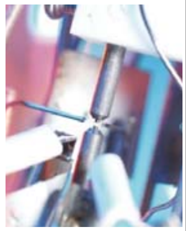


Operating parameters for $^{88}Sr^+$

- $z_o = 280$ μm
- $U_{RF} = 390$ V
- $\omega_{RF}/2\pi = 15.9$ MHz
- $(\omega_x, \omega_y, \omega_z)/2\pi = (1.94, 1.97, 3.96)$ MHz

AS, M A Wilson & P Gill, Opt. Comm. 190, 193 (2001)

Endcap trap

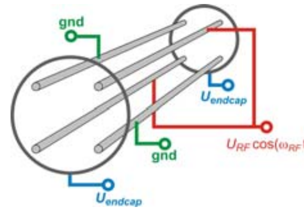


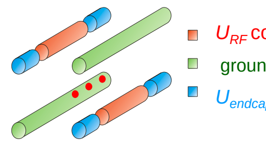
AS, M A Wilson & P Gill, Opt. Comm. 190, 193 (2001)

Linear trap

Radial symmetry

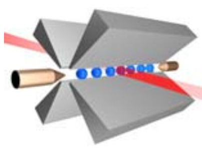
- single trapping zone
- ion strings
- $\omega_r > \omega_z$





Linear blade trap

U. Innsbruck design:



U. Innsbruck (R Blatt): $^{40}Ca^{+}$

- r_o = 800 μm
- Endcap separation = 5 mm
- U_{RF} ~ 1.4 kV,
- $\omega_{RF}/2\pi$ = 25.5 MHz
- U_{endcap} = 1000 V
- $(\omega_r, \omega_z)/2\pi$ = (4.0, 1.2) MHz

… more details from Christian Roos

Linear blade trap

U. Innsbruck (R Blatt): $^{40}Ca^{+}$

- r_o = 800 μm
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- U_{endcap} = 1000 V
- $(\omega_r, \omega_z)/2\pi$ = (4.0, 1.2) MHz

Operating parameters for $^{88}Sr^{+}$

- r_o = 350 μm
- Endcap separation = 2.7 mm
- U_{RF} ~ 390 V, $\omega_{RF}/2\pi$ = 16.3 MHz
- $\omega_r/2\pi$ = 0.9 MHz
- U_{endcap} = 250 V
- $\omega_z/2\pi$ = 0.4 MHz

3. Ion cooling basics

Doppler cooling

Ion's equilibriium energy

- Balance between cooling and heating

Average cooling force:

$$F = \hbar k \Gamma \left[\frac{\Omega^2}{\Gamma^2 + 4(\Delta - kv)^2} \right]$$

$$\frac{\Omega^2}{\Gamma^2} = \frac{I}{4I_S}$$

photon momentum; excited state decay rate; excited state probability

Cooling rate:

$$\dot{E}_{cooling} = \langle Fv \rangle = \left.\frac{dF}{dv}\right|_{v=0} \langle v^2 \rangle$$

Where for small velocities:
(Doppler broadening << Γ)

$$F = F_o + \left.\frac{dF}{dv}\right|_{v=0} v$$

Doppler cooling

Heating:

Spontaneously emitted photons: $\langle \Delta \mathbf{p} \rangle = 0$ but $\langle \Delta p^2 \rangle \neq 0$

Random walk process: $\langle \Delta p^2 \rangle \propto (\hbar k)^2 N$

Heating rate: $\dot{E}_{heating} = \frac{1}{2m}\frac{d}{dt}\langle p^2 \rangle$

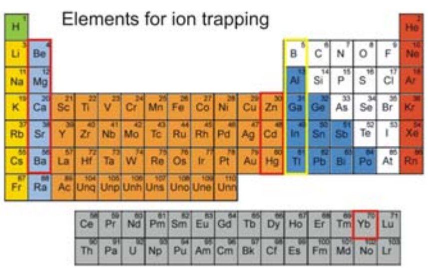
Equilibrium: $\dot{E}_{heating} + \dot{E}_{cooling} = 0$ $\qquad m\langle v^2 \rangle = k_B T$

Minimum energy: $k_B T_{\min} = \frac{\hbar\Gamma}{2}$ @ $\Delta = -\Gamma/2$

Full derivation & discussion, see: Leibfried *et al*, Rev Mod Phys, 281 75 (2003)

4. Elementary experimental techniques

Which ion?



Requirements

- Doppler cooling: optical transition with fast (~ns) decay:
- Qubit/clock: transition between long-lived states (hyperfine or optical)

See Monroe group website (www.iontrap.umd.edu) for interactive periodic table

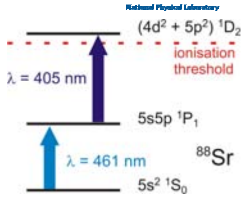
Which ion?

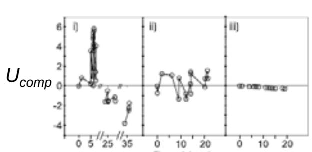
ion	cooling λ/nm	$\Gamma_{cooling}$ / MHz	qubit / clock transition	photoionisation λ/nm
$^{9}Be^{+}$	313	20	HF, 1.25 GHz	235
$^{25}Mg^{+}$	280	43		202
$^{40}Ca^{+}$	397	23	optical, 729 nm	422 + 389
$^{88}Sr^{+}$	422	23	optical, 674 nm	461 + 412
$^{138}Ba^{+}$	493.5	15	optical, 1760 nm	554
$^{111}Cd^{+}$	226.5	60	HF, 14.5 GHz	229
$^{199}Hg^{+}$	194	70	optical, 282 nm	185
$^{171}Yb^{+}$	369	20	HF, 12.6 GHz optical, 435 & 467 nm	399 + 394

Creating ions

Photoionisation

- Highly efficient
- Low atomic flux
- No charging of surroundings





Ca⁺

Kjærgaard *et al*, Appl. Phys. B 71, 207 (2000).

S. Gulde *et al*, Appl. Phys. B 73, 861 (2001).

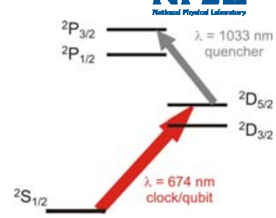
D.M. Lucas *et al*, Phys. Rev. A 69, 012711 (2004).

Sr+

M Brownnutt *et al*, Appl. Phys. B 87, 411 (2007).

$^{88}Sr^{+}$

- **422 nm**: laser cooling transition (frequency-doubled diode laser)
- **1092 nm**: repumper transition (Nd^{3+}-doped fiber laser)
- **674 nm**: narrow linewidth optical ***clock*** or ***qubit*** transition (highly-stable diode laser system)
- $^{2}D_{5/2}$ state lifetime = 390.8(1.6) ms *
- **1033 nm**: clearout transition (diode laser)



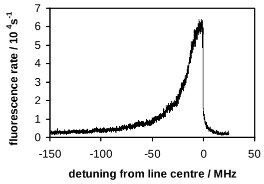
… and $^{40}Ca^{+}$ has similar energy level structure

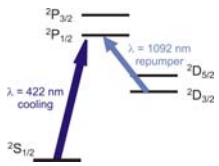
* Letchumanan, Wilson, Gill, Sinclair, PRA 72, 012509 (2005).

Single ion cooling

Doppler cooling

- intensity I_{cool} below saturation
- detuning $\Delta = \Gamma/2$
- detect blue fluorescence





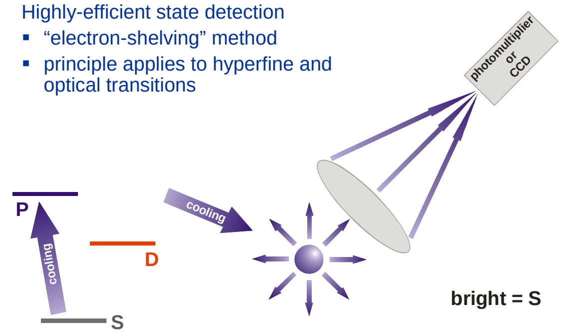
Scan cooling laser frequency

- Sharp drop due to ion heating
- Can use blue fluorescence to detect ion's state

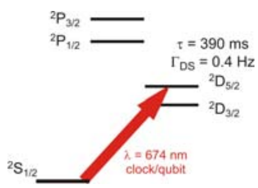
Pulsed-probe spectroscopy

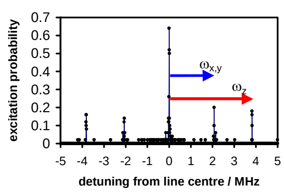
Highly-efficient state detection

- "electron-shelving" method
- principle applies to hyperfine and optical transitions



Single ion spectroscopy





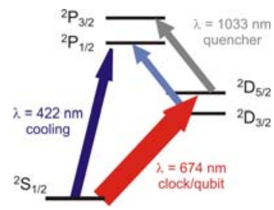
$^{88}Sr^+$ in endcap trap

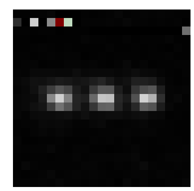
- Γ_{SD} ~ 0.4 Hz
- observe resolved sidebands: $\Gamma_{SD} << \omega_r, \omega_z$
- measure $\omega_r/2\pi$ = 2.1 MHz, $\omega_z/2\pi$ = 3.9 MHz
- Need $\Gamma_{laser} \leq$ 1 kHz (ideally << 1 kHz)

Electron shelving movie

Ion's equilibrium energy

- Balance between cooling and heating





5. Coherent optical interactions

Laser-ion interaction: 2-level atom

Total Hamiltonian*: $\hat{H} = \hat{H}_{motion} + \hat{H}_{electronic} + \hat{H}_{interaction}$

↑ describes laser-ion interaction

$$\hat{H}_{motion} = \frac{p^2}{2m} + \frac{1}{2} m \omega_m x^2$$

$$\hat{H}_{electronic} = \frac{1}{2} \hbar \omega_o \sigma_z$$

$$\hat{H}_{interaction} = \frac{1}{2} \hbar \Omega \left(\sigma^+ + \sigma^- \right) \left[e^{i(kx - \omega_L t + \phi)} + e^{-i(kx - \omega_L t + \phi)} \right]$$

Rabi frequency — Pauli matrices — laser frequency — *k*: laser wavevector

* Blockley, Walls, Risken, Europhys. Lett. 17, 509 (1992)

The Lamb-Dicke parameter: η

Important dimensionless quantity

$$\eta = k\sqrt{\frac{\hbar}{2m\omega_m}}$$

- ion oscillating at ω_m
- interacting with laser with wavevector k

Lamb-Dicke η parameter is:

- a measure of the spatial extent of the ion's ground state wavefunction
- desire η small, so try to make ω_m big

More generally: $$\eta = k\cos\theta\sqrt{\frac{\hbar}{2m\omega_m}}$$

…when $\underline{k}$ is at angle θ to motional axis

Interaction Hamiltonian: 2-level atom

- Express in terms of creation & annihilation operators
- Transform to interaction picture

$$\hat{H}_{interaction} = \frac{\hbar\Omega}{2}\left[e^{i\eta(\hat{a}+\hat{a}^\dagger)}\sigma^+ e^{-i\Delta t} + e^{-i\eta(\hat{a}+\hat{a}^\dagger)}\sigma^- e^{i\Delta t}\right]$$

where $\hat{a} = a e^{i\omega_m t}$ and $\Delta = \omega_L - \omega_o$

Laser couples electronic & vibrational states, depending on Δ

- laser couples $|g,n\rangle \to |e,n+m\rangle$

Rabi frequency*: $$\Omega_{n,n+m} = \Omega_o\left|\langle n+m|e^{i\eta(\hat{a}+\hat{a}^\dagger)}|n\rangle\right|$$

$$\Omega_{n,n+m} = \Omega_o e^{-\frac{\eta}{2}}\eta^{|m|}L_n^{|m|}(\eta^2)\left[\frac{n!}{(n+m)!}\right]^{\frac{m}{2|m|}}$$

* Wineland & Itano, Phys Rev A 20, 1521 (1979)

Lamb-Dicke regime

Lamb-Dicke regime

- extent of ion's wave function confined to < $1/k$
- inequality must hold:

$$\eta\sqrt{2n+1} << 1 \qquad \eta = k\sqrt{\frac{\hbar}{2m\omega_m}}$$

$\Omega_{n,n+m}$ simplifies: $$\Omega_{n,n+m} = \Omega_o\left|\langle n+m|(1+i\eta(\hat{a}+\hat{a}^\dagger))|n\rangle\right|$$

3 resonances exist:

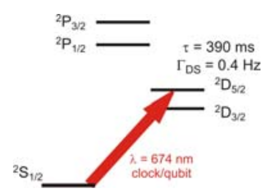
- carrier: $\Omega_{n,n} = \Omega_o(1-\eta^2 n)$ $\delta n = 0$ $\Delta = 0$
- red sideband: $\Omega_{n,n-1} = \Omega_o\eta\sqrt{n}$ $\delta n = -1$ $\Delta = -\omega_m$
- blue sideband: $\Omega_{n,n+1} = \Omega_o\eta\sqrt{n+1}$ $\delta n = +1$ $\Delta = +\omega_m$

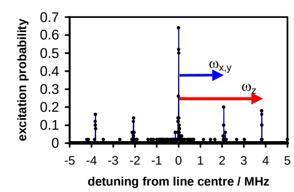
…..δn = 2 transitions are greatly suppressed

Lamb-Dicke regime

$^{88}Sr^+$ example

- λ = 674 nm, η_z = 0.035, η_r = 0.048, n ~ 10
- Resolved sidebands: $\Gamma_{SD} << \omega_{r,z}$

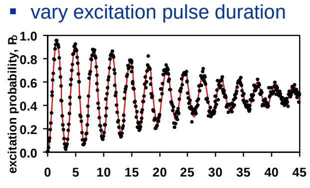




Carrier transition

Doppler-cool $^{88}Sr^+$ ion in 3D

- prepare ion in $|S, m_j = -\frac{1}{2}\rangle$ electronic state
- excite $|S, m_j = -\frac{1}{2}, n\rangle \to |D, m_j = -\frac{1}{2}, n\rangle$ transition (carrier)
- vary excitation pulse duration

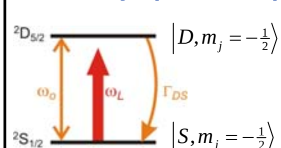


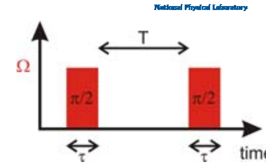
decay in contrast due to thermal distribution over vibrational levels

$$\Omega_{n,n} = \Omega_o(1-\eta^2 n)$$

$$P = \frac{A}{2}\left[1 - e^{-\gamma t}\sum_{n_r,n_z} P_{n_r}P_{n_z}\cos(\Omega_{n_r,n_z}t)\right] \longrightarrow \bar{n}_r = 14 \quad \bar{n}_z = 8$$

Ramsey spectroscopy





Prepare ion in state $|S\rangle$

- first π/2-pulse creates superposition $$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|S\rangle + |D\rangle\right]$$

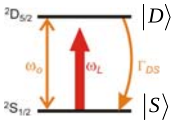
Free precession

- phase shift accumulates between atom and laser when $\omega_{laser} \neq \omega_o$

2nd π/2-pulse

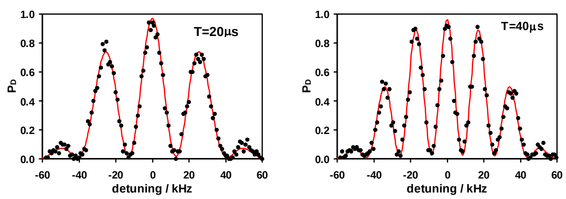
- atom-laser phase shift read out by atomic populations ρ_{SS} and ρ_{DD}
- fringes observed in spectral line shape

Ramsey spectroscopy in $^{88}Sr^{+}$



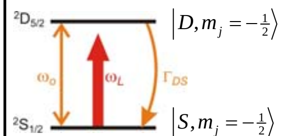
Doppler-cooled ion

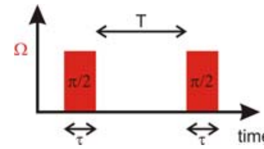
- prepare ion in state $\left|S, m_j = -\tfrac{1}{2}\right\rangle$
- $\pi/2$-pulse duration = 15 μs
- detect ion state via electron shelving
- tune ω_L



see V Letchumanan, *et al*, Phys Rev A 70,033419 (2004) for more details

Ramsey spectroscopy





Optical Bloch equations for single laser pulse:

$$\dot{\tilde{\rho}}_{DS} = \left[i(\omega_L - \omega_o) - \left(\Gamma_{laser} + \tfrac{1}{2}\Gamma_{DS}\right)\right]\tilde{\rho}_{DS} - i\frac{\Omega}{2}e^{-i\Phi}(\rho_{DD} - \rho_{SS})$$

$$(\dot{\rho}_{DD} - \dot{\rho}_{SS}) = -\Gamma_{DS}\left[1 + (\rho_{DD} - \rho_{SS})\right] + i\Omega\left(e^{-i\Phi}\tilde{\rho}^*_{DS} - e^{i\Phi}\tilde{\rho}_{DS}\right)$$

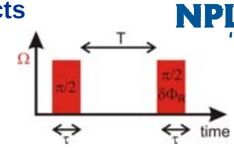
laser bandwidth: Γ_{laser}
laser phase: Φ

Initial conditions (t=0):

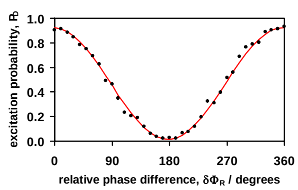
$$\rho_{SS} = 1 \quad \rho_{DD} = 0 \quad \tilde{\rho}_{DS} = 0$$

Oscillator phase effects

Relative phase shift between $\pi/2$-pulses: $\delta\Phi_R$



- record Ramsey spectrum
- P_D is maximum at ω_o
- set $\omega_{laser} = \omega_o$
- measure $P_D(\delta\Phi_R)$



Observe expected oscillatory behaviour in P_D as $\delta\Phi_R$ is varied

6. Ground state cooling

Quantum state preparation

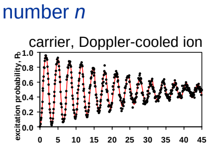
Coherent interactions

- Ω is a function of vibrational quantum number *n*

carrier: $\Omega_{n,n} = \Omega_o\left(1 - \eta^2 n\right)$

RSB: $\Omega_{n,n-1} = \Omega_o \eta \sqrt{n}$

BSB: $\Omega_{n,n+1} = \Omega_o \eta \sqrt{n+1}$



Need to prepare ion in well-defined quantum state:

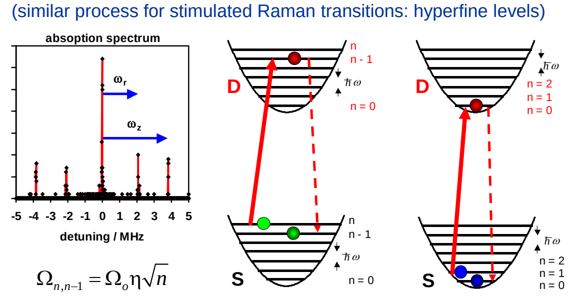
- electronic & vibrational ground state

$$\left|S, m_j = -\tfrac{1}{2}, n = 0\right\rangle$$

- require ground state cooling

Resolved sideband cooling

Example with optical transition
(similar process for stimulated Raman transitions: hyperfine levels)



$$\Omega_{n,n-1} = \Omega_o \eta \sqrt{n}$$

F. Diedrich, *et al*, PRL 62, 403 (1989), Ch Roos *et al*, PRL 83, 4713 (1999)

$^{88}Sr^{+}$ sideband cooling

Optically pump on cooling transition

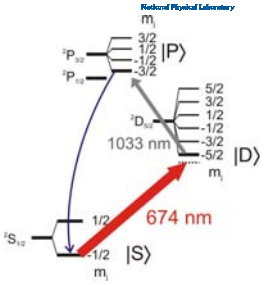
- prepare ion in $\left|S, m_j = -\tfrac{1}{2}, n\right\rangle$

Excite red sideband of qubit transition:

$$\left|S, m_j = -\tfrac{1}{2}, n\right\rangle \rightarrow \left|D, m_j = -\tfrac{5}{2}, n-1\right\rangle$$

Quench $\left|D\right\rangle$ state:

$$\left|D, m_j = -\tfrac{5}{2}, n-1\right\rangle \rightarrow \left|P, m_j = -\tfrac{3}{2}, n-1\right\rangle \rightarrow \left|S, m_j = -\tfrac{1}{2}, n-1\right\rangle$$



Optimum cooling rate

Optimal cooling rate

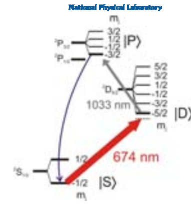
- tune effective decay rate Γ'_{DS}
- adjust saturation parameter of quencher

1) Quencher not intense enough

- R_{cool} limited by Γ_{DS}

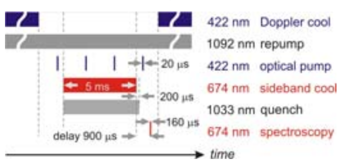
2) Quencher too intense

- increased spontaneous emission on $\left|P\right\rangle \rightarrow \left|D\right\rangle$ & $\left|P\right\rangle \rightarrow \left|S\right\rangle$
- increased momentum diffusion $\rightarrow$ heating!

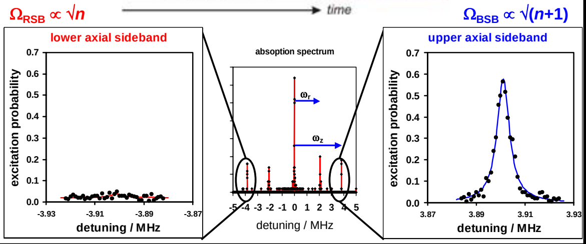


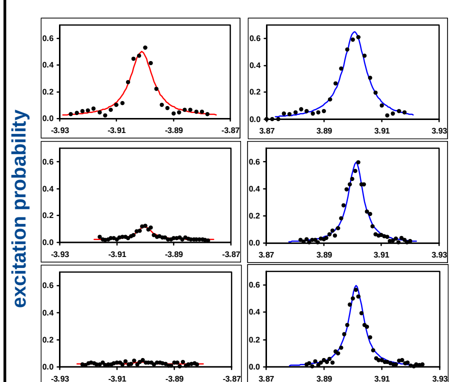
$^{88}Sr^{+}$ sideband cooling

Experiment sequence



$\Omega_{RSB} \propto \sqrt{n}$ $\Omega_{BSB} \propto \sqrt{(n+1)}$





0 ms cooling

2 ms cooling

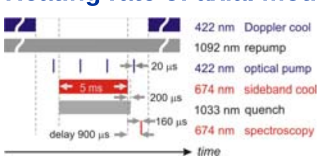
$\bar{n}_z = 0.19$

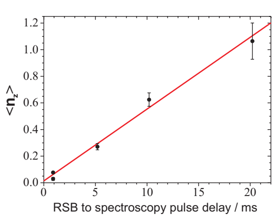
5 ms cooling

$\bar{n}_z = 0.03$

$\Omega_{RSB} \propto \sqrt{n}$ detuning from carrier / MHz $\Omega_{BSB} \propto \sqrt{(n+1)}$

Heating rate of axial motion





Heating rate of axial mode

$\dot{n}_z = 0.054\,/\,ms$

Extrapolating to $t = 0$

$\bar{n}_z(t=0) = 0.014$

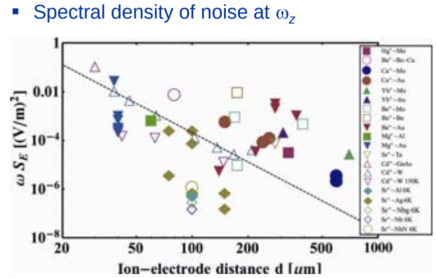
Ground state probability = 98.6%

V. Letchumanan, *et al*, PRA 75 063425 (2007)

Measured heating rates

Heating is due to electric field noise on electrodes

- Spectral density of noise at ω_z



$$\dot{n} = \frac{e^2 S_E(\omega_z)}{4m\hbar\omega_z}$$

$$S_E(\omega) \propto \frac{1}{d^4}$$

Graph courtesy of Winni Hensinger, Sussex

Turchette *et al*, Phys. Rev. A 61, 063418 (2000)
Deslauriers *et al*, Phys. Rev. A 70, 043408 (2004)

7. Summary

Summary: trapping 1

Trapping potential

- static & time-varying parts

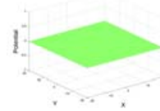
$$\Phi(x,y,z,t) = U_{DC}\tfrac{1}{2}\left(\alpha x^2+\beta y^2+\gamma z^2\right) + U_{RF}\cos(\omega_{RF}t)\tfrac{1}{2}\left(\alpha' x^2+\beta' y^2+\gamma' z^2\right)$$

constraint

- satisfy Laplace equation: $\nabla^2\Phi = 0$

2 configurations of Φ:

- 3D dynamical confinement in pure oscillating field
- 2D dynamical & static potential in 3rd dimension



Summary: trapping 2

Equation of motion: $\ddot{x} = -\frac{e}{m}\left[U_{DC}\alpha + U_{RF}\cos(\omega_{RF}t)\alpha'\right]$

- Transform to Mathieu equation

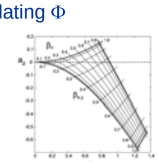
$$\frac{d^2x}{d\xi^2} + \left[a_x - 2q_x\cos(2\xi)\right]x = 0 \qquad \xi = \frac{\omega_{RF}t}{2} \qquad a_x = \frac{4eU_{DC}\alpha}{m\omega_{RF}^2} \qquad q_x = \frac{2eU_{RF}\alpha'}{m\omega_{RF}^2}$$

3D oscillating Φ

$a_z = -2a_x = -2a_y$

$q_z = -2q_x = -2q_y$

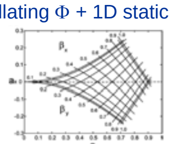
$\alpha = 2/\left(r_o^2 + 2z_o^2\right)$



2D oscillating Φ + 1D static

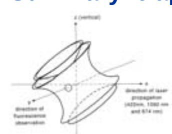
$q_y = -q_x$

$q_z = 0$

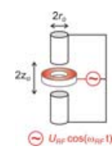


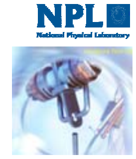
Ion trajectory: $r_i(t) = r_{oi}\cos(\omega_i t + \phi_i)\left[1 + \frac{q_i}{2}\cos(\omega_{RF}t)\right]$

Summary: traps

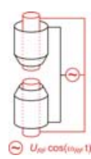


Hyperbolic electrodes
r_o = 5mm, $\omega_r/2\pi$ ~ 100 kHz





Ring trap: r_o ~ 500 μm, $\omega_r/2\pi$ ~ 1 MHz



Endcap trap: r_o ~ 500 μm, $\omega_r/2\pi$ ~ 100 kHz

Linear trap: r_o ~ 800 μm,
$(\omega_r, \omega_z)/2\pi$ = (4.0, 1.2) MHz

Summary: ion-laser interactions

Doppler cooling

- Minimum temperature $k_B T_{min} = \frac{\hbar\Gamma}{2}$

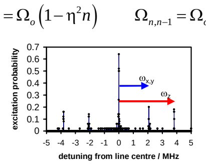
Lamb-Dicke parameter

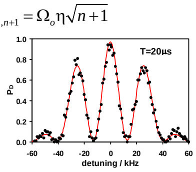
- Lamb-Dicke regime $\eta\sqrt{2n+1} \ll 1$

Coherent ion-laser interactions

- couples electronic and motional states

$$\Omega_{n,n} = \Omega_o\left(1-\eta^2 n\right) \qquad \Omega_{n,n-1} = \Omega_o\eta\sqrt{n} \qquad \Omega_{n,n+1} = \Omega_o\eta\sqrt{n+1}$$

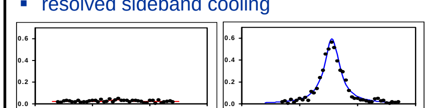




Summary: state preparation & heating

Motional ground state preparation

- resolved sideband cooling



detuning from carrier / MHz

Heating: the enemy!

- decoherence